Cumbernauld Academy

Physics Department

Unit 1 Our Dynamic Universe

Booklet 2

Mechanics

1.1 Equations of Motion

Vectors

Scalars are quantities which are defined by their ____size/magnitude____

Vectors are quantities which are defined by their size/magnitude and ___direction______.

Scalar Quantity	Vector Quantity
Speed	Velocity
Distance	Displacement
Mass	Weight
Time	Force
Energy	Acceleration

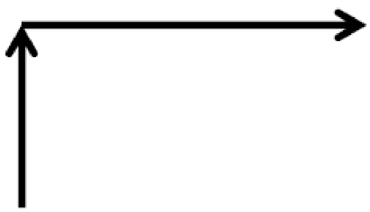
e.g force – 50N downwards and velocity 25ms^{-1} @ 090

Adding Vectors (Revision of National 5)

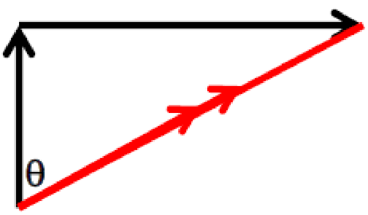
This is more difficult than adding scalars as the direction of the vectors must be taken into account.
The addition of two vectors is called the __***resultant***__ vector.
When you add vectors they have to be added __***tip-to-tail***__.
What does this mean?

- Each vector must be represented by a straight line of suitable scale.
- The straight line must have an arrow head to show its direction. i.e.

- The vectors must be joined one at a time so that the tip of the previous vector touches the tail of the next vector. i.e.



- A straight line is drawn from the starting point to the finishing point and the starting angle is marked.



- The resultant should have 2 arrow heads to make it easy to recognise.
- If using a scale diagram the length and direction of this straight line gives the resultant vector.
- Alternatively you can use trigonometry and SOHCAHTOA or the sine or cosine rule to calculate the resultant.

Distance and Displacement (Revision of National 5)

<u>Distance</u> is defined as the total ___length______ of a journey.

Distance is a ___Scalar___ quantity, which means it has a __size__ only.

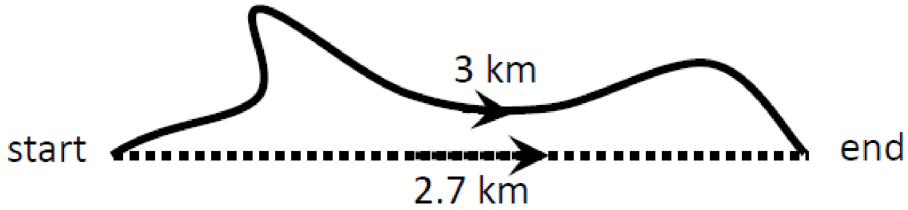
The symbol for distance is __d__. Distance is measured in __metres__ or __m__ for short.

<u>Displacement</u> is defined as the shortest ___path_____ between two points in a given ___direction___ from the starting point.

Displacement is a __vector__, which means it has a size and __direction__.

The symbol for displacement is __s__. Displacement is measured in __metres__ or __m__.

Consider the journey below. A person walks along a path (solid line) from start to end.

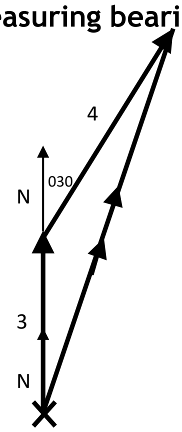


They will have walked further following the path than if they had been able to walk directly from start to end in a straight line (dashed line).
The solid line denotes the **distance** =_ 3km_. The dashed line denotes the **displacement** = _2.7 km East_

<u>Example</u>
A woman walks her dog 3 km due North (000) and then 4 km (030).
Find her
a) distance travelled
b) displacement.

<u>Solution</u> - **Use a ruler to measure the lengths of the vectors and a protractor to measure the bearing.**

- Choose an appropriate scale e.g. __ 1cm : 1km__
- Mark the start point with an X, draw a _ North line_ and draw the first vector.
- Draw a North line at the tip of this vector and now draw the second vector (tip to tail)
- Draw the resultant vector from start to end using the double arrow.
- Measure the length of the line and the bearing.
  When measuring bearings remember - from START - CLOCKWISE - from NORTH



Answer:

Speed and Velocity (Revision of National 5)

<u>Speed</u> is a measure of the distance an object covers in a set ____time____.

Speed is a ___Scalar___ quantity, which means it has a __size__ only.

The symbol for speed is __v__. Speed is measured in __metres per second__ or __m s^{-1}__.

From previous work in Maths and Physics we know that speed is calculated from the equation:

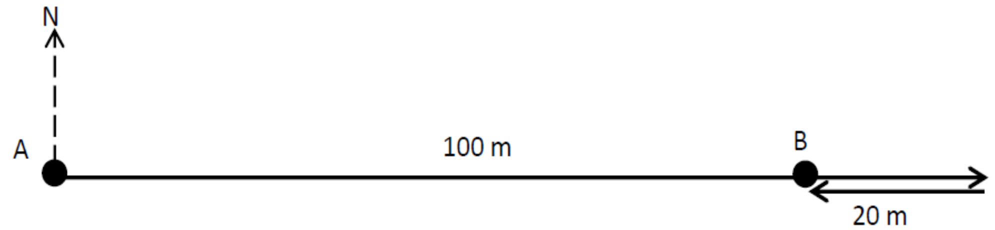
$$\boxed{v = d / t}$$

<u>Velocity</u> is a measure of the __displacement__ an object covers in a set time.
Velocity is a __ vector __ quantity, which means it has __size__ and __direction__.
The symbol for velocity is __v__, and is measured in __metres per second__ or __ms^{-1}__
The equation for velocity is:

$$\boxed{v = s / t}$$

<u>Example</u>
A runner sprints 100 m East along a straight track in 12 s and then takes a further 13 s to jog 20 m back towards the starting point.
(a) What distance does she run during the 25 s?
(b) What is her displacement from her starting point after the 25 s?
(c) What is her speed?
(d)What is her velocity?

<u>Solution</u> - always draw the vector diagram.



a) $d = 100 + 20$
$d = 120$ m

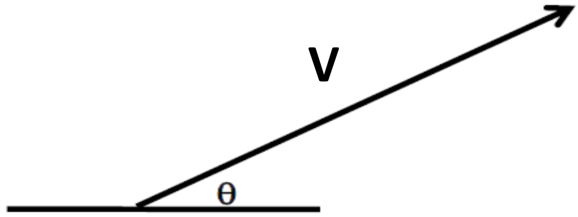
(b) $s = 100 + (-20)$
$s = 80$ m (090)

c) $v = d / t$
$v = 120/25$
$v = 4.8\ ms^{-1}$

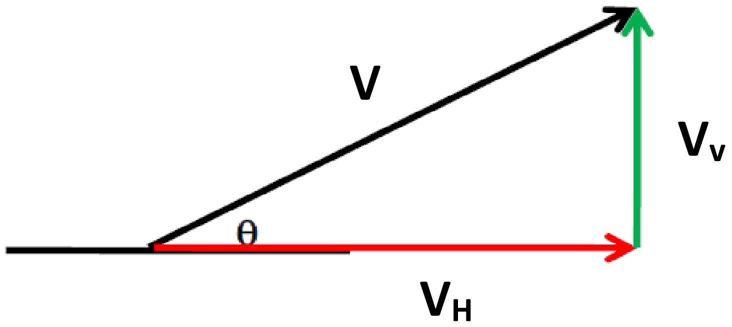
(d) $v = s / t$
$v = 80/25$
$v = 3.2\ ms^{-1}$ (090)

Resolving Vectors

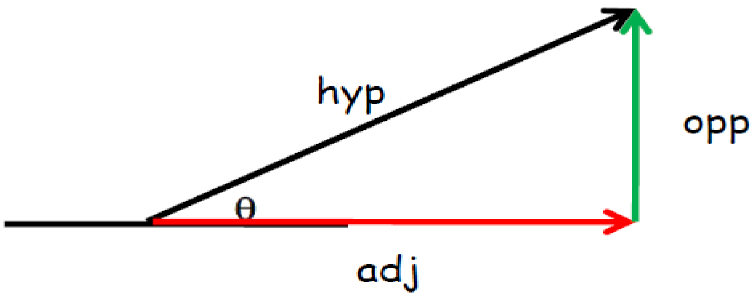
We have seen that two vectors can be added to give the resultant using vector addition.
Can we split a resultant vector into the two individual vectors that make it up?
Consider the following.



This shows a resultant vector, V, at an angle θ to the horizontal.
To travel to the end of the vector we could move in a straight line in the X direction and then a straight line in the Y direction as shown below.



But how do we find out the size of each line?
Since we have a right angled triangle with a known angle we can name the sides.



This means we can use Pythagoras' theroem to work out the unknown sides.

horizontal component	vertical component
$V_H = V\cos\theta$	$V_V = V\sin\theta$

<u>Example</u>
A football is kicked at an angle of 70° at 15 ms^{-1}.
Calculate:
a) the horizontal component of the velocity;
b) the vertical component of the velocity.

<u>Solution</u>
$V_H = V\cos\theta = 15\cos70 = 5.2\ ms^{-1}$
$V_v = V\sin\theta = 15\sin70 = 14.1\ ms^{-1}$

<u>Formal Homework 1 - Scalars and Vectors</u>

The 3 Equations of Motion

The 3 Equations of Motion (s u v a t)

The equations of motion can be applied to any object moving with constant acceleration in a straight line.

You must be able to:

- **select the correct formula;**
- **identify the symbols and units used;**
- **carry out calculations to solve problems of real life motion; and**
- **carry out experiments to verify the equations of motion.**

You should develop an understanding of how the graphs of motion can be used to derive the equations. This is an important part of demonstrating that you understand the principles of describing motion, and the link between describing it graphically and mathematically.

Equation of Motion 1 : ***v = u + at***

$$a = \frac{v-u}{t}$$

at = v - u

u + at = v

OR

v = u + at

Example

A racing car starts from rest and accelerates uniformly in a straight line at 12 ms^{-2} for 5.0 s. Calculate the final velocity of the car.

Solution **LIST s u v a t**

S =?

u = 0 ms^{-1} (rest)

v = ?

a = 12 ms^{-2}

t = 5.0 s

v = u + at

v = 0 + (12 x5.0)

v = 0 + 60

v = 60 ms^{-1}

The 3 Equations of Motion (ctd)

Equation of Motion 2: $s = ut + \frac{1}{2}at^2$

The displacement, s, is the area under the graph:

Area 1 = ut

Area 2 = ½ (v-u) t

But from equation 1 we get that (v-u) = at

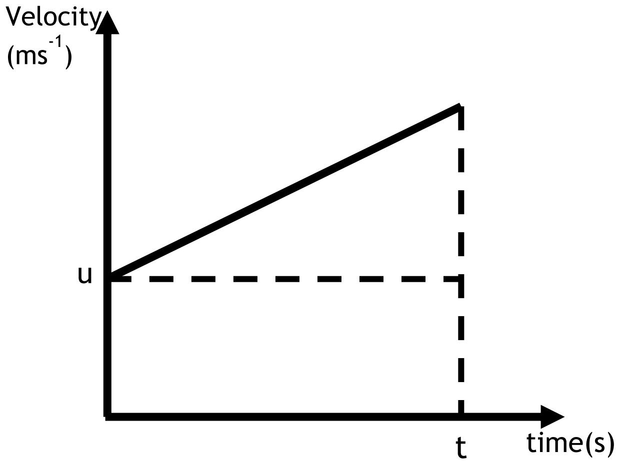
So Area 2 = ½ (at)t

Therefore,

$s = ut + \frac{1}{2}(at)t$

OR

$$s = ut + \frac{1}{2}at^2$$



Example

A speedboat travels 400 m in a straight line when it accelerates uniformly from 2.5 ms^{-1} in 10 s. Calculate the acceleration of the speedboat.

Solution

s = 400 m

u = 2.5 ms^{-1}

v

a = ?

t = 10 s

$s = ut + \frac{1}{2}at^2$

$400 = (2.5 \times 10) + (0.5 \times a \times 10^2)$

$400 = 25 + 50a$

$50a = 400 - 25 = 375$

$a = 375/50$

$a = 7.5$ ms^{-2}

Equation of Motion 3: $v^2 = u^2 + 2as$

We have already found that

$v = u + at$

$v^2 = (u + at)^2$

$v^2 = u^2 + 2uat + a^2t^2$

$v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

And since $s = ut + \frac{1}{2}at^2$

$$v^2 = u^2 + 2as$$

Example

A rocket is travelling through outer space with uniform velocity. It then accelerates at 2.5 ms^{-2} in a straight line in the original direction, reaching 100 ms^{-1} after travelling 1875 m.

Calculate the rocket's initial velocity.

Solution

s = 1 875 m

u = ?

v = 100 ms^{-1}

a = 2.5 ms^{-2}

t

$v^2 = u^2 + 2as$

$100^2 = u^2 + (2 \times 2.5 \times 1875)$

10 000 = u2 +9375

$u^2 = 10\,000 - 9375$

u = 25ms^{-1}

The 3 Equations of Motion with Decelerating Objects

When an object decelerates its velocity decreases. If the vector quantities in the equations of motion are positive, we represent the decreasing velocity by use of a _negative sign_ in front of the acceleration value.

Example 1

A car, travelling in a straight line, decelerates uniformly at 2.0 ms^{-2} from 25 ms^{-1} for 3.0 s. Calculate the car's velocity after the 3.0 s.

Solution

s

u = 25 ms^{-1} — $v = u + at$

v = ? — $v = 25 + (\mathbf{-2.0} \times 3.0)$

a = -2.0 ms^{-2} — $v = 25 + (-6.0)$

t = 3.0 s — $v = 19\ ms^{-1}$

Example 2

A greyhound is running at 6.0 ms^{-1}. It decelerates uniformly in a straight line at 0.5 ms^{-2} for 4.0 s. Calculate the displacement of the greyhound while it was decelerating.

Solution

s = ? — $s = ut + ½ at^2$

u = 6.0 ms^{-1} — $s = (6.0 \times 4.0) + (0.5 \times \mathbf{-0.5} \times 4.0^2)$

v — $s = 24 + (-4.0)$

a = -0.5 ms^{-2} — $s = 20\ m$

t = 4.0 s

Example 3

A curling stone leaves a player's hand at 5.0 ms^{-1} and decelerates uniformly at 0.75ms^{-2} in a straight line for 16.5 m until it strikes another stationary stone.
Calculate the velocity of the decelerating curling stone at the instant it strikes the stationary one.

Solution

s = 16.5 m — $v^2 = u^2 + 2as$

u = 5.0 ms^{-1} — $v^2 = 5.0^2 + (2 \times -0.75 \times 16.5)$

v = ? — $v^2 = 25 + (-24.75)$

a = -0.75 ms^{-2} — $v = \sqrt{0.25}$

t — $v = 0.5\ ms^{-1}$

Graphing Motion

Graphs

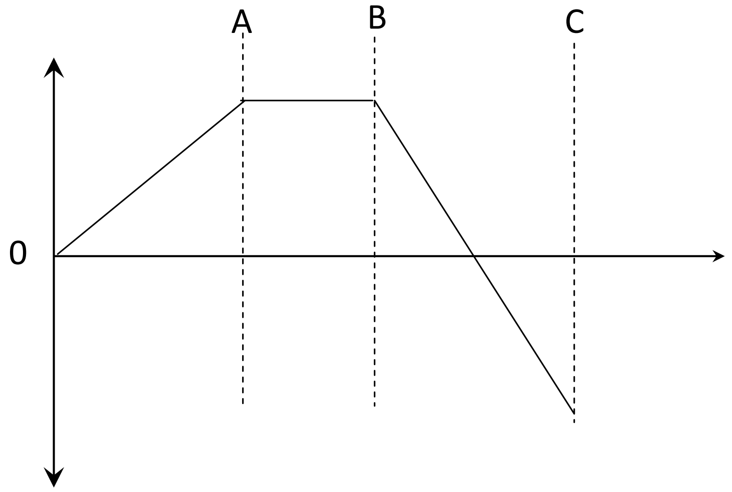
In all areas of science, graphs are used to display information.
Graphs are an excellent way of giving information, especially to show relationships between quantities.
In this section we will be examining three types of motion-time graphs.

Displacement-time graphs
Velocity-time graphs
Acceleration-time graphs

If you have an example of one of these types of graph then it is possible to draw a corresponding graph for the other two factors.

Displacement – time graphs

This graph represents how far an object is from its starting point at some known time. Because displacement is a vector it can have positive and negative values. (+ve and –ve will be opposite directions from the starting point).



OA – the object is moving away from the starting point. It is moving a constant displacement each second. This is shown by the constant gradient. What does this mean?

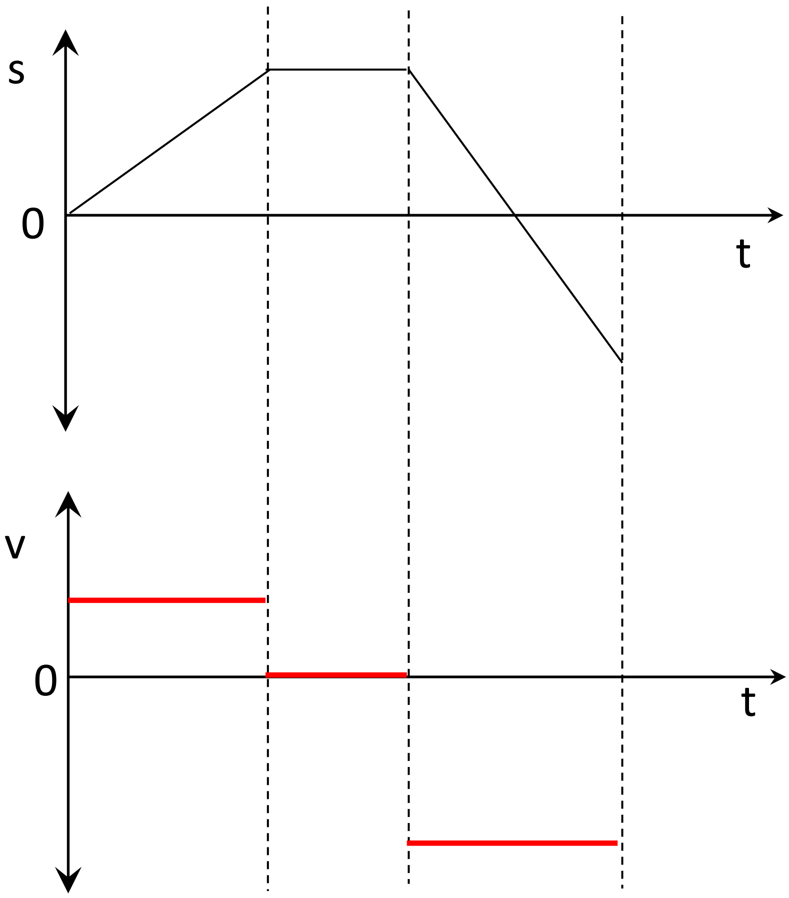
$$\text{gradient} = \frac{\text{displacement}}{\text{time}} = \text{velocity}$$

We can determine the velocity from the gradient of a displacement time graph.

AB – the object has a _constant displacement_ so is not changing its position, therefore it must be at _rest_. The gradient in this case is zero, which means the object has a velocity of zero [at rest]

BC – the object is now moving back towards the _starting point_, reaching it at time x. It then continues to move away from the _start_, but in the opposite direction. The gradient of the line is negative, indicating the _change in direction of motion_.

Converting Displacement - time Graphs to Velocity-time Graphs



A velocity time graph is the __gradient__ of a displacement time graph.

It is important to note if the gradient is __positive__ or __negative__.

The gradient gives us the information to determine the __direction__ of motion of the object.

Velocity – time Graphs

It is possible to produce a velocity time graph to describe the motion of an object.
We will only look at velocity time graphs with a constant acceleration.

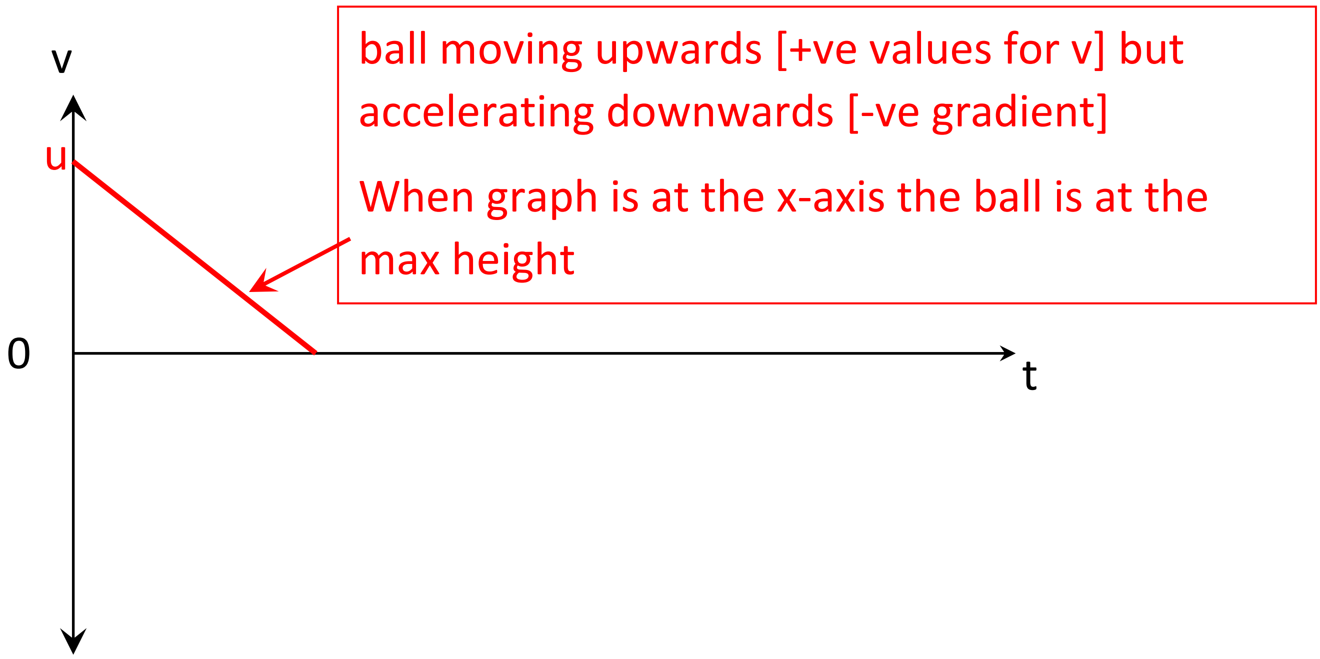
Scenario: The Bouncing Ball
Lydia fires a ball vertically into the air from the ground. The ball reaches its maximum height, falls, bounces, and then rises to a new, lower, maximum height.

What will the velocity time graph for this motion look like?

First decision: The original direction of motion is up so upwards is the positive direction

Part One of Graph

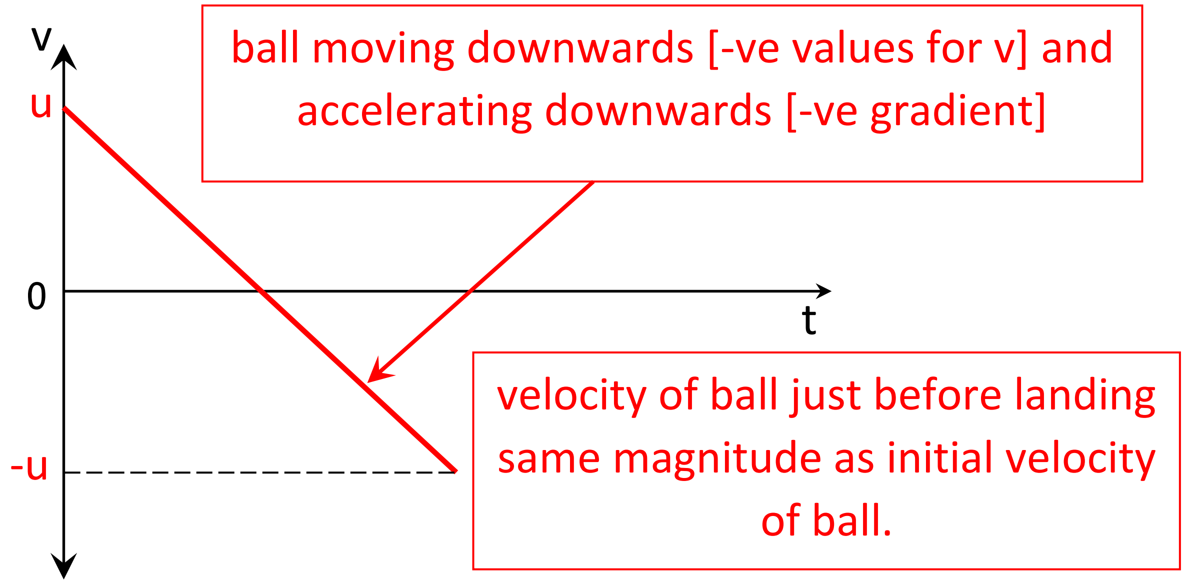
What is happening to the velocity?
The ball will be slowing down whilst it is moving upwards, having a velocity of zero when it reaches maximum height. The acceleration of the ball will be constant if we ignore air resistance.



Velocity - time Graphs (continued)

Part Two of Graph

Once the ball reaches its maximum height it will begin to fall __downwards__. It will have the same (negative) acceleration as when it was going up. The velocity of the ball just before it hits the ground will be the same __magnitude__ but opposite __direction__ as its initial velocity upwards.

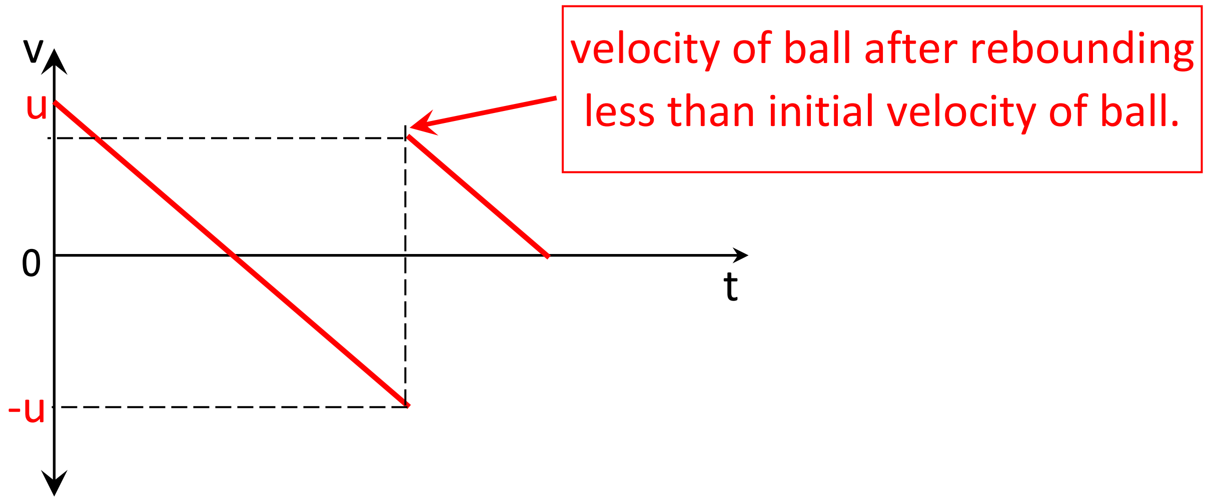


Part Three of Graph

The ball has now hit the ground. At this point it will rebound and begin its movement _upwards_.

In reality there will be a very small of time where the ball is in contact with the ground, where it compresses and bounces back up. In this example, we are taking this time to be 0s, which results in a disjointed graph (a break in the graph).

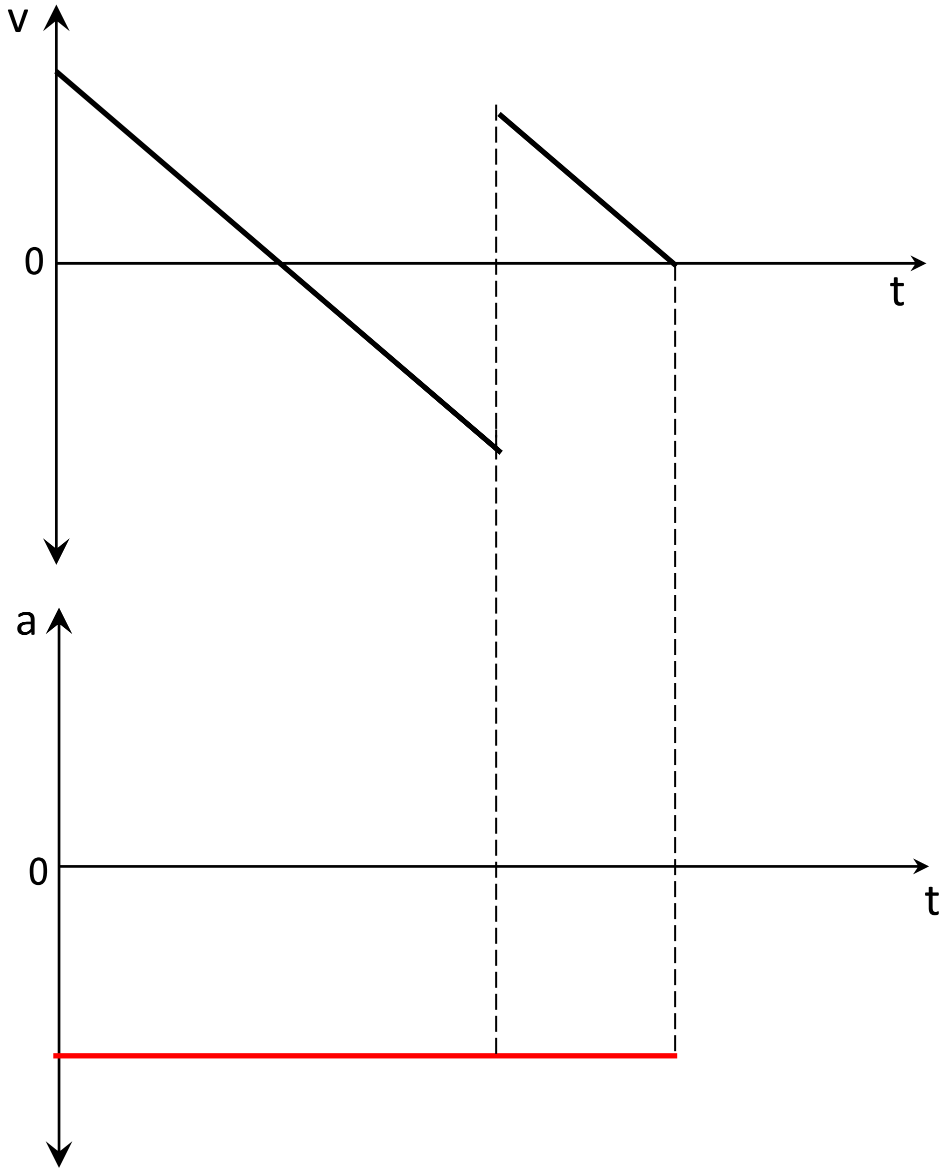
The acceleration of the ball after rebounding will be the same as the _initial acceleration_. The two lines will be parallel.



This now is the velocity time graph of the motion described in the original description.

Converting Velocity - time Graphs to Acceleration - time Graphs

What is important in this conversion is to consider the _gradient_ of the velocity-time graph line. In our example the gradient of the line is constant and has a _**negative**_ value. This means for the entire time sampled the acceleration will have a single _**negative**_ value.



All acceleration time graphs you are asked to draw will consist of _horizontal lines_, either above, below or on the time axis.

<u>Reminder from National 5</u>

The area under a speed time graph is equal to the _distance_ travelled by the object that makes the speed time graph.

In this course we are dealing with vectors so the statement above has to be changed to:

The area under a **velocity** time graph is equal to the _**displacement**_ **of** the object that makes the speed time graph. Any calculated areas that are below the time axis represent _**negative**_ displacements.

<u>Formal Homework 2 - Equations of Motion</u>

1.2 Forces, Energy and Power

Forces

Newton's 1st Law of Motion (Revision of National 5)

An object will remain at rest or travel in a straight line at a constant velocity (or speed) if the forces are balanced.

- If we consider the car moving in a straight line. If the engine force = friction, it will continue to move at a constant _velocity_ (or speed) in the same direction.
- If the same car is stationary (not moving) and all forces acting on it are _balanced_ (same as no force at all) the car will not move.

Newton's 2nd Law of Motion (Revision of National 5)

If the forces on an object are __unbalanced__ an object will accelerate or decelerate.

To work out the acceleration of an object the __unbalanced__ force must be calculated.

Newton's 2nd law states that

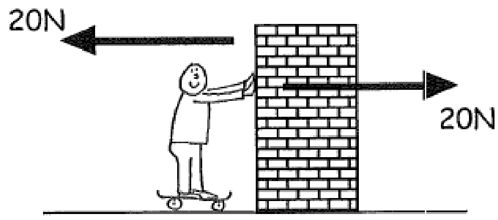
- "if an unbalanced force, $\mathbf{F_{un}}$, acts on a mass, **m**, the mass will accelerate at **a** m s^{-2}."

$$F_{un} = ma$$

Newton's 3rd Law of Motion (Revision of National 5)

If A exerts a force on B, B exerts an equal but opposite force on A. Or

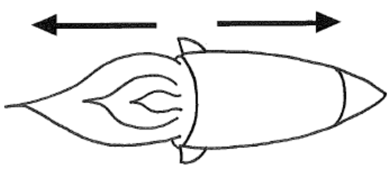
For every action there is an equal but opposite reaction.



A and B above are called __Newton__ Pairs.

Action - The boy on the skateboard exerts a force on **the wall**,

Reaction - the wall exerts an equal but opposite force on **the boy on the skateboard**



Action - If **the rocket engines** exert a force on the **exhaust gases**,

Reaction - __the exhaust gases _ exert an equal but opposite force on __the rocket engines_.

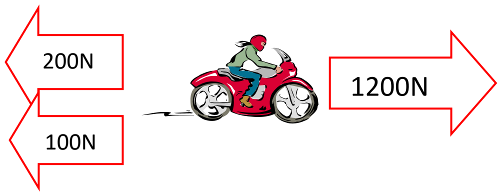
Note: Newton Pairs are **not** examples of balanced forces because the forces are acting on different objects.

Resultant Forces - Horizontal

When several forces act on one object, they can be replaced by one force which has the same effect. This single force is called the _**resultant**_ or _**unbalanced**_ force. Remember that **Friction** is a resistive force which acts in the _**opposite**_ direction to motion.

Example: Horizontal
A motorcycle and rider of combined mass 650 kg provide an engine force of 1200 N. The friction between the road and motorcycle is 100N and the drag value = 200N.
Calculate:

a) the unbalanced force acting on the motorcycle
b) the acceleration of the motorcycle



<u>Solution</u>

a) Draw a free body diagram

$F = 1200 - (200 + 100)$
$F = 900$ N   This 900 N force is the resultant of the 3 forces

b) $F = 900$ N $F = ma$
$a = ?$ $900 = 650 \times a$
$m = 650$ kg $a = 1.38\ ms^{-2}$

Resultant Forces – Vertical (Rocket)

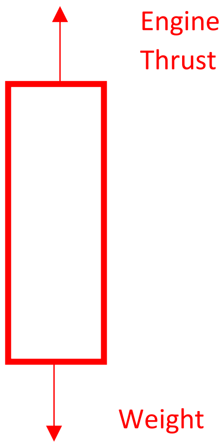
Example

At launch, a rocket of mass 20 000 kg accelerates off the ground at 12 ms^{-2} (ignore air resistance)

a) Use Newton's 3rd law of motion to explain how the rocket gets off the ground.
b) Draw a free body diagram to show all the vertical forces acting on the rocket as it accelerates upwards.
c) Calculate the engine thrust of the rocket which causes the acceleration of 12ms^{-2}.

Solutions

a) The rocket pushes the gas out the back downwards (action) and the gas pushes the rocket upwards (reaction).
b)

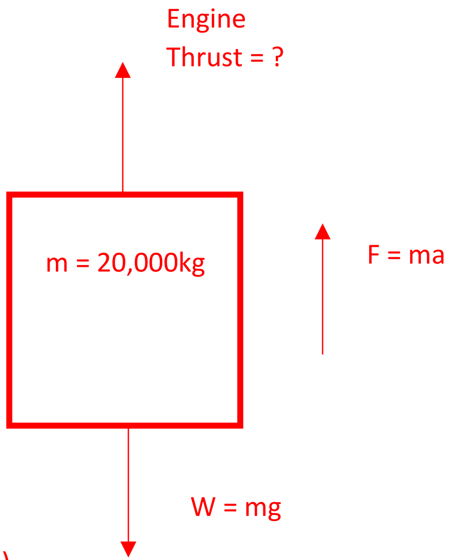


c) Calculate F and W

F = m a
F = 20 000 x 12
F = 240 000 N

W = m g
W = 20 000 x 9.8
W = 196 000 N

F = upward force (thrust) – downwards force (Weight)
240 000 = thrust – 196 000
<u>thrust = 436 000 N</u>

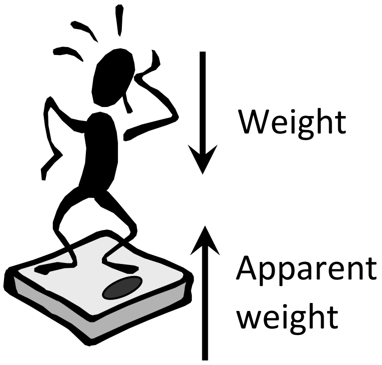


Resultant Forces - Vertical (Lift)

Have you noticed that when you are in a lift you experience a strange feeling when the lift starts to move and as it begins to slow to a stop. However, when the lift is in the middle of its journey you cannot tell if you are moving at all.

This is because at the start and end of the journey you will experience an _acceleration_ and consequently an _unbalanced_ force. This unbalanced force is what you 'feel'.

When you stand on a set of scales (Newton Balance) the reading on the scales is actually measuring the _**upwards**_ force.
This is the force the scales exert on you.
We will call this the _**Apparent Weight**_.
Now this is fine when you are in your bathroom trying to find your weight.



Normally, you and your bathroom scales will be stationary and so your weight will be equal to the upwards force (balanced forces).

When you weigh yourself when you are accelerating the reading on the scales will **<u>not</u>** be your weight. The reading will give you an indication of the unbalanced force acting on you, which could then be used to calculate an acceleration. This unbalanced force could be acting up or down depending on the magnitude and direction of the acceleration.

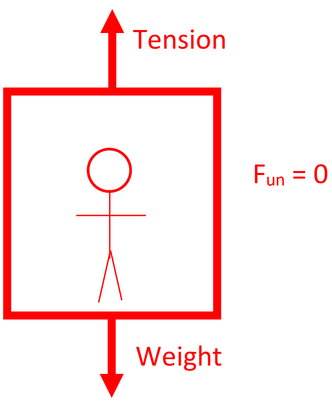
The value of the Apparent Weight will be equal to the _Tension, T_ in the cable of the lift.

There are 3 scenarios you can experience while in a lift:

(Remember that _weight_ will always be constant)

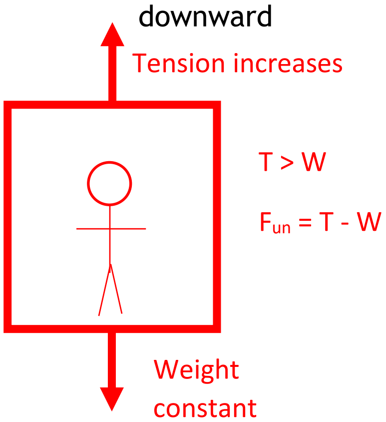
<u>No Acceleration</u>

Lift is stationary or travelling at a constant velocity



<u>Upward Acceleration</u>

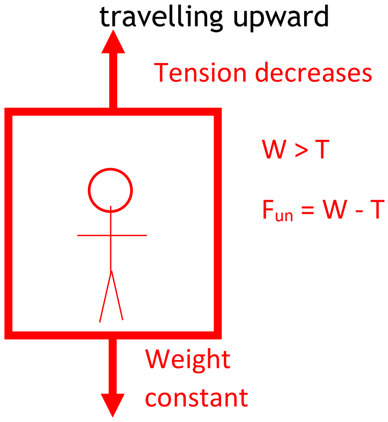
Lift is accelerating upward or decelerating while travelling downward



You feel heavier

<u>Downward Acceleration</u>

Lift is accelerating downward or decelerating while travelling upward



You feel lighter

Resultant Forces - Vertical (Lift)

Example
A man of mass 70 kg stands on a set of bathroom scales in a lift. Calculate the reading on the scales when the lift is accelerating downwards at 2 ms^{-2}.

Solution
Remember that the reading on the scales = apparent weight = tension in the cable

a) Calculate F and W
F = m a
F = 70 x 2
F = 140 N

W = m g
W = 70 x 9.8
W = 686 N

From the boxes on the previous page:
When the lift is **accelerating down**

F = W - reading
140 = 686 - reading
Reading on scale = 546 N

Example (2018 MC)

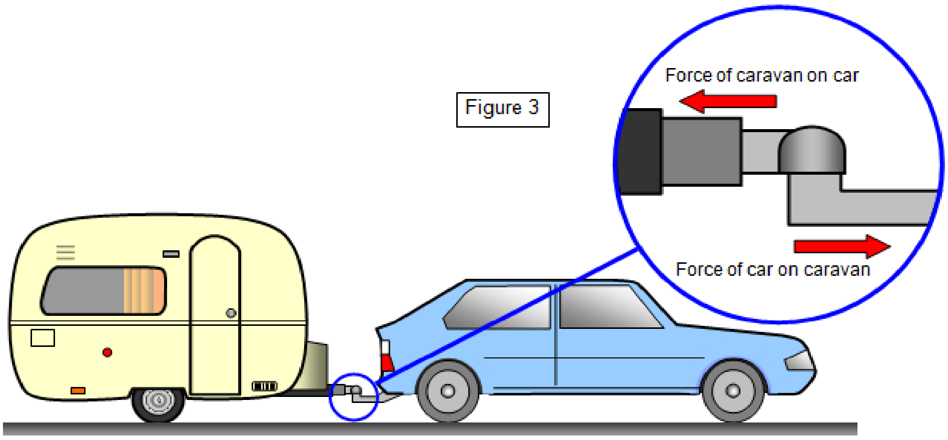
A person stands on a weighing machine in a lift. When the lift is at rest, the reading on the weighing machine is 700N. The lift now descends and its speed increases at a constant rate. The reading on the weighing machine:

A is a constant value higher than 700N
B is a constant value lower than 700N
C continually increases from 700N
D continually decreases from 700N
E remains constant at 700N.

Answer is B – Lift is accelerating downwards at a constant rate. This means tension in the wire holding the lift decreases and W > T. 'Apparent weight' = T, so is a constant value less than 700N

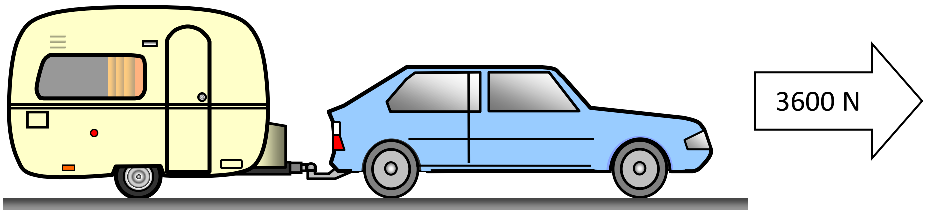
Internal Forces

An example of an internal force is the tension in the towbar (magnified below) when a car is pulling a caravan.



In higher physics, a common question in the SQA exam you are asked is to calculate the tension between the two objects.

Example
A car of mass 700 kg pulls a 500 kg caravan with a constant engine thrust of 3.6 kN.
Calculate the tension in the towbar during the journey (ignoring friction)



Solution: **HINT Calculate the acceleration of the whole system using F = ma**

F = 3600 N
m = 500 + 700 = 1200 kg
a = ?

a = F / m
a = 3600 / 1200
a = 3 ms^{-2}

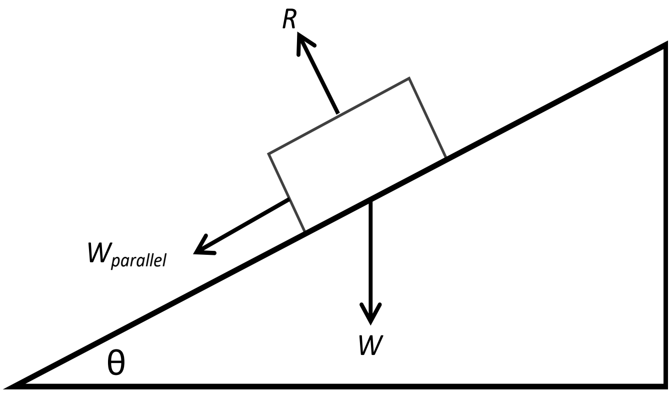
Use this acceleration to calculate the tension and use the mass of the caravan only as this is the mass of the object being pulled.

T = ?
m = 500 kg
a = 3ms^{-2}

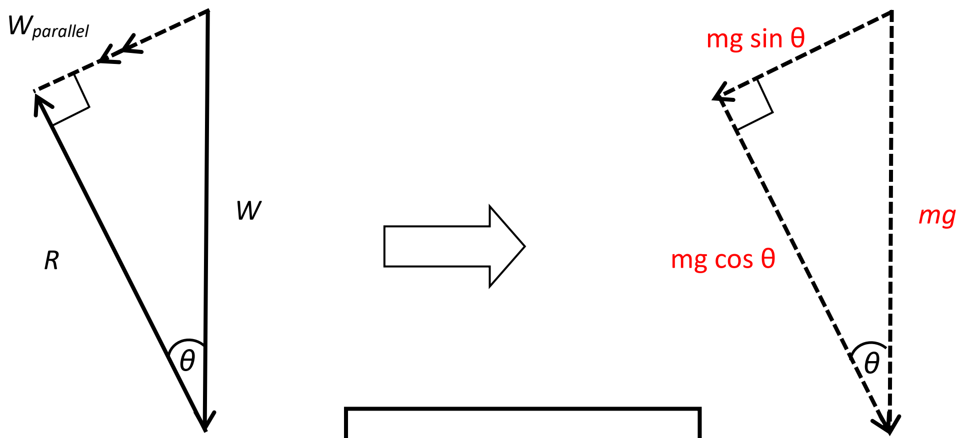
T = ma
T = 500 x 3
T = 1500 N

Forces on a Slope

Ever wondered why a ball rolls down a hill without being pushed or a skier can ski down a run without an initial force. In order to understand why this happens we need to look at the forces exerted on an object resting on a slope:



W is the weight of the object and R is the _reaction_ force acting perpendicular to the slope. If we draw these two forces tip to tail as described in section 1.1 we get the resultant force _$W_{parallel}$_ shown in the diagram below.



$W_{parallel}$ = **mgsinθ**

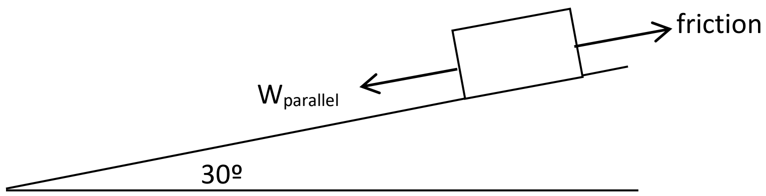
$W_{perpindicular}$ = **mgcosθ**

Forces on a Slope (continued)

Example

A car of mass 1000 kg is parked on a hill. The slope of the hill is 20° to the horizontal. The brakes on the car fail. The car runs down the hill for a distance of 75 m until it crashes into a hedge. The average force of friction on the car as it runs down the hill is 250 N.

(a) Calculate the component of the weight acting down(parallel to) the slope.
(b) Find the acceleration of the car.
(c) Calculate the speed of the car just before it hits the hedge.



Solution

(a) $W_{parallel} = mg\sin\theta$
$= 1000 \times 9.8 \sin 30$
$= 4900$ N

(b) $F = W_{parallel}$ - Friction
$= 4900 - 250$
$= 4650$ N

$a = F / m$
$a = 4650 / 1000$
$a = 4.65$ ms^{-2}

(c) $s = 75$ m
$u = 0$ (parked at rest)
$v = ?$
$a = 4.65$ ms^{-2}
$t =$

$v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 4.65 \times 75$
$v = 26.4$ ms^{-1}

Energy

Conservation of Energy

One of the fundamental principles of Physics is that of conservation of energy.

Energy cannot be created or destroyed, only converted from one form to another.

Work is done when converting from one form of energy to another. Power is a measure of the rate at which the energy is converted.

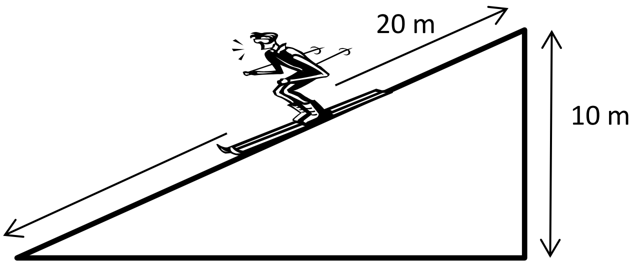
There are a number of equations for the different forms of energy:

E_w	=	Fd
E_k	=	$\frac{1}{2} mv^2$
E_p	=	mgh
E_h	=	$cm\Delta T$
E_h	=	ml
E	=	Pt

All forms of energy can be converted into any other form, so each of these equations can be equated to any other.

Example:

A skier of mass 60 kg slides from rest down a slope of length 20 m. The initial height of the skier was 10 m above the bottom and the final speed of the skier at the bottom of the ramp was 13 ms^{-1}.



Calculate:
(a) the work done against friction as the skier slides down the slope;
(b) the average force of friction acting on the skier.

Solution

(a) Calculate **E_p= mgh** to work out the amount of energy converted.

E_p= mgh
E_p= 60 x 9.8 x 10
E_p= 5880 J

(b) Use Ew = Fs
5880 = F x 20
F = 5880/20
F = 294N

Formal Homework 3 - Forces and Energy

1.3 Collisions, Explosions and Impulse

Momentum

Conservation of Momentum

Momentum (*p*) is the measure of an object's total motion and is the product of mass (m) and velocity (v).

$$p = mv$$

Momentum is a _vector_ quantity, so we must take a positive and negative _direction_.

An object can have a large momentum for two reasons, a large _mass_ or a large _velocity_.

The law of conservation of linear momentum can be applied to the interaction (collision) of two objects moving in one dimension:

In the absence of net external forces, total momentum before = total momentum after

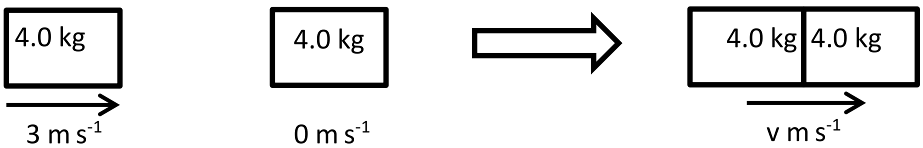
Collisions

The law of conservation of momentum can be used to analyse the motion of objects before and after a collision and an explosion. Let's deal with collisions first of all.

A collision is an event when two objects apply a force to each other for a relatively short time.

Example:
A trolley of mass 4.0 kg is travelling with a speed of 3 m s^{-1}. The trolley collides with a stationary trolley of equal mass and they move off together.
Calculate the velocity of the trolleys immediately after the collision.

Solution:



$$\text{Total } p_{before} = \text{Total } p_{after}$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$(4.0 \times 3) + (4.0 \times 0) = (4 + 4) \times v$$

$$12 = 8\text{ v}$$

$$v = \frac{12}{8}$$

$$v = 1.5 \text{ m s}^{-1}$$

Kinetic Energy – Elastic and Inelastic Collisions

Elastic and Inelastic Collisions

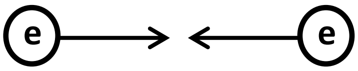
When two objects collide their _momentum_ is **always** conserved but, depending on the type of collision, their kinetic energy may or may not be. Take the two examples below:

1.

If you were to witness this car crash you would hear it happen. There would also be heat energy at the point of contact between the cars.
These two forms of energy will have come from the kinetic energy of the cars, converted during the collision.
Here, kinetic energy is not conserved as it is lost to sound and heat. This is an **_inelastic_ collision**.

In an **inelastic collision**
Total E_k before is greater than total E_k after

2.



When these two electrons collide they will not actually come into contact with each other, as their electrostatic repulsion will keep them apart while they interact.
There is no mechanism here to convert their kinetic energy into another form and so it is **_conserved_** throughout the collision. This is an _elastic_ collision.

In an **elastic collision**
Total E_k before is equal to total E_k after

Elastic and Inelastic Collisions (continued)

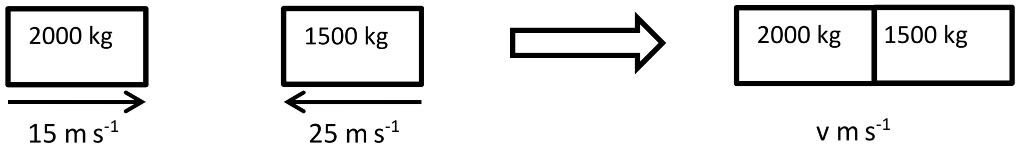
Example:

A car of mass 2000 kg is travelling at 15 m s^{-1}. Another car, of mass 1500kg and travelling at 25 m s^{-1} collides with it head on. They lock together on impact and move off together.

(a) Determine the speed and direction of the cars after the impact.

(b) Is the collision elastic or inelastic? Justify your answer.

Solution:



Total momentum before = Total momentum after

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$(2000 \times 15) + (1500 \times (-25)) = (2000 + 1500) \times v$$
$$3 \times 10^4 + (-37.5 \times 10^4) = 3500\,v$$
$$v = \frac{-7.5 \times 10^3}{3500}$$
$$= \underline{-2.1 \text{ m s}^{-1}}$$

The cars are travelling at 2.1 m s^{-1} to the left.

(b)

$$E_k \text{ before} = \tfrac{1}{2}m_1u^2 + \tfrac{1}{2}m_2u^2$$
$$= (\tfrac{1}{2} \times 2000 \times 15^2) + (\tfrac{1}{2} \times 1500 \times 25^2)$$
$$= 2.25 \times 10^5 + 4.69 \times 10^5$$
$$= 6.94 \times 10^5 \text{ J}$$

$$E_k \text{ after} = \tfrac{1}{2}m_{tot}v^2$$
$$= \tfrac{1}{2} \times 3500 \times 2.1^2$$
$$= 7.72 \times 10^3 \text{ J}$$

Kinetic energy has not been conserved, therefore the collision is inelastic.

Explosions

In a simple explosion two objects start together at rest then move off in _opposite_ directions. Momentum must still be _conserved_, as the total momentum before is zero, the total momentum after must also be _zero_.

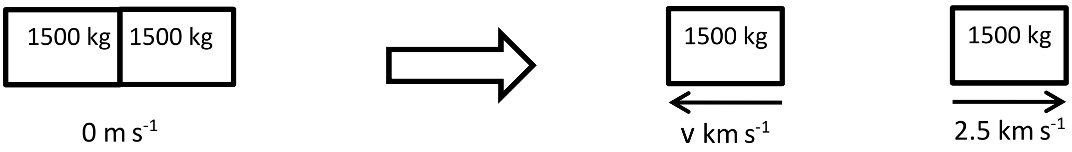
Example

An early Stark Jericho missile is launched vertically and when it reaches its maximum height it explodes into two individual warheads.

Both warheads have a mass of 1500 kg and one moves off horizontally, with a velocity of 2.5 km s^{-1} (Mach 9) at a bearing of 090°.

Calculate the velocity of the other warhead.

Solution:



$$0 = m_1v_1 + m_2v_2$$

$$0 = 1500 \times v_1 + 1500 \times 2.5 \times 10^3$$

$$1500 \times v_1 = -1500 \times 2.5\text{x}10^3$$

$$v_1 = -\frac{3.75 \times 10^6}{1500}$$

$$v_1 = \underline{-2.5 \times 10^3 \text{ m s}^{-1}}$$

The negative sign in the answer indicates the direction of v_1 is opposite to that of v_2, i.e. 270° rather than 090°.

Second warhead is travelling at 2.5 km s^{-1} on a bearing of 270°.

Impulse

Impulse

From Newton's Second Law:

$$F = ma = \frac{m(v - u)}{t}$$

$$F = ma = \frac{(mv - mu)}{t}$$

This expression states that the unbalanced force acting on an object is _equal_ to the rate of change of momentum of the object. This was how Newton first stated his 2nd Second Law.

Impulse is the product of _force_ and _time_, measured in **N s**. Impulse is the **cause** of a change in momentum. Rearranging Newton's Second Law from above:

$$Ft = mv - mu$$ (impulse has no symbol of its own)

$$Ft = \Delta p$$ (Δ means 'change in')

Impulse is _**equal**_ to the change in momentum which is measured in **kg m s^{-1}**.

This means you can calculate the impulse from:

$F \times t$ (N s), or
$mv - mu$ (kg m s^{-1}).

A change in momentum depends on:

- The size of the _force_
- The _time_ the force acts

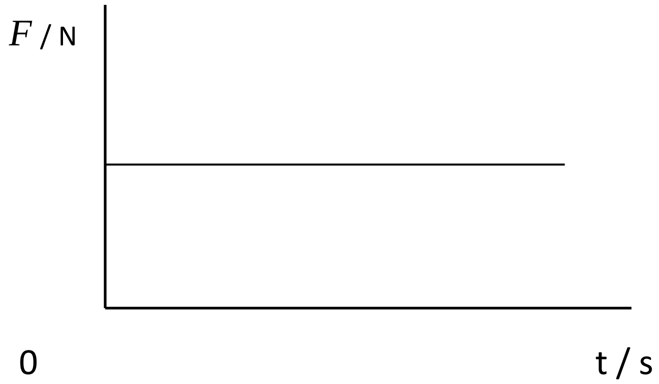
Example:
A force of 100 N is applied to a ball of mass 150 g for a time of 0.020 s.
Calculate the final velocity of the ball.

Solution:

F = 100N
m = 0.150 kg
t = 0.020 s
u = 0 m s^{-1}

$$\begin{aligned} Ft &= mv - mu \\ 100 \times 0.020 &= 0.150 \times (v - 0) \\ 2.0 &= 0.150\,v \\ v &= 2/0.15 \\ v &= 13.3 \text{ m s}^{-1} \end{aligned}$$

Impulse Graphs



Ft = impulse = change in momentum

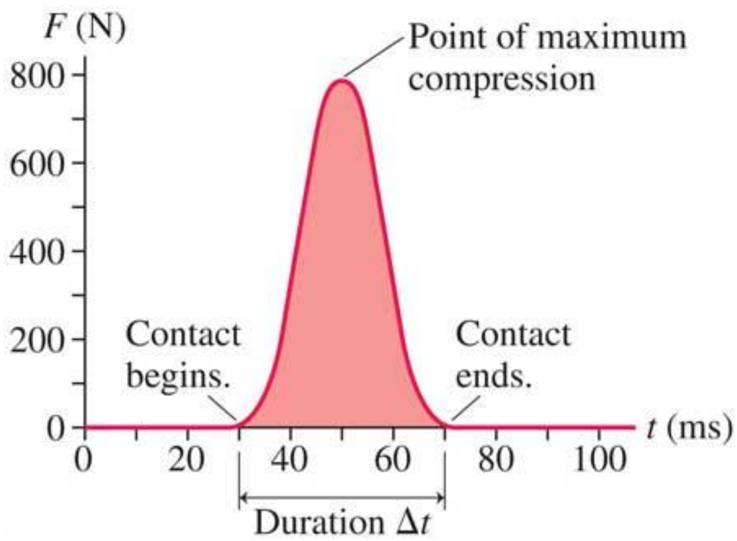
Ft = area under F - t graph

In reality, the force applied is not usually constant.
The analysis of the force acting on an object causing it to change speed can be complex.
Often we will examine the force over time in graphical form.

Consider what happens when a ball is kicked.

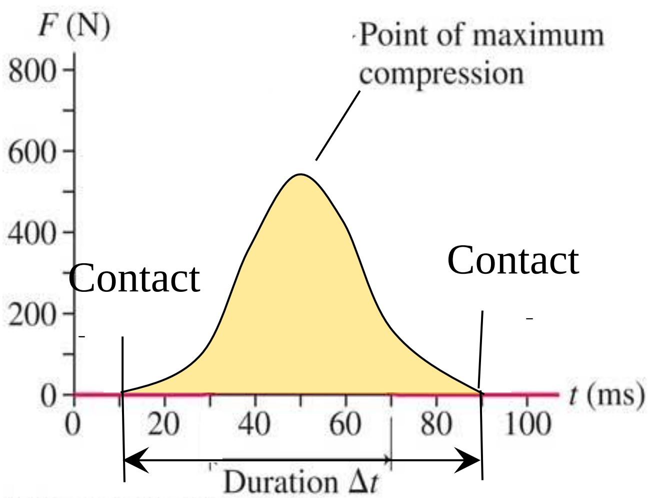
Once the foot makes contact with the ball a force is applied, the ball will compress as the force increases.
When the ball leaves the foot it will retain its original shape and the force applied will decrease.

This is shown in the graph below.



Impulse Graphs (continued)

If a ball of the same mass that is softer is kicked and moves of with the same speed as that above, then a graph such as the one below will be produced.



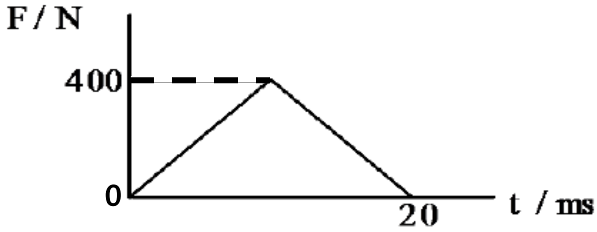
The maximum force applied is _smaller_ but the time it is applied has _increased_.

Both graphs have same area under the graph, so the same impulse (or change in momentum).

Example

A tennis ball of mass 100 g, initially at rest, is hit by a racquet.
The racquet is in contact with the ball for 20 ms and the force of contact varies over this period, as shown in the graph.

Determine the speed of the ball as it leaves the racquet.



Solution

Impulse = area under graph

$= \frac{1}{2} \times 20 \times 10^{-3} \times 400$

$= 4$ N s

$u = 0$

$m = 100$ g $= 0.1$ kg

$v = ?$

$Ft = mv - mu$

$4 = 0.1v - (0.1 \times 0)$

$4 = 0.1v$

$v = 40$ ms^{-1}

Practical Applications – Car Safety

Essentially the greater the time you can take to decelerate an object, the smaller the force you need to apply.

Airbags

The concept of the airbag - a soft pillow to land against in a crash - has been around for many years. The first patent on an inflatable crash-landing device for airplanes was filed during World War II. In the 1980s, the first commercial airbags appeared in cars. Stopping an object's momentum requires a force acting over a period of time. When a car crashes, the force required to stop an object is very large because the car's momentum has changed instantly while the passengers' has not, there is not much time to work with. The goal of any restraint system is to help stop the passenger while doing as little damage to him or her as possible. What an airbag wants to do is to slow the passenger's speed to zero with little or no damage. To do this it needs to **increase** the time over which the change in speed happens.

Crumple Zones

Placed at the front and the rear of the car, they absorb the crash energy developed during an impact. This is achieved by deformation. While certain parts of the car are designed to allow deformations, the passenger cabin is strengthened by using high-strength steel and more beams. Crumple zones delay the collision. Instead of having two rigid bodies instantaneously colliding, crumple zones **increase the time** before the vehicle comes to a halt. This **reduces the force** experienced by the driver and occupants on impact. The change in momentum is the same with or without a crumple zone.

Formal Homework 4 – Momentum and Impulse