CNHS Higher HW Solutions

Week 2 [15/02/19]

Qs 13 - 28

13. Use gradients to show that the points A(–2, –3), B(1, 1) and C(7, 9) are collinear.

mAB= $\frac{1+3}{1+2}=\frac{4}{3}$ mAC= $\frac{9+3}{7+2}=\frac{12}{9}=\frac{4}{3}$

mAB= mAC and common point A=>points A,B, and C are collinear.

14. P is the point (3, –3) and Q is (–1, 9).
The line l is parallel to PQ and passes through the point R(1, –2).
Find the equation of line l.

$m_l = m_{PQ} = \frac{9+3}{-1-3}=\frac{12}{-4}=-3$ and pt R (1,-2) y+2=-3(x-1)…y+2=-3x+3…**y= -3x + 1**

15. Triangle ABC has vertices A(4, 6), B(5, –1) and C(10, 4).

(a) Calculate the length of side AB.

(b) Show that triangle ABC is isosceles but not equilateral.

(a) $5\sqrt{2}$ (b) AB=$5\sqrt{2}$, BC=$5\sqrt{2}$, AC=$2\sqrt{10}$ …2 equal sides, isosceles.

16. A line joins the points P(–4, 3) and Q(2, –7).
Find the equation of the perpendicular bisector of PQ.

mPQ= $\frac{-7-3}{2+4}=\frac{-10}{6}=\frac{-5}{3}$ => m_{PERP}= $\frac{3}{5}$ and midpoint (-1,-2)

y+2= $\frac{3}{5}$(x+1)

5y+10=3(x+1)

5y+10=3x+3

4y - 3x= - 7 or equivalent

17. The line with equation $2y-3x=4$ makes an angle of $a°$ with the positive direction of the x-axis.
Calculate the value of a.

2y-3x=4…2y=3x+4…y= $\frac{3}{2}x+2$ …m=tan a = $\frac{3}{2}$ …a = $\tan^{-1}(\frac{3}{2})$ = 56.3°

18. (a) Find the gradient of the line with equation $3x + 4y = 2$.

(b) The line L passes through the point (1, 1) and is perpendicular to the line with equation $3x + 4y = 2$. Find the equation of line L.

(a) m=$\frac{-3}{4}$ (b) y-1=$\frac{4}{3}$(x-1)…3y-3=4(x-1)…3y-3=4x-4…**3y=4x-1**

19. A straight line makes an angle of $120°$ with the positive direction of the *x*-axis.
Find the exact value of the gradient of this line.

m=$\tan 120° = -\sqrt{3}$

20. (a) Sketch triangle ABC with vertices A(−4, 1), B(12, 3) and C(7, −7).

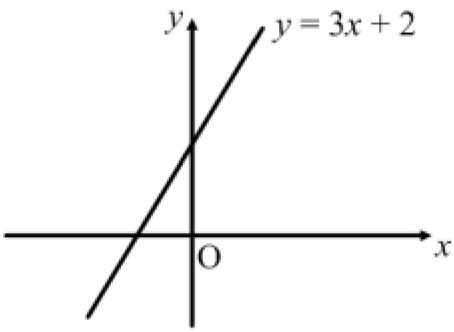
(b) Find the equation of the median CM, where M lies on AB.

(c) Find the equation of the altitude AD, where D lies on BC.

(d) Find the coordinates of the point of intersection of CM and AD.

(a) y = - 3x + 14 **(c) 2y = - x – 2** **(d) (6, - 4)**

21. Calculate the size of the **obtuse** angle between the line $y = 3x + 2$ and the *x*-axis.



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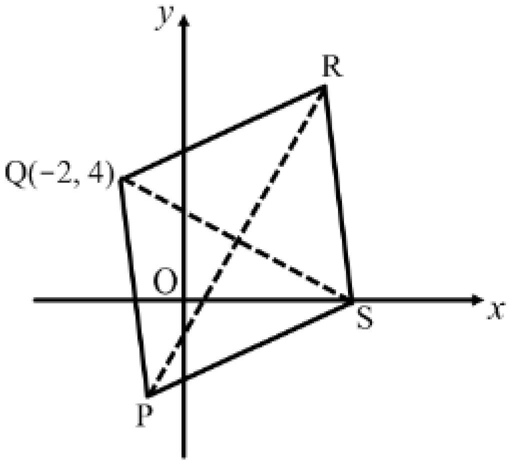
22. The diagram shows a rhombus PQRS with diagonals PR and QS.
Point Q has coordinates (–2, 4) and diagonal PR has equation $y = 3x - 1$.



Find the equation of diagonal QS.

3y = - x + 10

23. Functions f and g are defined on the set of real numbers by $f(x) = 2x + 4$ and $g(x) = 3x - 2$. Find simplified expressions for $f(g(x))$ and $g(f(x))$.

f(g(x))=f(3x-2)=2(3x-2)+4=6x-4+4=**6x**   g(f(x))=g(2x+4)=3(2x+4)-2=6x+12-2=**6x+10**

24. Functions f and g are defined on the set of real numbers by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$.

Find simplified expressions for:

(a) $f(g(x))$ (b) $g(f(x))$ (c) $f(f(x))$ (d) $g(g(x))$

(a) $f(g(x))=f(3x-4)=(3x-4)^2-1=9x^2-24x+16-1=\mathbf{9x^2-24x+15}$

(b) $g(f(x))=g(x^2+1)=3(x^2+1)-4=3x^2+3-4=\mathbf{3x^2-1}$

(c) $f(f(x))=f(x^2+1)=(x^2+1)^2+1=x^4+2x^2+1+1=\mathbf{x^4+2x^2+2}$

(d) $g(g(x))=g(3x-4)=3(3x-4)-4=9x-12-4=\mathbf{9x-16}$

25. (a) Express $x^2+6x+14$ in the form $(x+a)^2+b$.

(a) Hence **write down** the coordinates of the turning point on the parabola with equation $y=x^2+6x+14$.

(a) $(x+3)^2-9+14=(x+3)^2+5$ (b) Min TP (-3,5)

26. (a) The function *f* is defined on the set of real numbers by $f(x)=2x+3$.
Find an expression for the inverse function $f^{-1}(x)$.

(b) Find $f(f^{-1}(x))$.

(a) x → x2 +3 → f(x)…….-3 ÷2 →f^{-1}(x)… $f^{-1}(x)=\dfrac{x-3}{2}$

(b) x (remember, this is always true!)

27. The point with coordinates *A*(3, 2) is on the graph with equation $y=f(x)$.

Write down the image of the point *A* on the graph with equation:

(a) $y=-f(x-1)$ (b) $y=2f(x)+1$

(a) shifted right one then minus the y so **(4,-2)** (b) 2y then up 1 so **(3,5)**

28. Functions *f* and *g* are defined on the set of real numbers by $f(x)=x-1$ and $g(x)=x^2$.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) The function *h* is defined by $h(x)=f(g(x))+g(f(x))$.
Find an expression for $h(x)$ in its simplest form.

(a) x^2-1 and $(x-1)^2$ (b) $h(x) = x^2-1+ (x-1)^2 = x^2-1+ x^2-2x+1=\mathbf{2x^2-2x=2x(x-1)}$