# Mathématiques <br> Frontières 



UNIVERSITY OF THE
WEST of SCOTLAND
UWS


The Scottish Mathematical Council

Partial solutions and attempts can earn marks.
Neat and careful work is important.
Submit only one team answer sheet for each question

## Exercise 1 - AUX CARREAUX - 7 pts

Your solution is to be written in French, German, Spanish, or Italian using a minimum of 30 words.

Étienne y Gus están comiendo una tableta de chocolate: los dos son unos auténticos golosos pero, por educación, ninguno de los dos quería ser el egoísta que coja el último trozo.
La tableta inicial tiene veinticuatro cuadrados. Cada uno, por turno, rompe el chocolate en dos trozos rectangulares siguiendo la línea horizontal o vertical que separa los cuadrados. Se come uno de los trozos y le da el otro a su amigo.

Étienne empieza y se las arregla para que Gus se
vea obligado a coger el último cuadrado.
Describe su estrategia.

Étienne et Gus se font des politesses en mangeant une tablette de chocolat : tous deux sont d'authentiques gourmands mais aucun d'eux ne voudrait être l'égoïste qui prendra le dernier morceau.

La tablette initiale compte vingtquatre carreaux. Chacun, à tour de rôle, casse le chocolat en deux morceaux rectangulaires suivant une ligne horizontale ou verticale qui sépare les carreaux. Il mange l'un des morceaux et donne l'autre à son ami.

## Étienne commence et

 s'arrange pour que Gus soit obligé de prendre le dernier carreau.Décrire sa stratégie.


Étienne e Gus si scambiano delle gentilezze mentre mangiano una tavoletta di cioccolato: entrambi sono dei golosoni, ma non vorrebbero mai essere egoisti tali da prendere l'ultimo pezzetto.
La tavoletta iniziale è composta da ventiquattro quadratini. I due golosi, a turno, spezzano il cioccolato in due parti rettangolari secondo una delle linee verticalio orizzontali che separano i quadratini; mangiano una parte e passano la rimanente all'altro.

Étienne inizia e fa in modo che Gus sia costretto a prendere I'ultimo quadratino.

Descrivete la sua strategia.

Étienne und Gus essen zusammen eine Tafel Schokolade und zeigen dabei ihr gutes Benehmen: Beide lieben Schokolade, aber keiner möchte der Egoist sein, der das letzte Stück nimmt.

Die Tafel Schokolade hat insgesamt 24 Stücke. Nacheinander zerbricht jeder die Schokolade längs oder quer in zwei rechteckige Teile, immer entlang einer Linie zwischen den Schokoladenstücken. Den einen Teil isst er, den anderen Teil gibt er seinem Freund.
Étienne beginnt, und er erreicht, dass am Ende Gus das letzte Stück nehmen muss.

## Beschreibt Étiennes Strategie.

## Exercise 2 - ON HER SIDE - 5 pts

Six identical cubes, as shown below, are placed in front of Sophie.
Draw exactly on the answer sheet what Sophie sees on the side facing her.
Construct the net of one of these cubes.


## Exercise 3 - FOLD AND UNFOLD - 7 pts

Here is a method that can find one third of the length of a rectangular sheet of paper just by folding.


Fold and then unfold the sheet firstly along its diagonal (1), then its median (2), then along the folds (3) and (4) finally as shown in the figure above.

## Attach the folded paper on the answer sheet.

Show that fold (4) has one third of the length of the sheet of paper.

## Exercise 4 - DOUBLE PUZZLE - 5 pts

Léo cuts out four pieces $A, B, C, D$ from the puzzle below.


With the three pieces A, B and C, he forms a square. Suddenly, he exclaims: "But we can form another square with all the pieces of the puzzle!"

Draw the two squares that can be formed in this way, giving the details of their assembly.

## Exercise 5 - FOUNDATION - 7 pts

Numbers are written on bricks. The number written on each brick is equal to the sum of the numbers written on the two bricks located immediately below.

Find the missing numbers. Explain the process followed.


## Exercise 6 - PLAYING WITH MATCHES - 5 pts

Yaëlle has lined up matches.
If she takes them 2 at a time, 1 is left over;
If she takes them 3 at a time, 2 are left;
If she takes them 4 at a time, 3 are left;
If she takes them 5 at a time, 4 are left;
If she takes them 6 at a time, 5 are left;
If she takes them 7 at a time, there are none left over.
What is the minimum number of matches lined up?

## Explain your approach.



## Exercise 7 - STUCK - 7 pts

A small cube of volume $1 \mathrm{~cm}^{3}$ is wedged between large cubes.
Calculate the volume of the largest cube.


## Exercise 8 - CLEAR FOUNTAIN - 5 pts

The water from the fountain shown flows continuously. All its basins are full and overflowing. At each stage, half of the volume added to a basin flows into each of the two basins placed underneath. One cubic metre of water flows into the top basin.

Describe, in fractional form, how this cubic metre of water is distributed across the basins.

## Explain your approach.



## Exercise 9 - STICK IT ON - 7 pts

Audrey has five chopsticks of the same size as shown in the drawing below.


Each end of a chopstick is in contact with the end of another one. Points A, B and D are collinear, as are points A, E and C.

Calculate the angle at A, giving details of the calculations.

## Exercise 10 - AND ANOTHER ONE - 7 pts

Consider a right prism whose base is an equilateral triangle, and whose lateral faces are squares. All its edges are 6 cm long.

This prism is broken down into three pyramids of equal volume, two of them being identical.
The perspective drawing of such a prism and the pattern of one of the two identical pyramids is shown below.


On the answer sheet, trace, in real lengths, the net of the third pyramid.

## Exercise 11 - GARLIC! - 5 pts

Richard uses thirty whole heads of garlic each year in his cooking.
A head of garlic consists of six cloves. Each clove planted in the autumn produces a new head of garlic the following summer. He uses only his own cloves for replanting in the following season.

How many cloves will Richard have to plant to obtain, at the next harvest, a sufficient number of heads for his consumption, as well as maintaining his production level for the next harvest?

## Describe your reasoning.



## Exercise 12 - ENCIRCLED - 7 pts

Determine a positioning of the numbers $2,3,4,6,7,8$ and 12 such that the sum of the numbers placed on each of the four circles is equal to 39 .


## Exercise 13 - NO т, SHAME! - 10 pts

A rosette constructed from a regular hexagon is drawn below.


To calculate the area of the shaded region, it is cut into six pieces as shown in the sketch. Using pieces (1) - (6), we can form a rectangle.

Carry out the problem using a disc with radius $\mathbf{6 c m}$.
Stick the rectangle on the answer sheet.
Calculate the dimensions of the rectangle, and the area of the shaded region.

