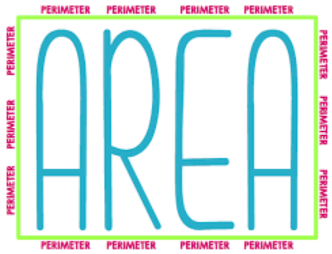
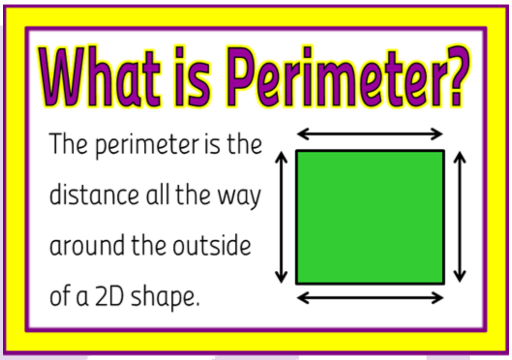
Unit 1: Numeracy (H225 74/75)



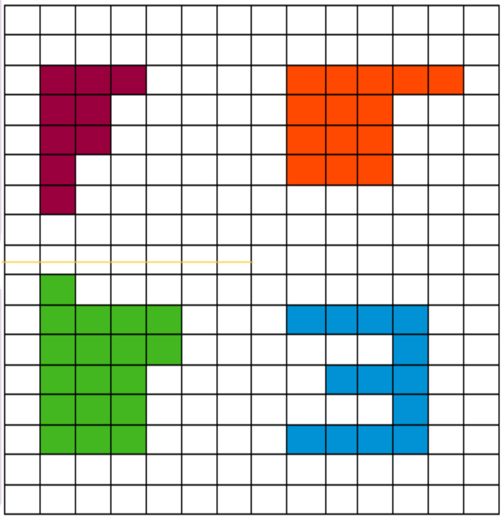
Area and Perimeter

Area: *Area is the amount of space taken up by a shape. You can tell that the picture on the right takes up more space than the picture on the left* ***but how much more space does it take up?***



As the picture on the left suggests, we use squares to answer this question.

The diagram on the right shows four shapes.



By counting the squares inside these shapes, we can identify that they have areas of 9 squares, 13 squares, 14 squares and 18 squares.

Can you match them up?

Which shape is the biggest and by how much?

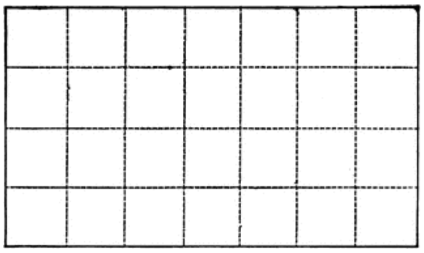
Counting squares can be time consuming and not all shapes have an exact number of squares. At N4 and N5 level, the shapes will not be drawn on a square grid and you will be expected to find their areas (the number of squares inside) using calculations.

Starting with Rectangles and Squares, it is possible to find the area of each one by **multiplying**.

If you look at this rectangle:

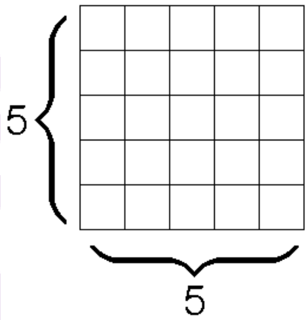
it has 4 rows of 7 squares (4×7) or

7 columns of 4 squares (7×4).



Either way, we can quickly find out that it has an area of 28 squares without the need to count them all.

Similarly, the square is 5 rows of 5 squares, so it has an area of 25 squares (5×5)



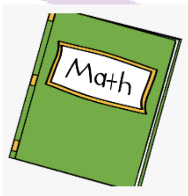
By naming the sides of a rectangle as LENGTH and BREADTH, we get the formula:

Area of a Rectangle = Length x Breadth

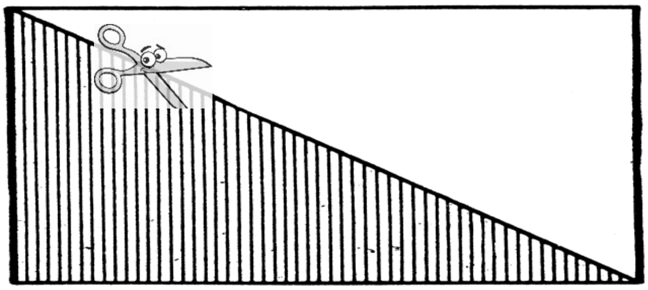
In shorthand: $A = L \times B$

There are three standard sized squares that are commonly used in N4 and N5 Applications of Mathematics to measure the area of a shape:

1. For tiny areas (e.g. a postage stamp) millimetre square - mm^2
2. Most common (e.g. text book) centimetre square - cm^2
3. Large areas (e.g. floor, wall, door) metre square - m^2

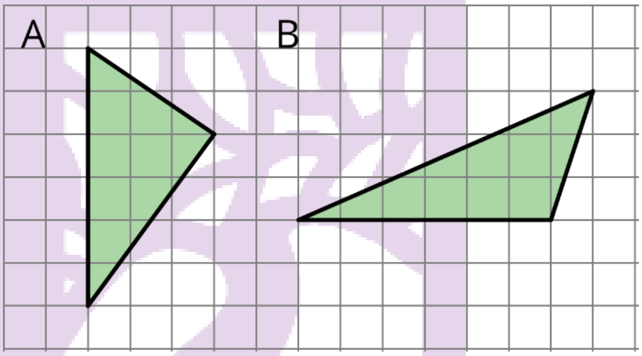


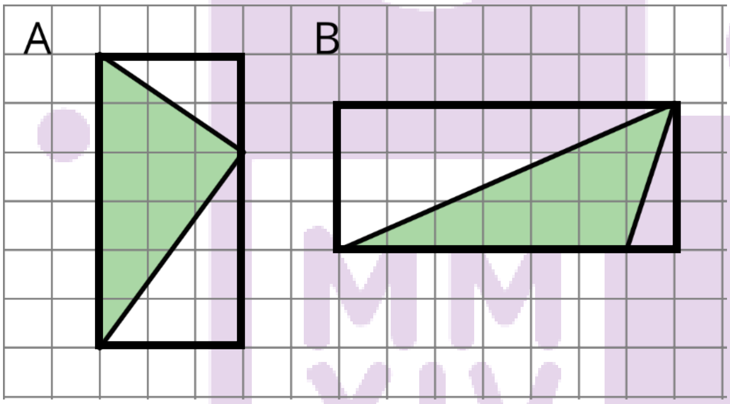
If we cut a rectangle along it's diagonal, we create two equally sized right-angled Triangles.



Thus the area of each TRIANGLE can be calculated by finding the area of the RECTANGLE first and then halving the answer.

This is true for all types of triangles. For the examples shown, counting squares is not going to be accurate enough . . .





but by drawing in the **surrounding rectangle**, we can calculate the number of squares being occupied by each shape:

Shape A - $3 \times 6 = 18$

then $18 \div 2 = 9$ squares

Shape B - $7 \times 3 = 21$

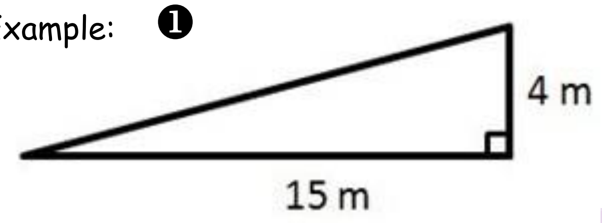
then $21 \div 2 = 10.5$ squares

Using the same names for the sides of a rectangle i.e. LENGTH and BREADTH, we get the formula:

Area of a Triangle = Length x Breadth divided by 2

In shorthand: $A = L \times B \div 2$

Example: ❶



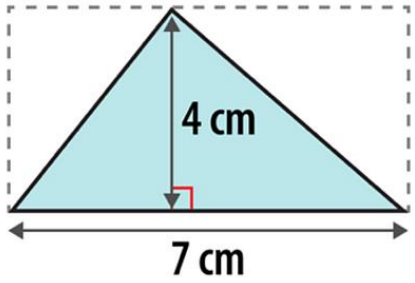
$A = L \times B \div 2$

$A = 15 \times 4 \div 2$

$A = 60 \div 2$

$A = 30\ m^2$

Example: ❷



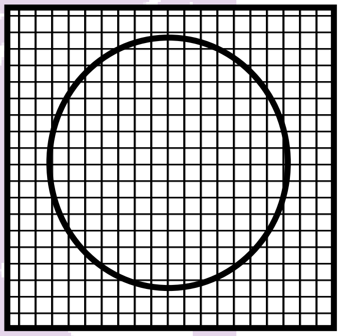
$A = L \times B \div 2$

$A = 7 \times 4 \div 2$

$A = 28 \div 2$

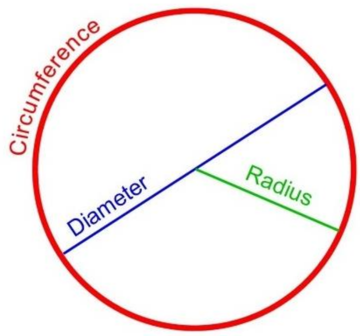
$A = 14\ cm^2$

You will also be expected to calculate the area of a circle. As you can hopefully see, counting squares is not really an option. However, at both N4 and N5 level, the formula sheet contains the required formula for us to calculate the area of a circle.

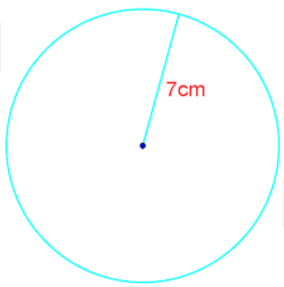


Area of a circle: $A = \pi r^2$

The A stands for Area and r stands for radius (the distance from the centre to the edge of the circle). r^2 means $r \times r$ and π (pi) stands for the number 3.14159… Almost all scientific calculators have a pi key



Example: ❶



$A = \pi r^2$

$A = \pi \times 7 \times 7$

$A = 153.93804\ldots$ *(using pi key)*

$A = 153.94\ cm^2$ *to 2 d.p.*

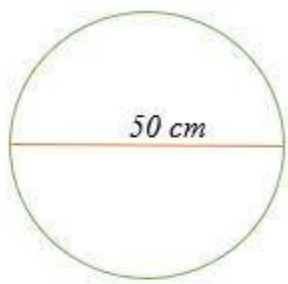
Example: ❷

$A = \pi r^2$

$A = \pi \times 25 \times 25$ *(since d=50)*

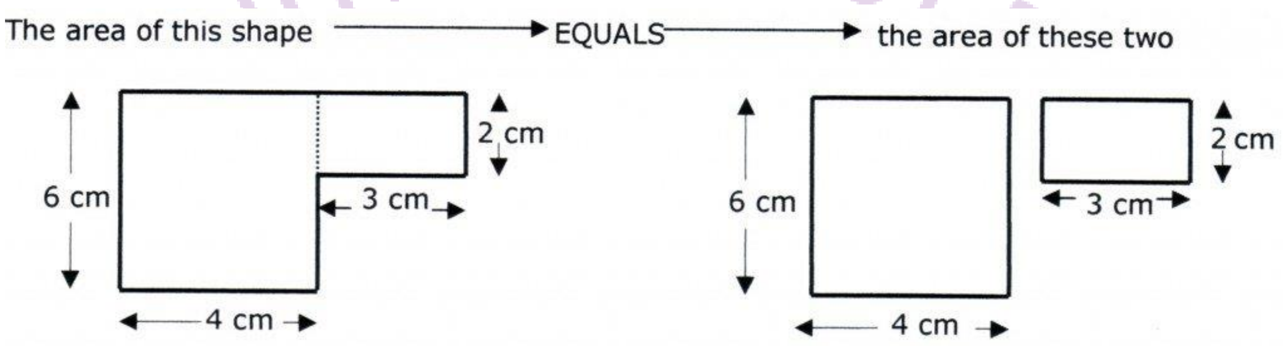
$A = 1963.495408\ldots..$

$A = 1963.5\ cm^2$ *to 1 d.p.*



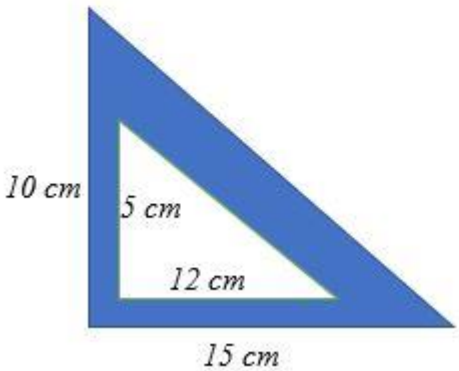
Area of Composite or Compound Shapes.

A composite/compound shape is one that is made up of two or more shapes joined together. The total area is calculated by finding the area of each part and adding them together:



The area of this shape $= (6 \times 4) + (2 \times 3)$
$= 24 + 6$
$= 30\ cm^2$

Alternatively, with a shape cut out, you would still find the area of each part but this time subtract:



Large triangle:

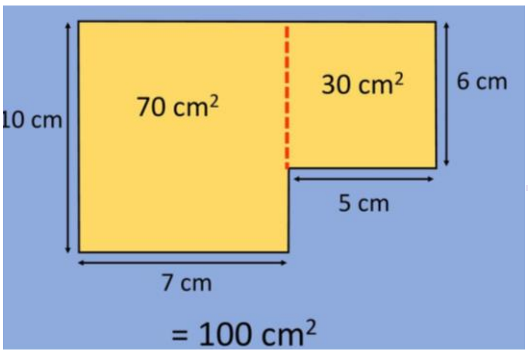
$$10 \times 15 \div 2 = 75cm^2$$

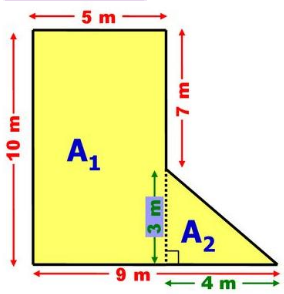
Small triangle:

$$5 \times 12 \div 2 = 30cm^2$$

Area of shaded part = $75 - 30 = 45cm^2$

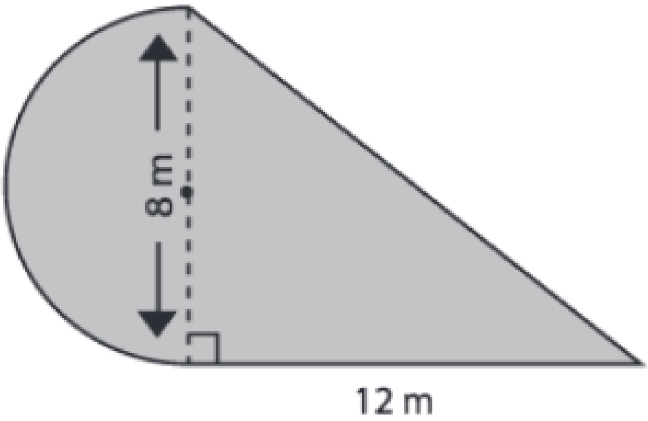
At N4 level, the shapes will be rectilinear (all straight edges):





$A_1 = 10 \times 5 = 50\ m^2$
$A_2 = 3 \times 4 \div 2 = 6\ m^2$
Total Area $= 56\ m^2$

However, at N5 level, parts of a circle can also be included:



Triangle:

$$8 \times 12 \div 2 = 48m^2$$

Semi-circle:

$$A = \pi r^2 \div 2$$

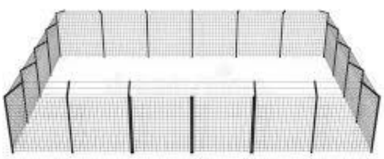
$$A = \pi \times 4 \times 4 \div 2$$

$$A = 25.1327 \ldots = 25.1m^2$$

Total Area: $48 + 25.1 = 73.1m^2$

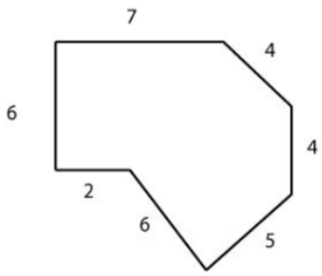
For the non-calculator paper, you would be expected to complete this calculation using the value 3.14 for π.

<u>Perimeter:</u> *Hopefully, you may have heard the word perimeter used in TV programmes and films. Expressions such as 'secure the perimeter' have been used in programmes involving armed forces and 'they set up a perimeter zone around the crime scene' in a Police series, perhaps.*

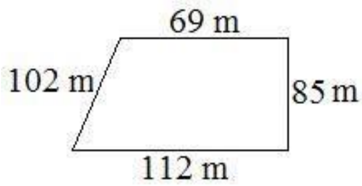


In Maths the perimeter is the ***<u>total length</u>*** *of the fence or the police tape.*

For shapes that are rectilinear (made up of straight edges), the perimeter is found by adding up the lengths of the sides:



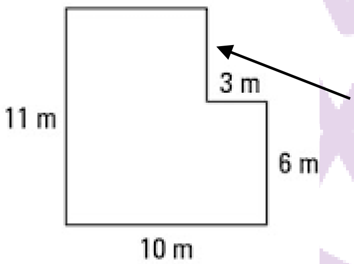
$$P = 7 + 4 + 4 + 5 + 6 + 2 + 6 = 34\ cm$$



$$P = 102 + 69 + 85 + 112 = 368\ m$$

At N4 and N5 level, however, not all the sides will be given. You will be expected to calculate the length of any missing sides before adding them up to find the perimeter of the shape:

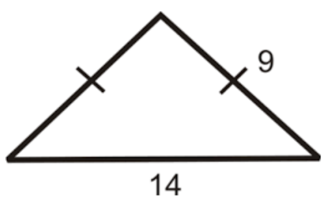
Examples: ❶



If we are to find the perimeter of the shape shown, we will need to find the length of the top edge first.
Can you see that this missing length is $7m\ (10 - 3)$*?*
There is also a missing length here.
Can you see that its length is $5m\ (11 - 6)$*?*

$$P = 11 + 10 + 6 + 3 + 5 + 7 = 42\ m$$

Example: ❷

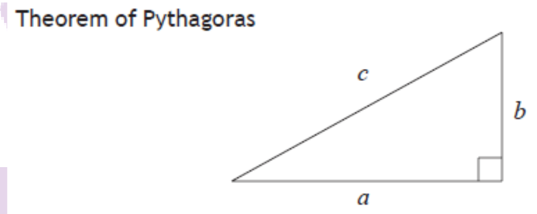


In this example the triangle is ISOSCELES – it has two sides the same length.
The two sides marked with \ and / are the same length.
Can you see that the missing side has length 9*?*

$$P = 14 + 9 + 9 = 32\ units$$

More complicated examples require the use of the Theorem of Pythagoras.

The theorem is given in the formulae list for both N4 and N5 as:

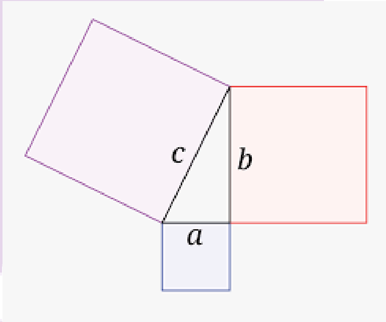


$$a^2 + b^2 = c^2$$

The Theorem is a formula that is used to find the missing side of a right-angled triangle when we know the lengths of the other two sides.

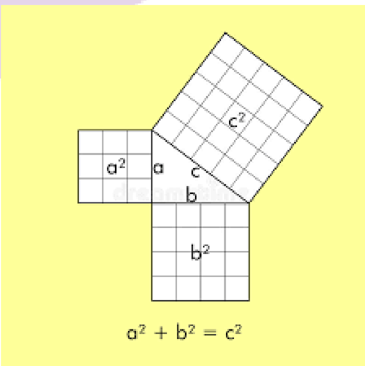
The side labelled c has is known as the HYPOTENUSE. It is the longest side. Pythagoras was a Greek Mathematician and was one of the first people to document that for any right-angled triangle:

If we draw a square on each side of the right-angled triangle then the area of the largest square is always the same as the area of the two smaller squares added together:

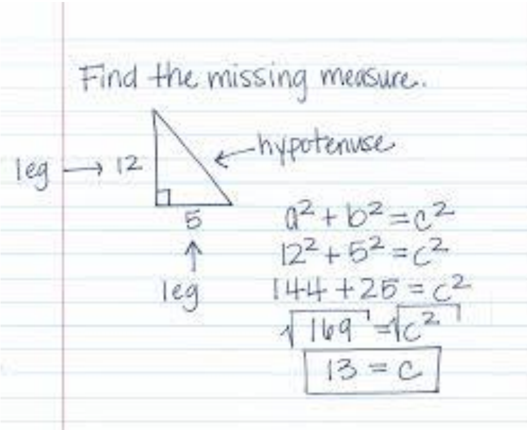


By counting squares, can you see that the smaller squares opposite have an area of 9 and 16? Can you also see that the large square has an area of 25 and that:

$$9 + 16 = 25$$

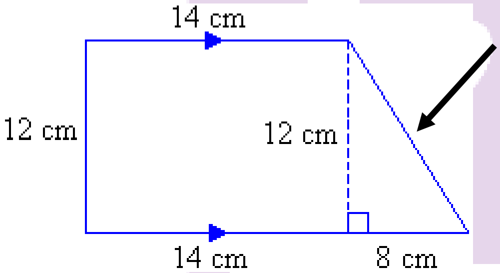


Pythagoras Calculations: *Pythagoras Calculations are generally set out as follows:*



- The square on the left side would be $12 \times 12 = 144$
- The square on the bottom side would be $5 \times 5 = 25$
- We can now work out that the bigger square must have an area of 169 since $144 + 25 = 169$
- If the large square has an area of 169 then we can use the √ key to find the length of its side i.e. $\sqrt{169} = 13$

Examples: ❶



If we are to find the perimeter of the shape shown, we will need to find the length of the missing side first.

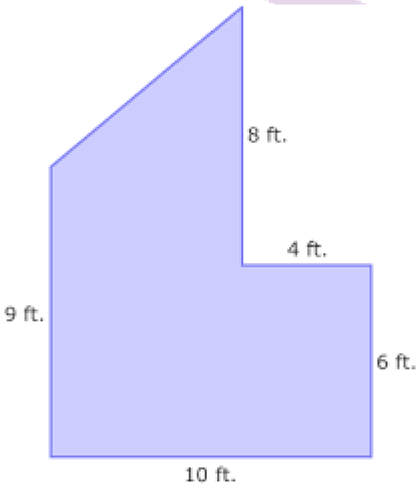
This missing length is can be calculated by substituting 8 and 12 into a Pythagoras calculation.

$$a^2 + b^2 = c^2$$
$$12^2 + 8^2 = c^2$$
$$144 + 64 = c^2$$
$$\sqrt{208} = \sqrt{c^2}$$
$$14.4 = c$$

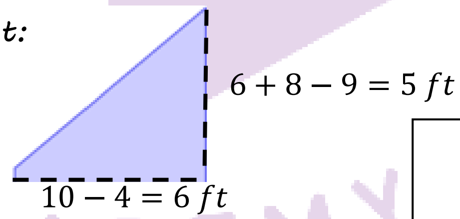
There allows us to calculate the perimeter:

$$P = 8 + 14 + 12 + 14 + 14.4 = 62.4\ cm$$

Example: ❷



In this example, we are not given the values for the Pythagoras calculation. We need to figure them out:

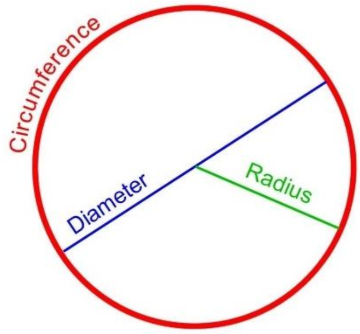


$$a^2 + b^2 = c^2$$
$$5^2 + 6^2 = c^2$$
$$25 + 36 = c^2$$
$$\sqrt{61} = \sqrt{c^2}$$
$$7.8 = c$$

$$P = 8 + 4 + 6 + 10 + 9 + 7.8 = 44.8\ ft.$$

Circumference of a Circle: *The perimeter of a circle is known as its CIRCUMFERENCE. A circle only has one side, and it would be difficult to measure it accurately with a ruler! As with the area of a circle, there is a formula for us to calculate the circumference of a circle.*

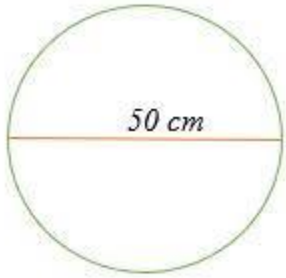
Circumference of a circle: $C = \pi d$



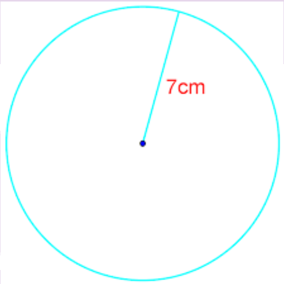
The C stands for Circumference and d stands for diameter (the distance across the circle passing through the its centre). π (pi) stands for the number 3.14159... Almost all scientific calculators have a pi key.

Example: ❶

$C = \pi d$
$C = \pi \times 50$
$C = 157.07963 \ldots\ldots$
$C = 157.1\ cm$ *to 1 d.p.*

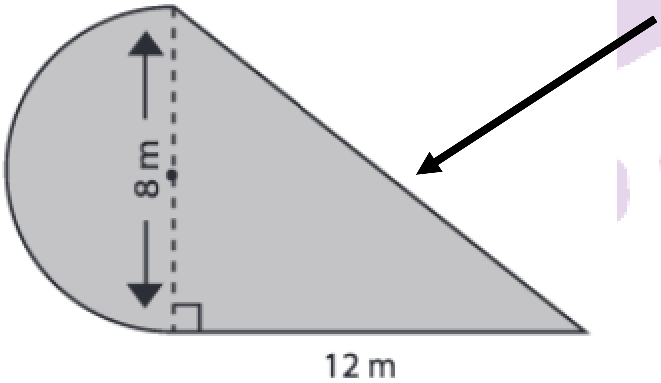


Example: ❷



$C = \pi d$
$C = \pi \times 14$ *(since r=7)*
$C = 43.982297 \ldots$ *(using pi key)*
$C = 43.98\ cm$ *to 2 d.p.*

Similar to AREA, at N4 level, the shapes will be rectilinear (all straight edges) but may have some sides missing and likewise, at N5 level, parts of a circle can also be included:



Hypotenuse:

$$a^2 + b^2 = c^2$$
$$12^2 + 8^2 = c^2$$
$$144 + 64 = c^2$$
$$\sqrt{208} = \sqrt{c^2}$$
$$14.422 \ldots = c$$
$$c = 14.4\ m$$

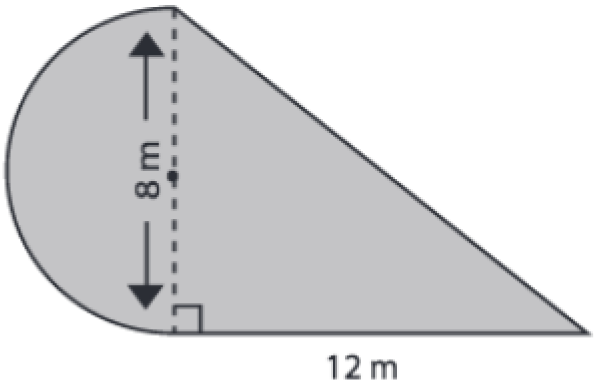
Semi-circle:

$C = \pi d \div 2$

$C = \pi \times 8 \div 2$

$C = 12.56637 \ldots = 12.6\, m$

Total Perimeter: $14.4 + 12.6 = 27\, m$



For the N5 non-calculator paper, you would be expected to complete this calculation using the value 3.14 for π.