Mathématiques SANS Frontières

2025

The Scottish Mathematical Council

Partial solutions and attempts can earn marks.

Neat and careful work is important.

Submit only one team answer sheet for each question

Mathématiques SANS Frontières

Exercise 1

7 pts

Apple Water

Your solution is to be written in French, German, Spanish, or Italian using a minimum of 30 words.

Richard hat Apfelsaft gemacht. Seine Enkelkinder trinken ihn gerne als Schorle, mit Mineralwasser gemischt. Richard nimmt eine Flasche mit 100 cl Apfelsaft, gießt 25 cl aus und füllt die Flasche wieder mit Mineralwasser auf. Danach gießt er 1/5 dieses Gemischs aus und ersetzt es durch Mineralwasser.

Wieviel Prozent Apfelsaft enthält die so entstandene Schorle?

Erklärt eure Antwort.

Richard acaba de hacer zumo de manzana. Los niños prefieren beberlo diluido con agua. Richard extrae 25 cl of zumo de manzana de una botella de 100 cl, luego la completa con agua. Después extrae 1/5 de esta mezcla y la sustituye por agua.

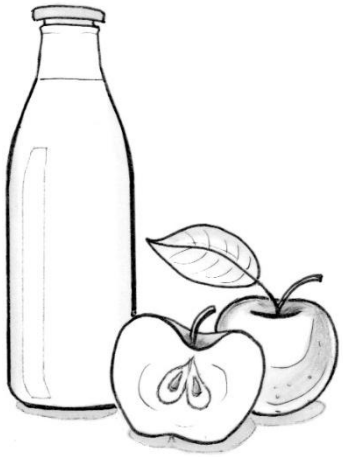
¿Cuál es el porcentaje de zumo de manzana de la mezcla así obtenida?

Justifica tu respuesta.

Richard vient de faire du jus de pommes. Les enfants préfèrent le boire dilué avec de l'eau. Richard prélève 25 cL de jus de pommes d'une bouteille de 100 cL, puis il complète avec de l'eau. Il prélève ensuite 1/5 de ce mélange et le remplace par de l'eau.

¿ Quel est le pourcentage de jus de pommes dans le mélange ainsi obtenu ?

Expliquer votre raisonnement.



Richard ha preparato del succo di mele. I bambini preferiscono berlo diluito in acqua.

Richard, perciò, preleva 25 cl di succo di mela da una bottiglia da 100 cl e, quindi, la riempie

aggiungendo dell'acqua. In seguito, preleva 1/5 di questa miscela e la sostituisce nella bottiglia con dell'acqua.

Qual è la percentuale di succo di mela nella miscela così ottenuta?

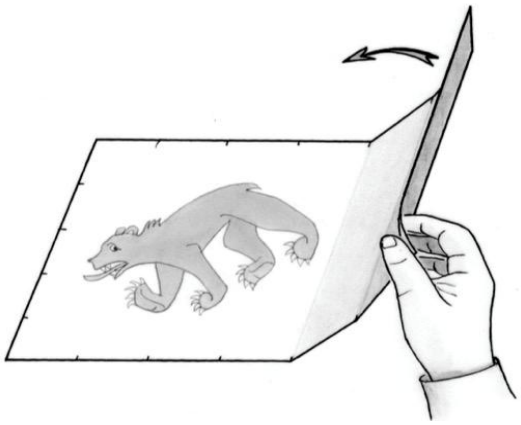
Illustrate il vostro ragionamento.

Exercise 2 5 pts

Multi-ply

The perimeter of a rectangular sheet of paper is 69.6 cm.

Maximus folds this sheet into seven equal parts in the direction of the length.



He obtains a rectangular strip which he folds into five equal parts in the direction of the initial width of the sheet. He obtains a square.

Calculate the dimensions of this sheet.

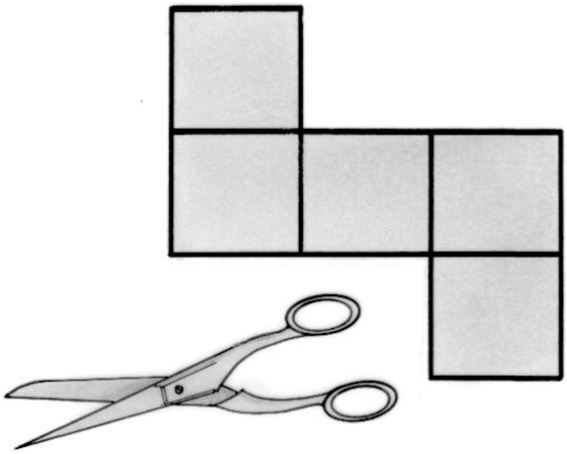
Explain your approach.

Exercise 3 7 pts

It's square

Propose two cuts of this figure to form a square of three pieces:

- ***one obtained in two perpendicular strokes of the scissors;***
- ***the other obtained in two parallel strokes of the scissors.***



Glue the two assemblies to the answer sheet.

Exercise 4 — 5 pts

To Termes

In the Middle Ages, the castle of Termes enjoyed a peaceful life. Enemy soldiers besieged it, preventing any supplies getting in. There were only 60 days of food left for those trapped in the castle. Through an underground passage, a group of 30 villagers took refuge in the castle. As a result, the supplies in the besieged castle were only enough for 50 days.

Determine the number of people in the castle of Termes before the arrival of the villagers.

Explain your approach.

Exercise 5 7 pts

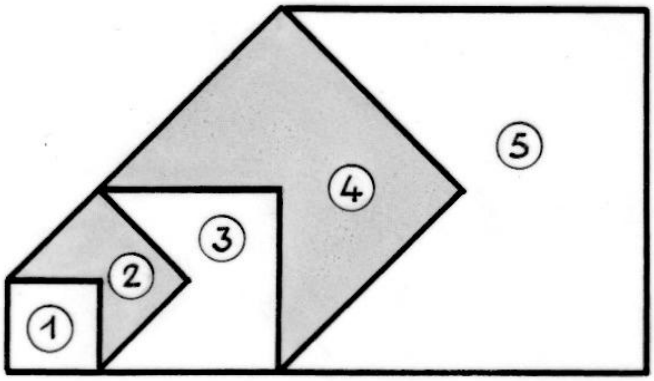
Reiterates

To construct the figure below, we start by drawing square ① of side length 1.

Then we construct the square ② on the side of the diagonal of the square ①.

Then we construct the square ③ on the side of the diagonal of the square ②.

Then we construct a square ④ on the side the diagonal of the square ③, etc.



Determine the length of the side of the square ⑨ as well as the total area of the figure obtained after constructing the square ⑨.

Exercise 6 — 5 pts

πramid

Today, to calculate the area of a disk, we use the number π whose approximate value is 3.1415926.

At the time of the construction of the pyramids, the Egyptians used another method to calculate the area of a disk:

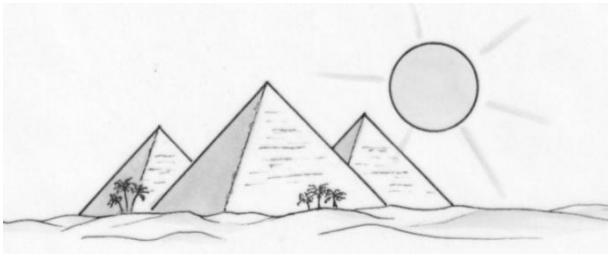
- Remove 1/9 of the disk diameter.
- Square the result.

This calculation gave a value close to the area of the disk.

In the formula used by the Egyptians, the number π does not appear.

What approximate value of π did the Egyptians use?

Explain your reasoning.



Exercise 7 7 pts

1 in front 1 behind

Mr. Reich wants to buy an apartment. He found the one he liked at a price written as a 6-digit number with first digit 1.

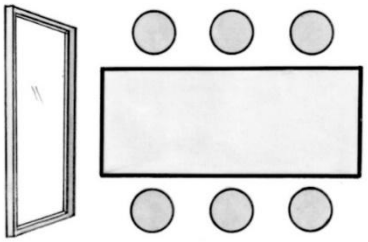
Oh amazing! When it was time to sign, the real estate agent made a mistake on the documents, and the first digit 1 went to the end of the number. The amount is then three times higher than the real price. Mr. Reich refused to sign.

Find the real price of Mr. Reich's apartment.
Explain your reasoning.

Exercise 8 5 pts

Seating Plan

Three couples dine around the same table: Mr. and Mrs. Hans; Mr. and Mrs. Klein; Mr. and Mrs. Muller.



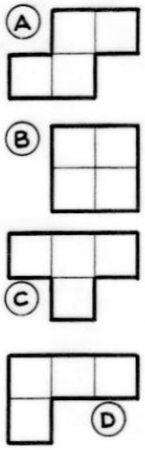
No man sat next to another man.
No man sits next to or opposite his wife.
Mr. Hans is neither next to nor opposite Mrs. Klein.
Mrs. Klein is talking with Mrs. Muller, next to whom she is sitting. Mr. Muller does not like drafts and is not next to the window.

Position the six guests, respecting these constraints.

Exercise 9 — 7 pts

Tetraminos

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



There are four tetraminos denoted (A)(B)(C)(D).

We place one of these four tetraminos, without changing its initial orientation, in the grid drawn opposite so that it covers exactly four integers. We then calculate the sum of the four integers that it covers.

Example: The tetramino **A** already placed covers the integers 13, 14, 22 and 23.

13 + 14 + 22 + 23 = 72.

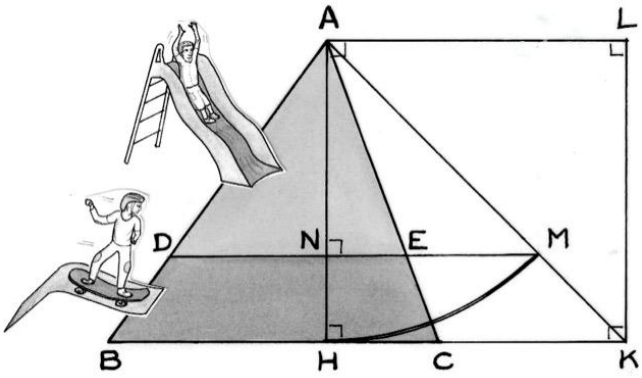
The sum of these four integers is 72

What are the four numbers covered by tetramino C if the sum is 318?
Which of these four tetraminos should be placed in the grid to obtain a sum of 121? What are the four numbers covered?
Only one of these tetraminos placed in the grid always gives an odd amount. Which one? Demonstrate why.

Exercise 10 10 pts

Same area

The municipality of Thalesheim has a triangular plot of land ABC shown on the map opposite. It wishes to divide this land into two zones of equal areas. In one of the areas, it is planned to develop a children's playground and in the other a skatepark.



To properly carry out this division of the land, the urban planner followed the following construction plan:

- Draw the height [AH] of the triangle ABC.
- Construct the square AHKL, then its diagonal [AK].
- Position the point M on the segment [AK], such as: AM = AH.
- Draw (MN) perpendicular to (AH) with N lying on [AH].
- Position the points D and E, the points of intersection of the line (MN) with the sides [AB] and [AC], respectively.

The measurements he took on the plan are: AH = 12 cm,

- CH = 5 cm, HB = 9 cm. The playground is represented by the triangle ADE and the skatepark by the trapezium BCED

Create this figure.

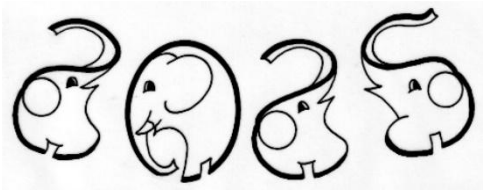
Show that the playground and the skatepark do indeed have the same area.

Exercise 11 5 pts

2025

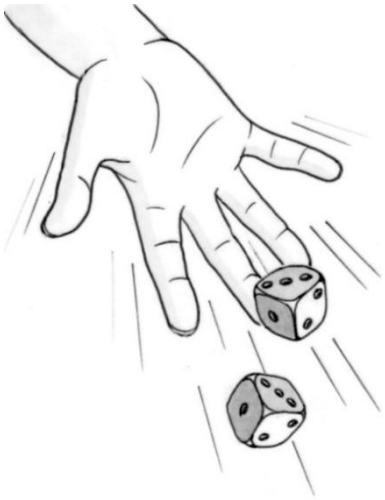
Find all positive integers a and b such that:

$$a^2 + b^2 = 2025 \times \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$



Exercise 12 — 7 pts

Probability



Two identical cubic dice are rolled simultaneously.

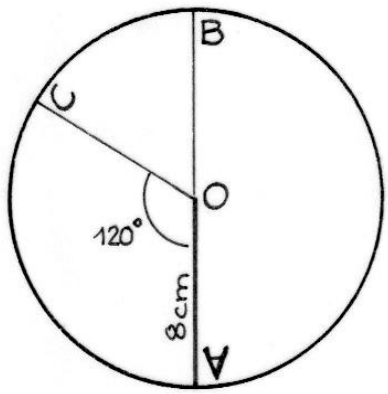
On the faces of each die, there are only the numbers 1, 2 and 3.

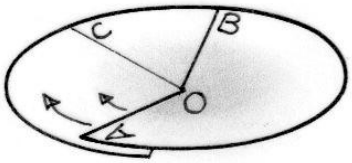
The probability of getting "a double 2" is $\frac{1}{9}$.

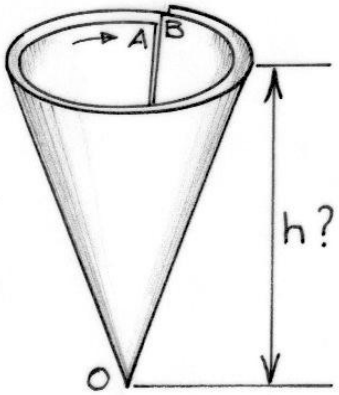
The probability of getting "1 and 2" is $\frac{1}{3}$.

How many 1s, 2s, 3s are on each die? Explain.

Exercise 13	10 pts	Cornet







Rémi wants to make a paper cone, without a base. He cuts a disk with a radius of 8 cm. He marks on the circle the diametrically opposed points A and B and the point C such that $\widehat{AOC} = 120°$.

Make this disk, then make a notch corresponding to the radius [OA] of the disk. Slide the paper, holding point A on the circumference of the disc, so as to superimpose two points to obtain a cone, as in the figure above

Determine the exact value of the height of the cone obtained in each of the following cases:

- ***when points A and B are superimposed,***
- ***when points A and C are superimposed.***