AH

National
Qualifications
SPECIMEN ONLY

SQ24/AH/01

Mathematics

Date — Not applicable

Duration — 3 hours

Total marks — 100

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQ24AH01

FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\dfrac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\dfrac{1}{a}\tan(ax)+c$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)+c$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)+c$
$\dfrac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\dfrac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f^{iv}(0)x^4}{4!}+\ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Total marks — 100 **MARKS**

Attempt ALL questions

1. Given $f(x)=\frac{x-1}{1+x^2}$, show that $f'(x)=\frac{1+2x-x^2}{(1+x^2)^2}$. **3**

2. State and simplify the general term in the binomial expansion of $\left(2x-\frac{5}{x^2}\right)^6$.

Hence, or otherwise, find the term independent of x. **3**

3. Find $\int \frac{2}{\sqrt{9-16x^2}}dx$. **3**

4. Show that the greatest common divisor of 487 and 729 is 1.

Hence find integers x and y such that $487x + 729y = 1$. **4**

5. Find $\int x^2e^{3x}dx$. **5**

6. Find the values of the constant k for which the matrix $\begin{pmatrix} 3 & k & 2 \\ 3 & -4 & 2 \\ k & 0 & 1 \end{pmatrix}$ is singular. **4**

7. A spherical balloon is being inflated. When the radius is 10 cm the surface area is increasing at a rate of $120\pi\,\text{cm}^2\,\text{s}^{-1}$.

Find the rate at which the volume is increasing at this moment. **5**

(Volume of sphere = $\frac{4}{3}\pi r^3$, surface area = $4\pi r^2$)

8. (a) Find the Maclaurin expansions up to and including the term in x^3, simplifying the coefficients as far as possible, for the following: **5**

(i) $f(x)=e^{3x}$

(ii) $g(x)=(x+2)^{-2}$

(b) Given that $h(x)=\frac{x\,e^{3x}}{(x+2)^2}$ use the expansions from (a) to approximate the value of $h\left(\frac{1}{2}\right)$. **3**

MARKS

9. Three terms of an arithmetic sequence, u_3, u_7 and u_{16} form the first three terms of a geometric sequence.

Show that $a = \frac{6}{5}d$, where a and d are, respectively, the first term and common difference of the arithmetic sequence with $d \neq 0$.

Hence, or otherwise, find the value of r, the common ratio of the geometric sequence. **4**

10. Using logarithmic differentiation, or otherwise, find $\frac{dy}{dx}$ given that

$e^y = \frac{(3x+2)e^{2x}}{(2x-1)^2}$, $x > \frac{1}{2}$. **3**

11. Find the exact value of $\int_1^2 \frac{x+4}{(x+1)^2(2x-1)}\,dx$. **7**

12. (a) Given that m and n are positive integers state the negation of the statement:

m is even or n is even. **1**

(b) By considering the contrapositive of the following statement:

if mn is even then m is even or n is even,

prove that the statement is true for all positive integers m and n. **3**

13. Consider the curve in the (x, y) plane defined by the equation $y = \frac{4x-3}{x^2-2x-8}$.

(a) Identify the vertical asymptotes to this curve and justify your answer. **2**

Here are two statements about the curve:

(1) It does not cross or touch the x-axis.

(2) The line $y = 0$ is an asymptote.

(b) (i) State why statement (1) is false.

(ii) Show that statement (2) is true. **3**

MARKS

14. The lines L_1 and L_2 are given by the following equations.

L_1: $\frac{x+6}{3} = \frac{y-1}{-1} = \frac{z-2}{2}$

L_2: $\frac{x+5}{4} = \frac{y+4}{1} = \frac{z}{4}$

(a) Show that the lines L_1 and L_2 intersect and state the coordinates of the point of intersection. **5**

(b) Find the equation of the plane containing L_1 and L_2. **3**

A third line, L_3, is given by the equation $\frac{x-1}{2} = \frac{y+7}{4} = \frac{z-3}{-1}$.

(c) Calculate the acute angle between L_3 and the plane. Give your answer in degrees correct to 2 decimal places. **4**

15. (a) Given that $f(x) = \ln\left(\frac{1+x}{1-x}\right)$, find $f'(x)$, expressing your answer as a single fraction. **2**

(b) Solve the differential equation

$$\cos x\frac{dy}{dx} + y\tan x = \frac{\cos x}{e^{\sec x}}$$

given that $y = 1$ when $x = 2\pi$. Express your answer in the form $y = f(x)$. **7**

16. Let $S_n = \sum_{r=1}^{n} \frac{1}{r(r+1)}$ where n is a positive integer.

(a) Prove that, for all positive integers n, $S_n = \frac{n}{n+1}$. **5**

(b) Find

(i) the least value of n such that $S_{n+1} - S_n < \frac{1}{1000}$

(ii) the value of n for which $S_n \times S_{n-1} \times S_{n-2} = S_{n-8}$. **5**

MARKS

17. (a) Given $z=\cos\theta+i\sin\theta$, use de Moivre's theorem and the binomial theorem to show that:

$$\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$$

and

$$\sin 4\theta=4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta.$$ **5**

(b) Hence show that $\tan 4\theta=\dfrac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$. **3**

(c) Find algebraically the solutions to the equation

$$\tan^4\theta+4\tan^3\theta-6\tan^2\theta-4\tan\theta+1=0$$

in the interval $0\le\theta\le\dfrac{\pi}{2}$. **3**

[END OF SPECIMEN QUESTION PAPER]

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SQ24/AH/01 **Mathematics**

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

General Marking Principles for Advanced Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(d) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(e) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(f) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.

(g) Unless specifically mentioned in the Detailed Marking Instructions, do not penalise:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in solutions
- repeated errors within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper

Determine: determine an answer from given facts, figures, or information.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Express: use given information to rewrite an expression in a specified form.

Find: obtain an answer showing relevant stages of working.

Hence: use the previous answer to proceed.

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

Prove: use a sequence of logical steps to obtain a given result in a formal way.

Show that: use mathematics to show that a statement or result is correct (without the formality of proof) — all steps, including the required conclusion, must be shown.

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features.

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
1			Ans: demonstrate result •¹ know and start to use quotient rule •² complete differentiation •³ simplify numerator	3	 •¹ $\frac{(1+x^2)\times 1-\ldots}{(1+x^2)^2}$ •² $\frac{(1+x^2)\times 1-2x(x-1)}{(1+x^2)^2}$ •³ $\frac{1+x^2-2x^2+2x}{(1+x^2)^2}=\frac{1+2x-x^2}{(1+x^2)^2}$
Notes:					
2			Ans: 6000 •¹ correct substitution into general term •² simplify •³ identify r and find coefficient	3	 •¹ $\binom{6}{r}(2x)^{6-r}\left(-\frac{5}{x^2}\right)^r$ •² $\binom{6}{2}2^{6-r}(-5)^r x^{6-3r}$ •³ $\binom{6}{2}(2)^4(-5)^2=6000$
Notes: 1 Accept $\binom{6}{6-r}(2x)^{6-r}\left(-\frac{5}{x^2}\right)^r$ or correct equivalent for •¹. 2 If coefficient is found by expanding the expression, only •³ is available.					
3			Ans: $\frac{1}{2}\sin^{-1}\left(\frac{4x}{3}\right)+c$ •¹ evidence of identifying an appropiate method •² re-write in standard form •³ final answer with constant of integration	3	 •¹ eg identify standard integral $\int\frac{1}{\sqrt{a^2-x^2}}dx$ •² $2\int\frac{1}{4\sqrt{\left(\frac{3}{4}\right)^2-x^2}}dx$ or equivalent •³ $2\times\frac{1}{4}\sin^{-1}\left(\frac{4x}{3}\right)+c=\frac{1}{2}\sin^{-1}\left(\frac{4x}{3}\right)+c$
Note: For •¹ accept any appropriate evidence eg using substitution $u=4x$.					

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
4			Ans: $x=244, y=-163$	4	
			•[1] start correctly		•[1] $729=487\times1+242$
			•[2] show last non-zero remainder = 1		•[2] $487=242\times2+3$ $242=80\times3+2$ $3=2\times1+1$ $2=2\times1+0, \quad GCD=1$
			•[3] evidence of two correct back substitutions using $2=242-3\times80$ or $3=487-242\times2$ or $242=729-487\times1$		$1=3-2\times1=3-(242-80\times3)=81\times3-242$ $=81(487-2\times242)-242$ •[3] $=81\times487-163\times242$ $=81\times487-163(729-487)$ $=244\times487-163\times729$ carefully check for equivalent alternatives
			•[4] values for x and y		•[4] $1=487\times244-729\times163$ So, $x=244, y=-163$
Notes:					
5			Ans: $\frac{x^2e^{3x}}{3}-\frac{2xe^{3x}}{9}+\frac{2e^{3x}}{27}+c$	5	
			•[1] evidence of application of integration by parts		•[1] $(x^2\int e^{3x}dx-\int\left(\int e^{3x}.\frac{d}{dx}x^2dx\right)dx)$
			•[2] correct choice of u and v'		•[2] $u=x^2 \quad v'=e^{3x}$
			•[3] correct first application		•[3] $\frac{1}{3}x^2e^{3x}-\frac{2}{3}\int xe^{3x}dx$ or equivalent
			•[4] start second application		•[4] $\int xe^{3x}dx=\frac{xe^{3x}}{3}-\frac{e^{3x}}{9}$ or equivalent
			•[5] final answer with constant of integration		•[5] $\frac{x^2e^{3x}}{3}-\frac{2xe^{3x}}{9}+\frac{2e^{3x}}{27}+c$ or equivalent
Notes:					
6			Ans: $k=\frac{3}{2}, -4$	4	
			•[1] starts process for working out determinant		•[1] $3\begin{vmatrix}-4 & 2\\ 0 & 1\end{vmatrix}-k\begin{vmatrix}3 & 2\\ k & 1\end{vmatrix}+2\begin{vmatrix}3 & -4\\ k & 0\end{vmatrix}$
			•[2] completing process correctly		•[2] $-12-k(3-2k)+8k$
			•[3] simplify and equate to 0		•[3] $2k^2+5k-12=0$

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
			•4 find values of k		•4 $k=\frac{3}{2}$, $k=-4$
Note: Accept answer arrived at through row and column operations.					
7			Ans: $\frac{dV}{dt}=600\pi\,\text{cm}^3\text{s}^{-1}$	5	
			•1 interprets rate of change		•1 $\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}=120\pi$
			•2 correct expression for $\frac{dA}{dr}$		•2 $A=4\pi r^2$, $\frac{dA}{dr}=8\pi r$
			•3 find $\frac{dr}{dt}$		•3 $\frac{dr}{dt}=\frac{120\pi}{80\pi}=\frac{3}{2}$
			•4 correct expression for $\frac{dV}{dt}$		•4 $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}=4\pi r^2\times\frac{3}{2}$
			•5 evaluates $\frac{dV}{dt}$		•5 $\frac{dV}{dt}=4\pi(10)^2\times\frac{3}{2}=600\pi\,\text{cm}^3\,\text{s}^{-1}$
Notes:					
8	a	i	Ans: $f(x)=1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3+\ldots$	2	
			•1 state Maclaurin expansion for e^{3x} up to x^3		•1 $f(x)=1+\frac{3x}{1!}+\frac{(3x)^2}{2!}+\frac{(3x)^3}{3!}+\ldots$
			•2 correct expansion		•2 $f(x)=1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3+\ldots$
8	a	ii	Ans: $g(x)=\frac{1}{4}-\frac{1}{4}x+\frac{3}{16}x^2-\frac{1}{8}x^3+\ldots$	3	
			•3 correct differentiation of $g(x)$		•3 $g(x)=(x+2)^{-2}$, $g'(x)=-2(x+2)^{-3}$, $g''(x)=6(x+2)^{-4}$, $g'''(x)=-24(x+2)^{-5}$
			•4 correct evaluations of g functions		•4 $g(0)=\frac{1}{4}$, $g'(0)=-\frac{1}{4}$, $g''(0)=\frac{3}{8}$ $g'''(0)=-\frac{3}{4}$
			•5 correct expansion		•5 $g(x)=\frac{1}{4}-\frac{1}{4}x+\frac{3}{16}x^2-\frac{1}{8}x^3+\ldots$
8	b		Ans: $h\left(\frac{1}{2}\right)=0\cdot 327$	3	
			•6 connection between $h(x)$, $f(x)$ and $g(x)$		•6 $h(x)=x\,f(x)g(x)$

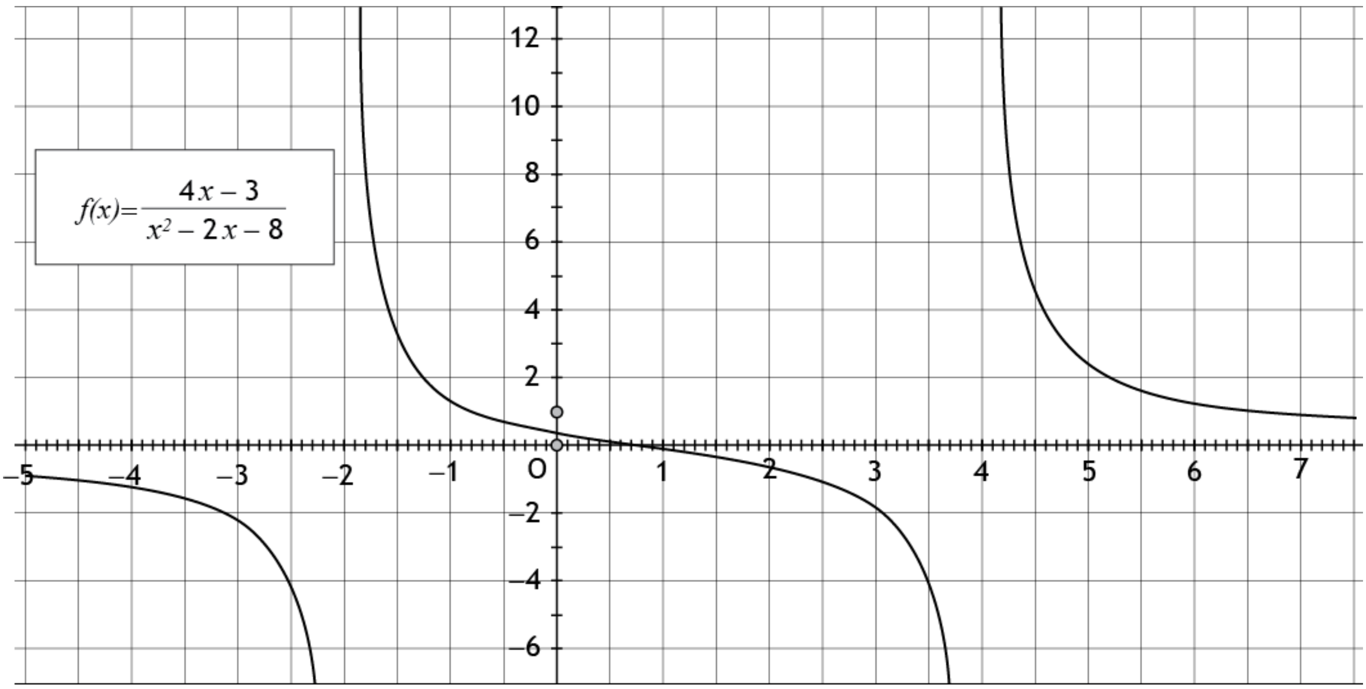
Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
			•7 approximate $f\left(\frac{1}{2}\right)$ and $g\left(\frac{1}{2}\right)$ •8 evaluate $h\left(\frac{1}{2}\right)$		•7 $f\left(\frac{1}{2}\right)=4\cdot1875,\ g\left(\frac{1}{2}\right)=0\cdot15625$ •8 $h\left(\frac{1}{2}\right)=0\cdot327$
Note: Accept answer given as a fraction. For aii) award full credit for answers arrived at using a binomial expansion. For b) evidence of use of the expansions from a) must be evident. Candidates who simply calculate the value of $h\left(\frac{1}{2}\right)$ directly without using the approximations from a) receive no marks for b).					
9			Ans: proof, $r=\frac{9}{4}$ •1 create term formulae •2 form ratios for r •3 complete proof •4 evaluate r	4	•1 $u_3=a+2d$, $u_7=a+6d$, $u_{16}=a+15d$ •2 $\frac{a+15d}{a+6d}=\frac{a+6d}{a+2d}$ $(a+15d)(a+2d)=(a+6d)^2$ •3 $a^2+17ad+30d^2=a^2+12ad+36d^2$ $5a=6d$ $a=\frac{6}{5}d$ •4 $r=\frac{\frac{6}{5}d+6d}{\frac{6}{5}d+2d}=\frac{9}{4}$
Notes:					
10			Ans: $\frac{dy}{dx}=\frac{3}{3x+2}+2-\frac{4}{2x-1}$ •1 introduction of $\log_e$ •2 express function in differentiable form •3 differentiate	3	•1 $y=\ln\left(\frac{(3x+2)e^{2x}}{(2x-1)^2}\right)$ •2 $y=\ln\lvert 3x+2\rvert+2x-2\ln\lvert 2x-1\rvert$ •3 $\frac{dy}{dx}=\frac{3}{3x+2}+2-\frac{4}{2x-1}$
Note: In this question the use of modulus signs is not required for the award of •1 and •2.					

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
11			Ans: $\ln 2 - \frac{1}{6}$ •[1] correct form of partial fractions •[2] 1st coefficient correct •[3] 2nd coefficients correct •[4] 3rd coefficients correct •[5] integrate any two terms •[6] integrate all three terms •[7] evaluate	7	•[1] $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1}$ •[2] $A = -1$ •[3] $B = -1$ •[4] $C = 2$ •[5] $\int_1^2 \left(\frac{2}{2x-1} - \frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx$ $= \left[\ln\lvert 2x-1 \rvert - \ln\lvert x+1 \rvert + (x+1)^{-1} \right]_1^2$ •[6] $= \left[\ln\lvert 2x-1 \rvert - \ln\lvert x+1 \rvert + (x+1)^{-1} \right]_1^2$ •[7] $\ln 2 - \frac{1}{6}$
Note: do not penalise the omission of the modulus sign at •[5] and •[6]					
12	a		Ans: m is odd and n is odd •[1] correct statement	1	•[1] m is odd and n is odd
	b		Ans: proof •[2] contrapositive statement •[3] begin proof •[4] complete proof	3	•[2] If m and n are both odd then mn is odd •[3] Let $m = 2p - 1$, $n = 2q - 1$ where p, q are positive integers. Then, $mn = 2(2pq - p - q) + 1$ where $2pq - p - q$ is clearly an integer therefore mn is clearly odd. •[4] And so the contrapositive statement is true and it follows that the original statement, 'if mn is even then m is even or n is even', that is equivalent to the contrapositive, is true.
Note: For •[1] accept an equivalent statement, eg **'neither *m* nor *n* is even'** but do not accept any other answer, eg 'It is not true to say that m is even or n is even.					
13	a		Ans: $x = 4, x = -2$ with explanation	2	

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
			•1 correct asymptotes •2 suitable explanation		•1 $x^2-2x-8=0 \Leftrightarrow x=4 \text{ or } x=-2$ •2 y tends towards $\pm\infty$ as $x \to 4$ and $x \to -2$
13	b	i	Ans: false with explanation •3 suitable explanation	1	•3 The statement is false because the graph meets the x-axis when $x=\frac{3}{4}$.
13	b	ii	Ans: proof •4 method •5 complete proof	2	•4 eg $f(x)=\dfrac{\frac{4}{x}-\frac{3}{x^2}}{1-\frac{2}{x}-\frac{8}{x^2}}$ •5 As $x \to \pm\infty, f(x) \to \frac{0}{1}=0$ ie the line y = 0 is a horizontal asymptote

Note:

For •2 accept $4x-3 \neq 0$ at $x=4$ or $x=-2$.
Graph of function. For marking guidance — not required by candidate.



Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
14	a		Ans: (3, -2, 8) •1 write lines in parametric form	5	•1 $\begin{array}{l} x=3t-6 \\ y=-t+1 \\ z=2t+2 \end{array}$ and $\begin{array}{l} x=4p-5 \\ y=p-4 \\ z=4p \end{array}$

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
			•2 create equations for intersection •3 solve a pair of these equations (eg the first two) for p and t •4 check that the third equation is satisfied •5 state coordinates of point of intersection		•2 $4p-5=3t-6$ $p-4=-t+1$ $4p=2t+2$ •3 $t=3$ and $p=2$ •4 eg $4(2)=2(3)+2$ •5 evidence of substitution into third equation and (3, −2, 8)
14	b		Ans: $-6x-4y+7z=46$ •6 use vector product to find normal to the plane •7 evaluate normal vector •8 form equation of plane	3	•6 $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 4 & 1 & 4 \end{vmatrix}$ •7 $-6\mathbf{i}-4\mathbf{j}+7\mathbf{k}$ •8 $-6x-4y+7z=46$
14	c		Ans: 49° •9 select correct vectors •10 complete calculations of $\lvert a\rvert, \lvert b\rvert$ and $a \cdot b$ •11 evaluate acute angle between normal to plane and line •12 calculate angle between line and plane	4	•9 $\begin{pmatrix} -6 \\ -4 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ •10 $\left\lvert \begin{pmatrix} -6 \\ -4 \\ 7 \end{pmatrix} \right\rvert = \sqrt{101}$, $\left\lvert \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \right\rvert = \sqrt{21}$ and $\begin{pmatrix} -6 \\ -4 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} = -35$ •11 $40{\cdot}54^\circ$ •12 $90^\circ - 40{\cdot}54^\circ = 49{\cdot}46^\circ$
Notes:					
15	a		Ans: $\dfrac{2}{\left(1-x^2\right)}$ •1 express function in differentiable form •2 complete process	2	•1 $\ln(1+x)-\ln(1-x)$ •2 $\dfrac{1}{1+x}+\dfrac{1}{1-x}=\dfrac{2}{(1-x^2)}$
15	b		Ans: $y=\dfrac{x+e-2\pi}{e^{\sec x}}$	7	

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
			•3 express in standard form •4 form of integrating factor •5 find integrating factor •6 state modified equation •7 integrate both sides •8 substitute in for x and y and find c •9 state particular solution		•3 $\frac{dy}{dx}+y\frac{\tan x}{\cos x}=\frac{1}{e^{\sec x}}$ •4 IF= $e^{\int\frac{\tan x}{\cos x}dx}$ •5 IF $=e^{\sec x}$ •6 $\frac{d}{dx}\left(ye^{\sec x}\right)=1$ •7 $e^{\sec x}y=x+c$ •8 $e^{\sec 2\pi}\cdot 1=2\pi+c$, $c=e-2\pi$ •9 $y=\frac{x+e-2\pi}{e^{\sec x}}$
Notes:					
16	a		Ans: proof •1 strategy use partial fractions •2 find A and B •3 state result and start to write out series •4 strategy •5 complete proof	5	•1 $\frac{1}{r(r+1)}=\frac{A}{r}+\frac{B}{r+1}$ •2 A = 1, B = -1 •3 $1-\frac{1}{2}\ +\frac{1}{2}-\frac{1}{3}\ +\frac{1}{3}-\frac{1}{4}\ +\frac{1}{4}-\frac{1}{5}....$ $+\frac{1}{n-1}-\frac{1}{n}\ +\frac{1}{n}-\frac{1}{n+1}$ •4 Note that successive terms cancel out (telescopic series) $1+\left(-\frac{1}{2}+\frac{1}{2}\right)+\left(-\frac{1}{3}+\frac{1}{3}\right)+\left(-\frac{1}{4}+\frac{1}{4}\right)+\left(-\frac{1}{5}+...\right)$ $+\left(...+\frac{1}{n-1}\right)+\left(-\frac{1}{n}+\frac{1}{n}\right)-\frac{1}{n+1}$ •5 cancels terms and $1-\frac{1}{n+1}=\frac{n}{n+1}$
16	a		Ans: proof (alternative) •1 state hypothesis and consider $n=k+1$		•1 Assume $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ true for some n = k, **and** consider $n=k+1$ ie $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\sum_{r=1}^{k}\frac{1}{r(r+1)}+\frac{1}{(k+1)(k+2)}$

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
			•[2] start process for $k+1$ •[3] complete process •[4] show true for $n=1$ •[5] state conclusion		$\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$ •[2] $=\frac{k(k+2)}{(k+1)(k+2)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k^2+2k+1}{(k+1)(k+2)}$ $=\frac{(k+1)^2}{(k+1)(k+2)}$ •[3] $\frac{(k+1)}{(k+1)+1}$ •[4] For n = 1 LHS = $\frac{1}{1(1+1)}=\frac{1}{2}$ RHS = $\frac{1}{1+1}=\frac{1}{2}$ LHS = RHS so true for n = 1 •[5] Hence, if true for $n=k$, then true for $n=k+1$, but since true for $n=1$, then by induction true for all positive integers n.
	b	i	Ans: n = 31 •[6] set up equation and start to solve •[7] process •[8] obtain solution	3	•[6] eg $\frac{n+1}{n+2}-\frac{n}{n+1}<\frac{1}{1000}$ and evidence of strategy •[7] $n^2+3n-998>0$ •[8] n = 31
	b	ii	Ans: n = 11 •[9] set up equation •[10] solve for n	2	•[9] $\left(\frac{n}{n+1}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n-1}\right)=\frac{n-8}{n-7}$ •[10] n = 11

Notes:

1 •[1] is only available for induction hypothesis and stating that $k+1$ is going to be considered.

2 •[3] is only awarded if final line shows results required in terms of $k+1$ and is arrived at by appropriate working, including target/desired result approach, from the •[3] stage.

3 •[5] is only awarded if the candidate shows clear understanding of the logic required.

17	a		Ans: proof $\bullet^1$ use de Moivre's theorem $\bullet^2$ start process using binomial theorem $\bullet^3$ complete expansion $\bullet^4$ identify and match real terms $\bullet^5$ identify and match imaginary terms	5	$\bullet^1$ $z^4 = \cos 4\theta + i\sin 4\theta$ $\bullet^2$ $(\cos\theta + i\sin\theta)^4 = \cos^4\theta + 4\cos^3\theta(i\sin\theta) + 6\cos^2\theta(i\sin\theta)^2 + \ldots$ $\bullet^3$ $\cos^4\theta + 4\cos^3\theta(i\sin\theta) - 6\cos^2\theta\sin^2\theta + 4\cos\theta(i\sin\theta)^3 + \sin^4\theta$ $\bullet^4$ $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ $\bullet^5$ $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$
17	b		Ans: proof $\bullet^6$ strategy $\bullet^7$ divide numerator and denominator by $\cos^4 x$ $\bullet^8$ complete	3	$\bullet^6$ $\tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta} = \dfrac{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}$ $\bullet^7$ $\dfrac{\dfrac{4\cos^3\theta\sin\theta}{\cos^4\theta} - \dfrac{4\cos\theta\sin^3\theta}{\cos^4\theta}}{\dfrac{\cos^4\theta}{\cos^4\theta} - \dfrac{6\cos^2\theta\sin^2\theta}{\cos^4\theta} + \dfrac{\sin^4\theta}{\cos^4\theta}}$ $\bullet^8$ $\dfrac{\dfrac{4\sin\theta}{\cos\theta} - \dfrac{4\sin^3\theta}{\cos^3\theta}}{1 - \dfrac{6\sin^2\theta}{\cos^2\theta} + \dfrac{\sin^4\theta}{\cos^4\theta}} = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$
17	c		Ans: $\theta = \frac{\pi}{16}$ and $\frac{5\pi}{16}$ $\bullet^9$ strategy $\bullet^{10}$ complete process and find a solution for 4θ $\bullet^{11}$ find both solutions	3	$\tan^4\theta + 4\tan^3\theta - 6\tan^2\theta - 4\tan\theta + 1 = 0$ $\bullet^9$ $4\tan\theta - 4\tan^3\theta = 1 - 6\tan^2\theta + \tan^4\theta$ $\dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} = 1$ $\bullet^{10}$ $\tan 4\theta = 1,\ 4\theta = \dfrac{\pi}{4}$ $\bullet^{11}$ $\theta = \dfrac{\pi}{16}$ and $\dfrac{5\pi}{16}$

[END OF SPECIMEN MARKING INSTRUCTIONS]