Unit 1: Numeracy (H225 74/75)

Fractions

Fraction of an amount: *To calculate a fraction of an amount we divide the amount by the denominator (bottom value) then multiply the answer by the numerator (top value):*

e.g. $\frac{2}{3}\ of\ £18 = £18\ \div 3 \times 2 = £12$

$$\frac{4}{5}\ of\ 25kg = 25kg\ \div 5 \times 4 = 20kg$$

Equal Fractions: *Groups of equal fractions can be created by multiplying the numerator and denominator by the **same** value:*

e.g. $\frac{3}{5}\frac{\times 2}{\times 2} = \frac{6}{10}$ $\quad \frac{6}{10}\frac{\times 3}{\times 3} = \frac{18}{30}$ $\quad \frac{18}{30}\frac{\times 5}{\times 5} = \frac{90}{150}$

Simplify a Fraction: *When we divide the numerator and denominator by the **same** value, this is known as simplifying the fraction:*

e.g. $\frac{3}{9}\frac{\div 3}{\div 3} = \frac{1}{3}$ $\quad \frac{6}{8}\frac{\div 2}{\div 2} = \frac{3}{4}$ $\quad \frac{20}{50}\frac{\div 10}{\div 10} = \frac{2}{5}$

Add/Subtract Fractions: *We can only add or subtract fractions if they have the same denominator. If the denominators are not the same, then we use **equal fractions** (see above) to make them the same:* e.g. $\frac{1}{2} + \frac{1}{3}$

- *We use the lcm (lowest common multiple) of the denominators to identify the new denominator. In the above example the lcm of 2 and 3 is 6 as 6 is the smallest number that appears in both the 2- and 3-times tables.*
- $\frac{1}{2}\frac{\times 3}{\times 3} = \frac{3}{6}$ *and* $\frac{1}{3}\frac{\times 2}{\times 2} = \frac{2}{6}$
- *Now that the denominators are the same, we can make the calculation:*

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Percentage Fractions: *Percentages are a special fraction that mean out of 100. The % symbol is based on a 1 and two 0s* $^{0}{}_{0}$:

e.g. $23\% = \frac{23}{100}$ $\quad 11\% = \frac{11}{100}$ $\quad 71\% = \frac{71}{100}$

Percentage of an Amount: *To calculate a percentage of an amount we convert the percentage to a fraction then divide the amount by the denominator (always 100) and multiply the answer by the numerator (top value)*:

e.g. $7\% \; of \; £800 = \frac{7}{100} \; of \; £800 = £800 \div 100 \times 7 = £56$

$11\% \; of \; 200kg = \frac{11}{100} \; of \; 200kg = 200kg \div 100 \times 11 = 22kg$

- *There are many percentages that when they are written as a fraction can be simplified.*
- *Simplifying the fraction first, can make the calculation easier for us:*

e.g. $30\% \; of \; £80 = \frac{30}{100} \; of \; £80$ *but* $\frac{30}{100}$ *can be simplified* $\frac{30}{100} \frac{\div 10}{\div 10} = \frac{3}{10}$

We can now change the calculation to $\frac{3}{10} \; of \; £80 = £80 \div 10 \times 3 = £24$

- *Below is a table of commonly used percentages in society and their equivalent simplified fraction. Try to learn these:*

Percentage	10%	20%	25%	$33\frac{1}{3}\%$	50%	$66\frac{1}{3}\%$	75%	100%
Simplified Fraction	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	1

Increase/Decrease an amount by a Percentage: *To perform this type of calculation, we find the percentage of the amount then add (if it's an increase) or subtract (if it's a decrease) this amount from the original value:*

Examples: ❶ Find the new price of a pair of jeans costing £25 that are in a sale with 10% off.

$$10\%\ of\ £25 = \frac{1}{10}\ of\ £25 = £25\ \div 10 \times 1 = £2.50$$

New price $= £25 - £2.50 = £22.50$

❷ A 500ml bottle of juice is offered in a new size bottle with 20% extra free. Calculate how much juice is in the new bottle.

$$20\%\ of\ 500ml = \frac{1}{5}\ of\ 500ml = 500ml\ \div 5 \times 1 = 100ml$$

New volume of juice $= 500ml + 100ml = 600ml$

Percentage Change: *To calculate the percentage change between two quantities, we use the formula:*

$$\%\,Change = \frac{difference}{original\ amount} \times\ 100$$

Examples: ❶ A shop increases the cost of a camera from £350 to £420. Express this increase as a percentage.

$$difference = £420 - £350 = £70$$

$$\%\,Change = \frac{difference}{original\ amount} \times\ 100$$

$$\%\,Change = \frac{70}{350} \times\ 100 = 70 \div 350 \times\ 100 = 20\%$$

❷ A car costing £5000 was sold a year later for £4750. Express this loss as a percentage.

$$difference = £5000 - £4750 = £250$$

$$\%\,Change = \frac{difference}{original\ amount} \times\ 100$$

$$\%\,Change = \frac{250}{5000} \times\ 100 = 250 \div 5000 \times\ 100 = 5\%$$

Decimal Fractions: *This is another way that we can write a fraction. It is based on the columns we learned at primary school – Thousands, Hundreds, Tens, Units, Tenths, Hundredths, etc:*

Units 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
0	•	2		
1	•	3	5	
1	•	8		
4	•	2	0	5

e.g. $0.2 = \frac{2}{10}$ $\quad$ $0.07 = \frac{7}{100}$ $\quad$ $0.37 = \frac{37}{100}$

We can be asked at N5 level to convert any fraction to a decimal as well. To complete this we need to remember that the line that separates the numerator (top value) and the denominator means divide:

Examples: ❶ Convert $\frac{5}{8}$ to a decimal fraction.

$$\frac{5}{8} = 5 \div 8$$

Using the 'bus shelter' technique and adding a decimal point and some zeros:

$$\begin{array}{r|l} & 0.\;\;6\;\;2\;\;5\;\;\cancel{} \\ \hline 8 & 5.\;\;{}^{5}0\;\;{}^{2}0\;\;{}^{4}0\;\;\cancel{0} \end{array}$$

NB – the last zero was not required

$$\frac{5}{8} = 0.625$$

❷ Convert $\frac{4}{7}$ to a decimal fraction to 3 decimal places.

$$\frac{4}{7} = 4 \div 7$$

We need four zeros so we can round to 3 dp:

$$\begin{array}{r|l} & 0.\;\;5\;\;7\;\;1\;\;4 \\ \hline 7 & 4.\;\;{}^{4}0\;\;{}^{5}0\;\;{}^{1}0\;\;{}^{3}0 \end{array}$$

$$\frac{4}{7} = 0.5714\ldots\ldots = 0.571$$

Using decimals is a powerful way in which we can perform complicated percentage calculations much more efficiently.

<u>Using Decimals to Perform Percentage Calculations:</u> *Instead of converting the percentage to a fraction and dividing by the denominator and multiplying the answer by the numerator, we can convert the percentage to a decimal and multiply. Using the same examples above:*

Percentage of an Amount:

$$11\%\ of\ 200kg = 200kg \times 0.11 = 22kg$$

$$7\%\ of\ £800 = £800\ \times 0.07\ = £56$$

Increase/Decrease an amount by a Percentage:

Examples: ❶ Find the new price of a pair of jeans costing £25 that are in a sale with 10% off.

$$100\% -\ 10\% = 90\% = 0.9$$

$$\text{New price} = £25 \times 0.9 = £22.50$$

❷ A 500ml bottle of juice is offered in a new size bottle with 20% extra free. Calculate how much juice is in the new bottle.

$$100\% +\ 20\% = 120\% = 1.2(0)$$

$$\text{New volume of juice} = 500ml \times 1.2 = 600ml$$

<u>Repeated Percentage Change (Compound Interest):</u> *This is when we are required to perform the same percentage calculation several times*

Examples: ❶ £500 is invested with a bank that is offering an interest rate of 2% p.a. (per annum/year) for 3 years

	Old Method:	New Method:
Year 1	$2\%\ of\ £500 = \frac{2}{100}\ of\ £500$ $= £500\ \div 100 \times 2 = £10$ New balance $= £500 + £10 = £510$	$100\% +\ 2\%\ = 102\% = 1.02$ New balance $= £500\ \times 1.02 = £510$
Year 2	$2\%\ of\ £510 = \frac{2}{100}\ of\ £510$ $= £510\ \div 100 \times 2 = £10.20$	$100\% +\ 2\%\ = 102\% = 1.02$ New balance $= £510\ \times 1.02 = £520.20$

	New balance $= £510 + £10.20 = £520.20$	
Year 3	$2\%\ of\ £520.20 = \frac{2}{100}\ of\ £520.20$ $= £520.20 \div 100 \times 2 = £10.40$ New balance $= £520.20 + £10.40 = £530.60$	$100\% + 2\% = 102\% = 1.02$ New balance $= £520.20 \times 1.02 = £530.60$

- *As you can see above, using decimals saves a line of working for each year the money was invested.*
- *Since we are multiplying by the same decimal each time, we can use indices to shorten these calculations even further:*

$$100\% + 2\% = 102\% = 1.02$$
$$\text{New balance} = £500 \times 1.02^3 = £530.60$$

❷ A car costing £25000 depreciates (decreases in value) at a rate of 7.1% per year. Calculate how much the car is worth in 7 years.

$$100\% - 7.1\% = 92.9\% = 0.929$$

$$\text{New car value} = £25000 \times 0.929^7 = £14929.6630\ldots = £14929.66$$

<u>Reverse Percentage Change:</u> *This is when we are given the new value along with the percentage change and are required to calculate the original amount. Once again, using decimals allows us to make this type of calculation more efficiently. Using the same examples as in "Increase/Decrease an amount by a Percentage" but turned around:*

Examples: ❶ After a 10% discount, the new price of a pair of jeans is £22.50. Find the original price.

$$100\% - 10\% = 90\% = 0.9$$
$$\text{Original price} = £22.50 \div 0.9 = £25$$

❷ A new size bottle of juice with 20% extra free. If the new bottle holds 600ml of juice, how many millilitres of juice did the old bottle hold?

$$100\% + 20\% = 120\% = 1.2$$
$$\text{Original bottle} = 600ml \div 1.2 = 500ml$$

Ordering Fractions: *It is not always obvious which fraction is the bigger when they are in numerator/denominator format (e.g. $\frac{3}{8}$ or $\frac{2}{5}$). Similarly, when written in decimal format (e.g. 0.78 or 0.8). It is much more common to convert all the types of fractions into percentages with this type of question and then deciding.*

e.g. Arrange in order of size: $0.388, \frac{3}{8}, 38.3\%, 0.39$

- $0.388 = 38.8\%$ by multiplying the decimal by 100
- $\frac{3}{8} \frac{\times 125}{\times 125} = \frac{375}{1000}$ using "equal fractions" and getting 10, 100, 1000, etc as the new denominator.

  Using the columns to convert to a decimal, we get $\frac{375}{1000} = 0.375$

  $0.375 = 37.5\%$ by multiplying by 100
- 38.3% - already in percentage format
- $0.39 = 39\%$ by multiplying the decimal by 100

The question now becomes:

Arrange in order of size: $38.8\%, 37.5\%, 38.3\%, 39\%$

This gives 37.5%, 38.3%, 38.8%, 39% from smallest to largest.

Don't forget to write the answer in the original format:

$$\frac{3}{8}, \quad 38.3\%, \quad 0.388, \quad 0.39$$