Mathématiques SANS Frontières

2024

The Scottish Mathematical Council

Partial solutions and attempts can earn marks.

Neat and careful work is important.

Submit only one team answer sheet for each question

Exercise 1 –  **– 7 pts**

Your solution is to be written in French, German, Spanish, or Italian using a minimum of 30 words.

Jacquot va manger tous les jours dans le même restaurant. Il décide de faire une cagnotte pour le serveur. Il met 3 € dans la cagnotte s'il est content du service, il enlève 4 € de la cagnotte s'il n'est pas satisfait. Pendant quelques semaines, il a été très content de chaque repas et la cagnotte est ainsi bien remplie. Puis, au bout des quatorze repas suivants, le serveur n'a rien gagné en plus mais n'a rien perdu non plus.

Combien de fois Jean a-t-il été content au cours des quatorze derniers repas ? Expliquer votre raisonnement.

Jacquot va a comer todos los días al mismo restaurante. Decide guardar un bote para el camarero. Mete 3 € en el bote si está contento con el servicio, quita 4 € del bote si no está satisfecho. Durante algunas semanas, ha estado muy contento con la comida y por lo tanto, el bote está bien lleno. Y después, tras las catorce comidas siguientes, el camarero no ha ganado ni ha perdido nada más.

¿Cuántas veces Jacquot ha estado contento a lo largo de las catorce últimas comidas? Justifica tu respuesta.

Jacquot mangia tutti i giorni nello stesso ristorante. Decide di preparare un salvadanaio per le mance per il cameriere. V'inserisce 3 € se è soddisfatto del servizio, toglie 4 € se non lo è. Dopo alcune settimane è stato molto soddisfatto di ogni pasto e il salvadanaio è stato così ben riempito. Poi, alla fine dei quattordici pasti seguenti, il cameriere non ha né perso né guadagnato altro.

Quante volte, durante i quattordici ultimi pasti, Jacquot è rimasto soddisfatto? Motivate la vostra risposta.

Jacquot geht jeden Tag im selben Restaurant essen. Er beschließt, eine Trinkgeld-Kasse für den Kellner einzurichten. Er legt 3 € in die Kasse, wenn er zufrieden ist, und nimmt 4 € heraus, wenn er nicht zufrieden ist.

Einige Wochen lang war er sehr zufrieden gewesen, und so war die Trinkgeld-Kasse gut gefüllt. Bei den folgenden 14 Restaurant-Besuchen hat der Kellner dann nichts verdient, aber auch nichts verloren.

Bei wie vielen der 14 Restaurant-Besuche war Jacquot zufrieden? Erklärt eure Antwort.

Exercise 2 – AGE PROFILE – 5 pts

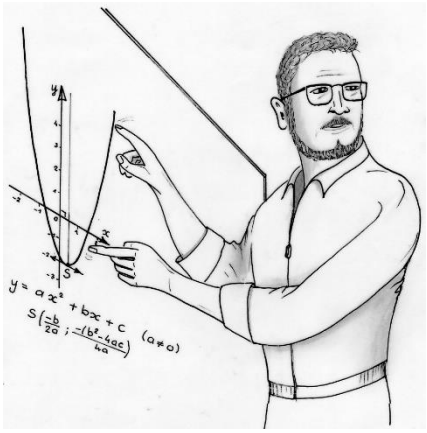
When asking a person's year of birth, they often only give the last two digits of the year. For example, they say:

"I was born in 11, or in 92, or in 78."

This is how John, a mathematics teacher, born before the year 2000, answers:

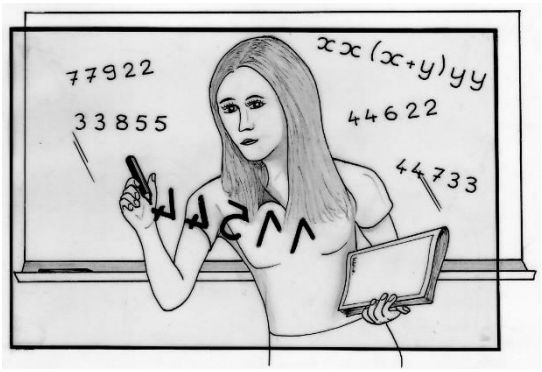
"This year, in 2024, I am the age of my year of birth".

Calculate John's age. In 2024, can John's grandson say the same thing? Explain your reasoning.



Exercise 3 – CXI – 7 pts

x and y are natural integers whose sum is less than 10.



“Stunning numbers” are numbers whose writing is of the form xx(x+y)yy. Here are two examples: 33,855 and 77,922.

Give two more examples and verify that these four stunning numbers are divisible by 111.

Establish a conjecture on the form of the result of these divisions.

Prove the conjectured property.

Exercise 4 – ON THE WAY TO 2025 – 5 pts

In this square, we may move from one square to an adjacent square but not diagonally. We can not go through the same box twice and the intermediate results are always completed.

2024	x 3	: 23	x 26	: 88
: 19	x 20	x 17	: 35	: 10
x 5	: 11	x 27	: 31	x 25
x 2	: 8	: 17	x 21	x 14
x 29	: 37	x 5	: 2	2025

Starting from 2024, find a path which respects the operations and which leads to the 2025 box.

Explain your approach.

(Note for UK pupils – In mainland Europe, the colon sign (:) is used for division)

Exercise 5 – ONLINE PARCEL – 7 pts

MsF prepares and ships packages all over the world.

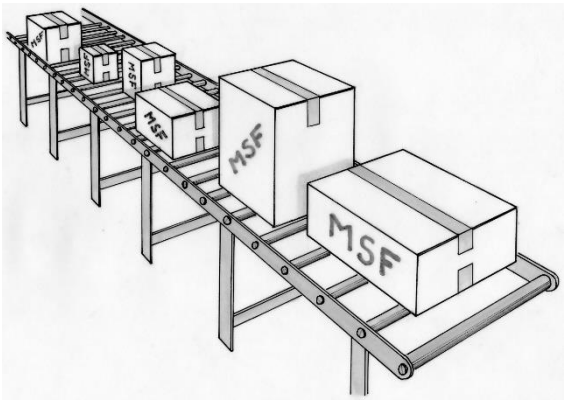
40 packages numbered from 1 to 40 are placed, one behind the other, on a conveyor belt which transports them to the delivery truck.

The total mass of these 40 packages is 106 kg.

The sum of the masses of any three parcels which follow one another is always equal to 8kg.

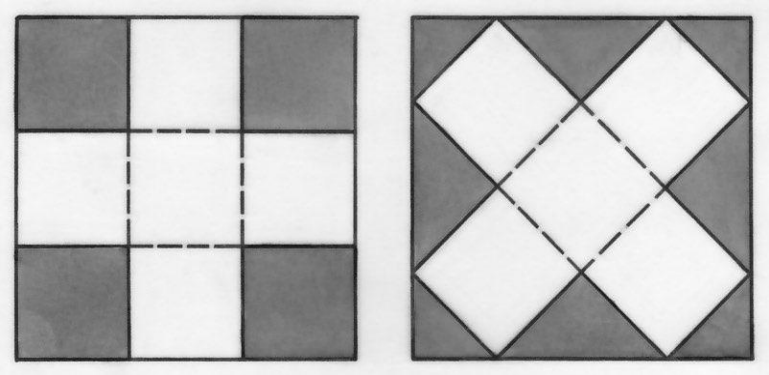
Packages numbered 20 and 21 have exactly the same mass.

Determine the masses of packages numbered 20 and 21. Explain your approach.



Exercise 6 – OFFCUTS, BOSS! – 5 pts

The Inzeboîte company manufactures and sells cube-shaped cardboard boxes without lids. Two box patterns are offered. They are drawn on square sheets of the same dimensions. Then they are cut and folded.



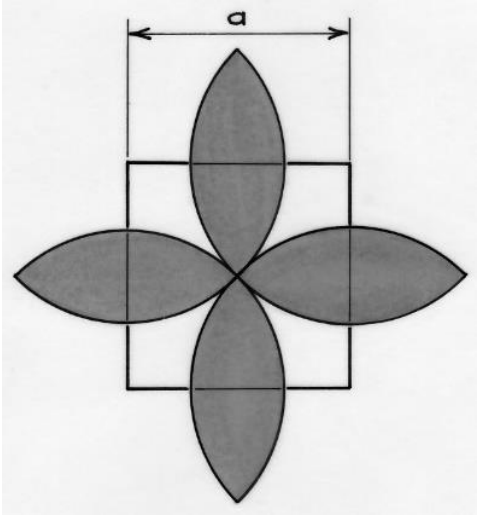
Choose the pattern that produces the least cardboard scraps.

Explain your approach.

Exercise 7 – MAKE A FLOWER – 7 pts

Here is a construction program:

- draw a square and mark its center;
- draw the four circles having as centers the vertices of the square and passing through the center of this square.



Part of the figure has been colored.

Make this construction from a square with a side of 6 cm.

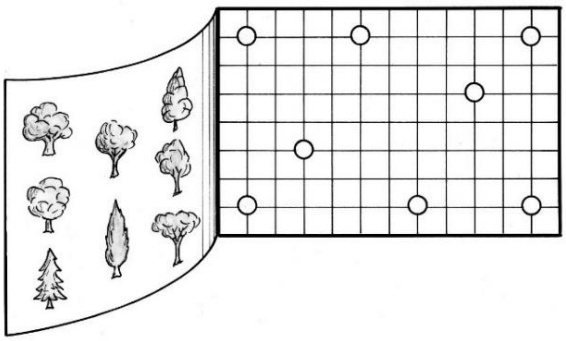
We label α as the length of the side of the square. Determine the area of the colored part expressed using α.

Exercise 8 – TREE SHARING – 5 pts

A town decides to create a subdivision of eight identical plots.
The land has eight remarkable trees that absolutely must be preserved.
A plan of this land is given in the drawing below where the position of the trees is indicated.
The surveyor proposes to subdivide the land while respecting the following constraints:

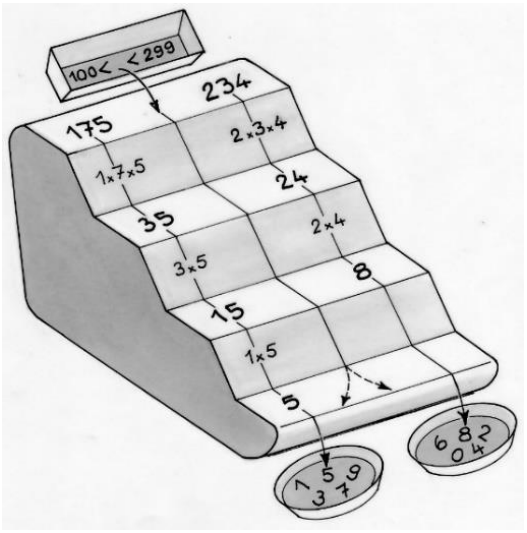
- the eight plots must have the same shape;
- each plot must contain a remarkable tree.

Give two possible divisions of this land into eight plots.



Exercise 9 – NUMBER SLIDE – 7 pts

Here is a three-step algorithm:
Step 1: Choose a starting whole number between 100 and 299 inclusive.
Step 2: Multiply the digits of this number.
Step 3: If the previous result is less than 10, the algorithm stops, otherwise repeat step 2.



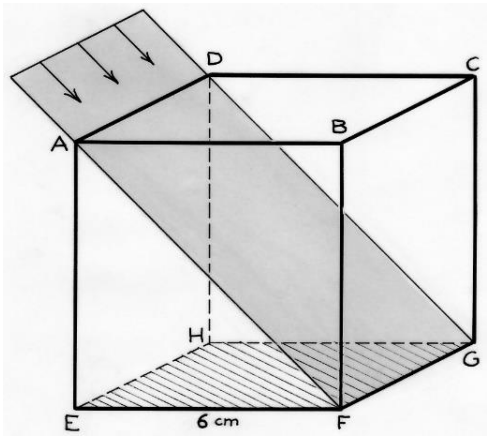
Here are two examples:
- by choosing 234 as the starting number:
$234 \rightarrow 24 \rightarrow 8$
the result of the algorithm is 8;
- by choosing 175 as the starting number:
$175 \rightarrow 35 \rightarrow 15 \rightarrow 5$
the result of the algorithm is 5.

When running this algorithm with several different numbers, it is rare for the result to be odd: 1; 3; 5; 7 or 9.

Find all the numbers that give an odd result. Explain your approach.

Exercise 10 – CUTTING PLANS – 10 pts

The cube with an edge of 6 cm is cut successively according to the ADGF, BCHE, ABGH planes and finally according to the CDEF plane, each time keeping the solid which has the square EFGH as its base.



Represent the cube in cavalier perspective with the solid obtained inside. Colour the edges of the solid again.

Make a full-size pattern of this solid.

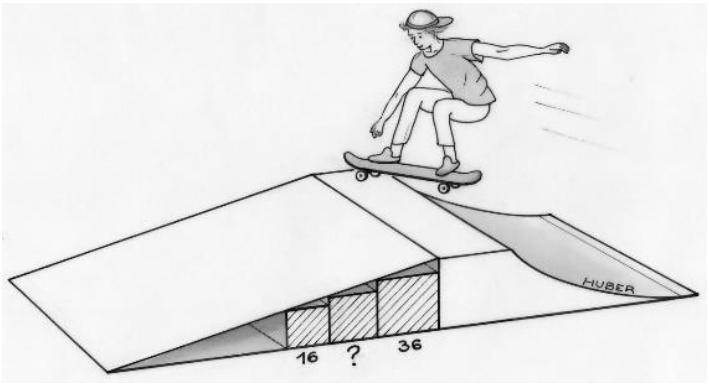
Express the volume of this solid in terms of the volume of the cube.

How many identical solids of this type can we put in this cube?

Exercise 11 – SKATE SLOPE – 5 pts

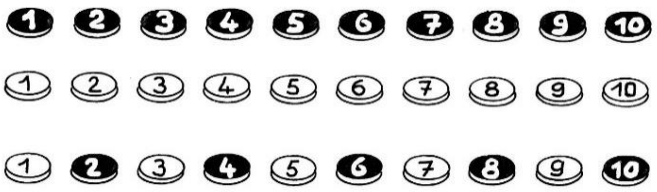
Maxence built himself a skateboard ramp with three blocks in the shape of straight paving stones with a square base that he joined together and a piece of flat sheet metal. His friend Anatole would like to make the same ramp at home. Maxence explains to him: “The most important thing is that the three blocks have one of their edges in good contact with the ramp! The largest block has a square base with a side of 36 cm and the smallest has a square base with a side of 16 cm. But I no longer remember the length of the side of the intermediate square.”

Help Anatole find the missing dimension. Explain the approach.



Exercise 12 – TO SEE THE TOKENS – 7 pts

Agathe places ten tokens, each having a black side and a white side next to each other with the black side visible. These tokens are numbered from 1 to 10 on both sides.



She performs several successive manipulations on the tokens by proceeding as follows:

First manipulation: she turns over each of the ten tokens.

Second manipulation: she turns all the tokens on which a multiple of 2 is written.

Third manipulation: she turns all the tokens on which a multiple of 3 is written.

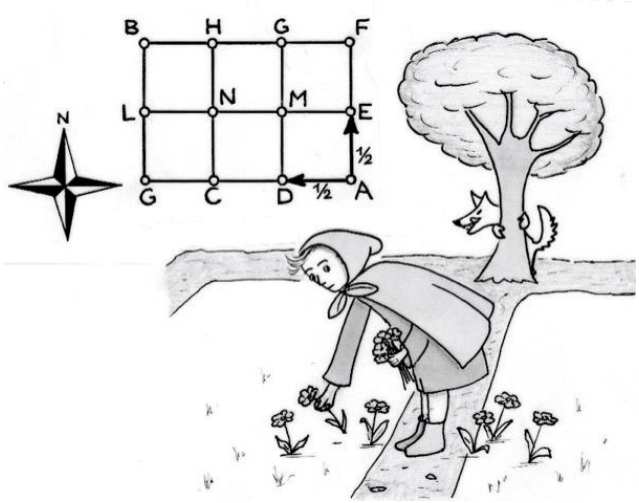
Etc…, until the tenth and final manipulation, where she turns only the token numbered 10.

Give the numbers of the tokens showing a white side after ten manipulations.

If Agathe had 100 tokens, numbered from 1 to 100, which ones would show a white side after a hundred manipulations? What is their common characteristic?

Exercise 13 – WOLF, ARE YOU THERE?– 10 pts

Little Red Riding Hood goes alone into the forest to visit her sick grandmother to bring her a pot of butter and a pancake. The grid below represents the different possible paths through the forest. Each point represents the intersection between two paths. Little Red Riding Hood moves from one intersection to another, but only to the north or west. If two paths are possible, the probability that she moves north or west is ½. So the probability that Little Riding Hood goes from point A to point C is ½ x ½ = ¼ . Unfortunately for her, the wolf has hidden in the forest and, if she comes across it, she will be devoured. Little Red Riding Hood is at A and her grandmother's house is at B. The wolf is hiding at point L.



Calculate the probability that Little Red Riding Hood passes through point L and is thus devoured by the wolf. Justify your approach.

At which intersection does the wolf have the greatest probability of encountering Little Red Riding Hood, knowing that he cannot hide in A or in B?

Justify your approach.