GEOMETRY & MEASURE

TOPICS

- Using formulae
- Calculating gradient
- Shape
  - Perimeter
  - Area
  - Volume
- Speed, distance and time
- Pythagoras
- Container packing
- Precedence tables
- Scale drawings
- Bearings

STARTER

1. Michael researches the cost of a turkey in some local butcher shops:

Cost (£)	15	18	23	19	31	18	30

a) Calculate the mean and standard deviation for the butcher shop costs.

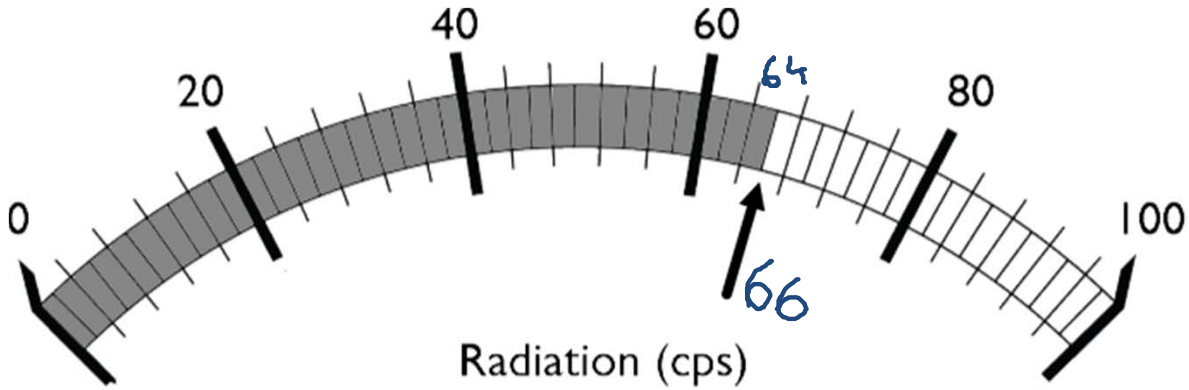
b) In supermarkets, a similar survey found that the mean price of a turkey was £19 and the standard deviation was £1·70.

Make two comments comparing the prices of turkeys between butcher shops and supermarkets.

2. A machine is used to measure radiation. The units of the reading are 'cps'.

An item is tested for radiation. The diagram below shows the reading on the machine. If the reading is over 65 cps, the item is classed as dangerous.

Is the item dangerous?



FORMULAE

GEOMETRY & MEASURE

FORMULAE

- *In this topic you will be assessed on how you substitute values into a given formula.*

When we **substitute** we are replacing letters in a formula for given numbers.

You have already used different formulae:

- Area of a circle = π x radius^2
- Distance = Speed x Time
- Volume cuboid = length x height x breadth

It is important to follow rules of bodmas when using formula.

EXAMPLES

1) Calculate the area of a circle using the formula $A = \pi r^2$ when the r = 3cm.

$A = \pi \times 3^2$

$= \pi \times 9$

$= 28.27$

2) Body Mass Index is given by the formula $\text{BMI} = \frac{weight\ (kg)}{height\ (m)^2}$.

a) Find the BMI of someone weighing 45kg with a height of 154cm.

$= 1.54m$

$\text{BMI} = \frac{45}{1.54^2}$

$= 18.97$

b) Find the weight of a person with a BMI of 22 and a height of 1m 76cm.

$22 = \frac{\text{weight}}{1.76^2}$

$22 \times 1.76^2 = 68.1472$

$= 68.15\ kg$

3) The speed, *v* m/s, of a car accelerating is given by the formula v = u + at, where u = initial speed in m/s, a = acceleration in m/s^2 and t = time in seconds.

Calculate the speed when u = 15 m/s, a = 0.2 m/s^2 and t = 30 seconds.

$v = u + at$
$= 15 + 0.2 \times 30$
$= 15 + 6$
$= 21$ m/s

4) It is important for new drivers to learn about the overall stopping distance when driving their car. Stopping distance is a combination of 'thinking' distance and braking distance.

$$\text{stopping distance} = \text{thinking distance} + \frac{speed^2}{20}$$

On wet roads this is multiplied by 2 and in icy condition the stopping distance is 10 times greater.

Calculate:

- Stopping distance for a car travelling at 36mph when raining.

Thinking distance is 30ft.

$S.d = 30 + \frac{36^2}{20}$
$= 30 + 64.8 = 94.8$ ft.

In rain
$= 94.8 \times 2 = 189.6$ ft

- Stopping distance for a car travelling at 63mph in icy weather.

Thinking distance is 60ft.

$S.d = 60 + \frac{63^2}{20}$
$= 60 + 198.45$
$= 258.45$

Icy conditions
$= 258.45 \times 10$
$= 2584.5$ ft.

STARTER

1. Using the formula $a = \sqrt{8b + c^2}$ work out a when b = -3 and c = 5.

2. Jenny works as a travel agent. She lives in Musselburgh and needs to catch the train to work from Haymarket Station.

   It takes her 15 minutes to walk from her flat to Musselburgh Police Station where she catches the bus. She gets off the bus at Haymarket Station.

   The bus timetable is shown below. She must be at Haymarket Station by 7:25 am.

   What is the latest time she can leave her flat to make it to the station on time?

Tranent Windygoul	-	-	-	0455	-	0519	-	0543	-	0603	-	0621	-	0637	-	
Johnny Cope Stone	-	-	-	0505	-	0529	-	0553	-	0613	-	0631	-	0647	-	
Seton Sands	-	0436	0451	|	0515	|	0539	|	0601	|	0621	|	0637	|	0653
Prestonpans Ayres Wynd	-	0445	0500	0512	0524	0536	0548	0600	0610	0620	0630	0638	0646	0654	0702	
Levenhall Roundabout	-	0451	0506	0518	0530	0542	0554	0606	0616	0626	0636	0644	0652	0700	0708	
Musselburgh Police Station	-	0457	0512	0524	0536	0548	0600	0612	0622	0633	0643	0651	0659	0707	0715	
Eastfield (Musselburgh Road)	0440	0503	0518	0530	0542	0554	0606	0618	0628	0640	0650	0658	0706	0714	0722	
Portobello King's Road	0446	0510	0525	0537	0549	0601	0613	0625	0636	0648	0658	0706	0714	0722	0730	
Meadowbank House	0453	0517	0532	0544	0556	0608	0620	0632	0643	0655	0705	0713	0721	0729	0739	
Leopold Place	0459	0523	0538	0550	0602	0614	0626	0639	0650	0702	0712	0720	0728	0737	0747	
Haymarket Station	0513	0537	S	0604	0616	0628	0641	0654	0705	0717	0727	0736	0746	0756	0807	
Drum Brae Roundabout	0523	0547	-	0614	0626	0639	0652	0705	0716	0728	0739	0748	0759	0809	0820	
Clerwood	0531	0555	-	0622	0634	0647	0700	0713	0724	0737	0748	0758	0809	0819	0830	

STARTER

1. A cylinder box of biscuits with a radius of 10cm has a volume of 2198cm^2. Use the formula $h = \frac{V}{\pi r^2}$ to work out the height.

$h = \frac{2198}{\pi \times 10^2}$

$= 6.999\ldots = 7\text{cm}$

2. As part of Jenny's job she often has to call offices in different countries. She uses the world clock app on her phone to check the local times for the offices she must call. The following times are displayed below.

Glasgow	12:04	
Honolulu	04:04	− 8hr
New York	07:04	− 5hr
Tokyo	21:04	+9hr

All offices are open 8:30 am – 7 pm. Tomorrow she must call all three offices. She has planned to call the offices at the times shown below.

09:35 – Tokyo, 18:35 ✓ 13:15 – New York, 08:15 ✗ 16:45 – Honolulu 08:45 ✓

Which of the three business calls will she be able to make? Explain your answer.

GRADIENT

GEOMETRY & MEASURE

GRADIENT

- *In this topic you will be assessed on you calculate gradient and apply it to problems where gradient is a limitation or requirement.*

The gradient of a slope is a measure of the steepness. It is found by comparing vertical height to horizontal height.

The gradient formula is:

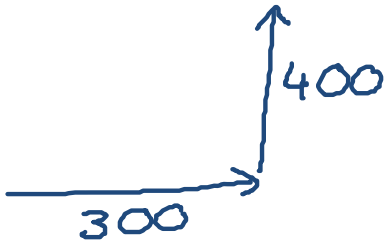
$$Gradient = m = \frac{vertical\ change}{horizontal\ change} = \frac{rise}{run}$$

Gradients can be given as whole numbers, decimals, fractions and percentages.

EXAMPLES

1. A resort grades its ski slopes according to their gradient.
One slope rises by 400m over a distance of 300m.
How would the resort classify the slope?

Green	Less than 20%
Blue	20 - 40%
Red	Over 40%

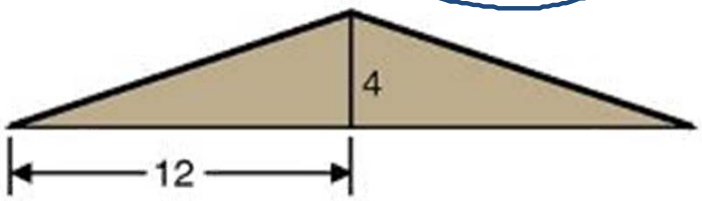


$m = \frac{\text{rise}}{\text{run}} = \frac{400}{300} = 1.3$

$= 130\%$

This slope is a Red slope.

2. Building regulations require a roof to have a minimum gradient of 0·3 ± 15%.
Does the roof meet the requirements?



15% of 0·3

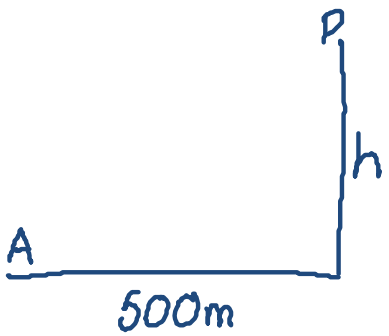
$= 0.3 \div 100 \times 15 = 0.045$

$0.3 - 0.045 = 0.255$ ← minimum gradient

$m = \frac{\text{rise}}{\text{run}} = \frac{4}{12} = 0.333...$

As $0.333 > 0.255$ it meets requirements.

3. Planes must land at no more than a 2·5% gradient. If a plane is landing at 2·0% and is 500m away from the airport, what is its height off the ground?



$2\% = 0.02$

$0.02 = \frac{h}{500}$

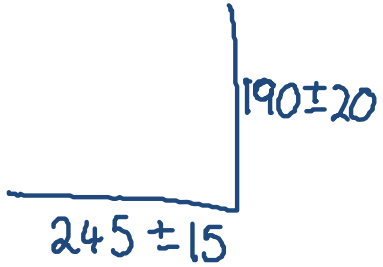
$500 \times 0.02 = h$

height = 10m

4. A carpenter is making stairs. Each stair has a rise of (190 ± 20)mm and a run of (245 ± 15)mm.

What is the maximum gradient of the staircase?

↘ when height is at max and width is at min



$m = \frac{190+20}{245-15} = \frac{210}{230} = 0.913\ldots = \underline{\underline{0.91}}$

STARTER

Emma and Andrew want to buy a new bathroom suite.
To pay for the new suite they need to borrow £4500.

They are offered two different repayment options at a fixed simple interest rate of 12.5% per annum:

	Loan term	Monthly repayment
Option 1	12 months → 1 year	£421.88
Option 2	24 months → 2 year	£234.38
Option 3	36 months → 3 year	£171.88

Calculate the monthly repayments for each of the loan options.

12.5% = 4500 ÷ 100 × 12.5 = £562.50

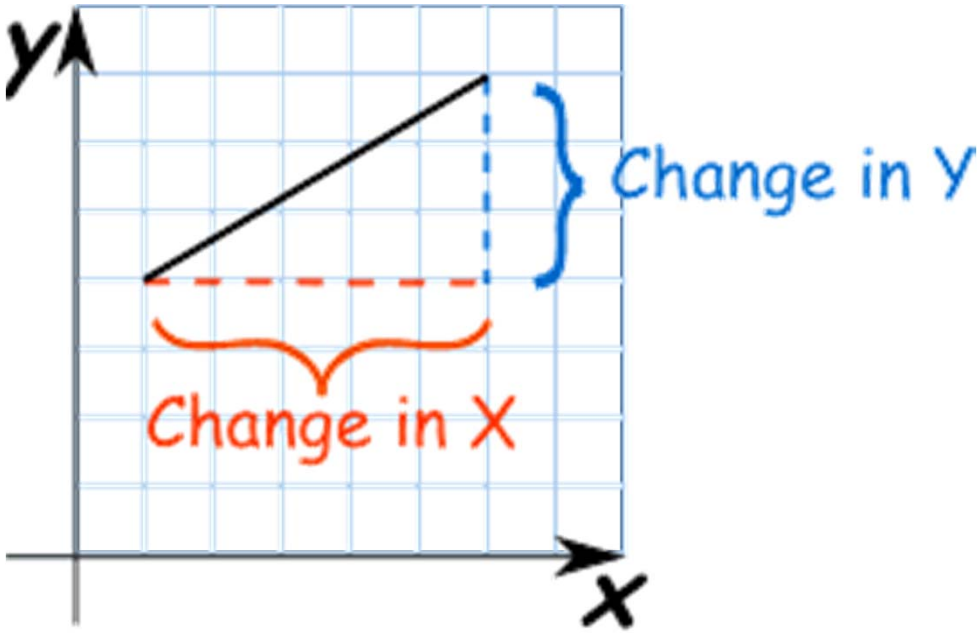
Op 1: £562.50 + 4500 = 5062.50 ÷ 12 = 421.875

Op 2: £1125 + 4500 = 5625 ÷ 24 = 234.375

Op 3: £1687.50 + 4500 = 6187.50 ÷ 36
= 171.875

GRADIENT WITH COORDINATES

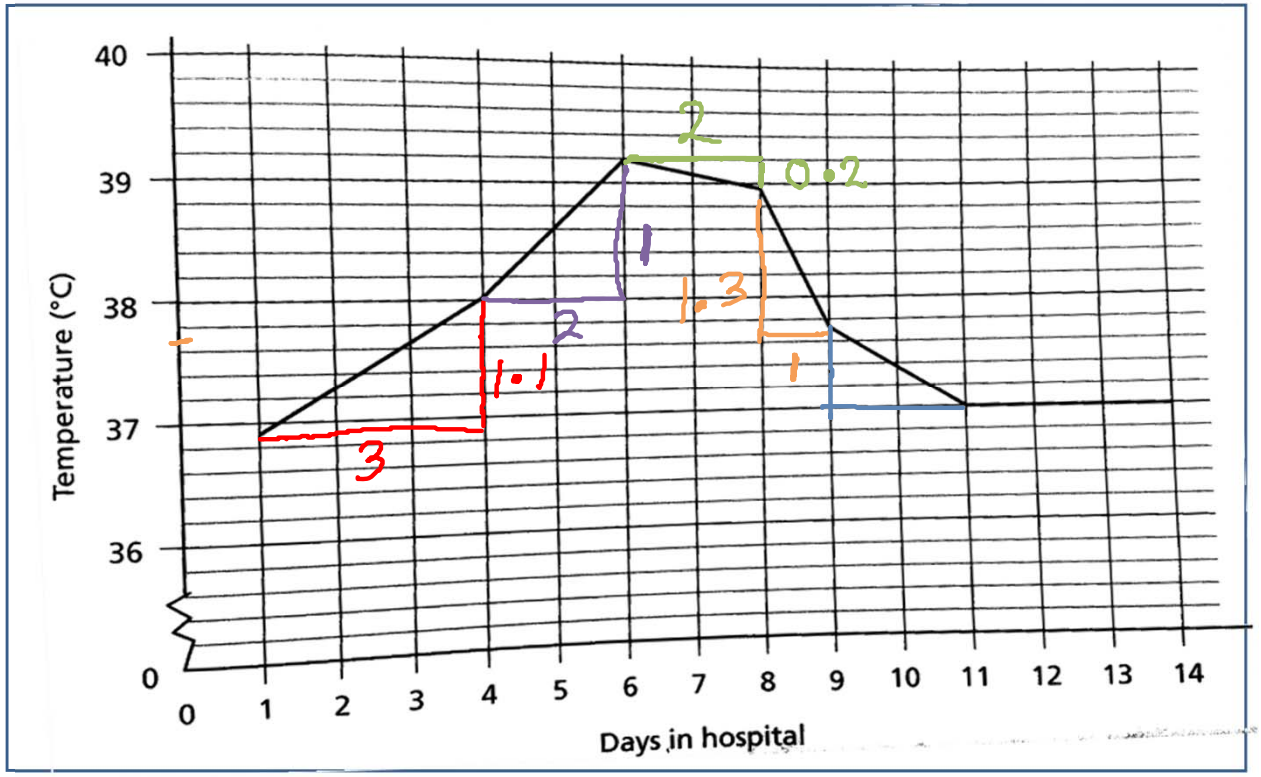
- *You need to be able to read a vertical and horizontal distance using a graph.*
- *You will be asked to make comparisons between different sections of a graph or chart*



EXAMPLE

The chart show's Harry's temperature during a 2 week hospital stay.

a) Calculate the gradient of each section.



Day 1 - 4

$m = \frac{1.1}{3}$

$= 0.37$

Day 4 - 6

$m = \frac{1}{2}$

$= 0.5$

Day 6 - 8

$m = \frac{0.2}{2}$

$= 0.1$

Day 8 - 9

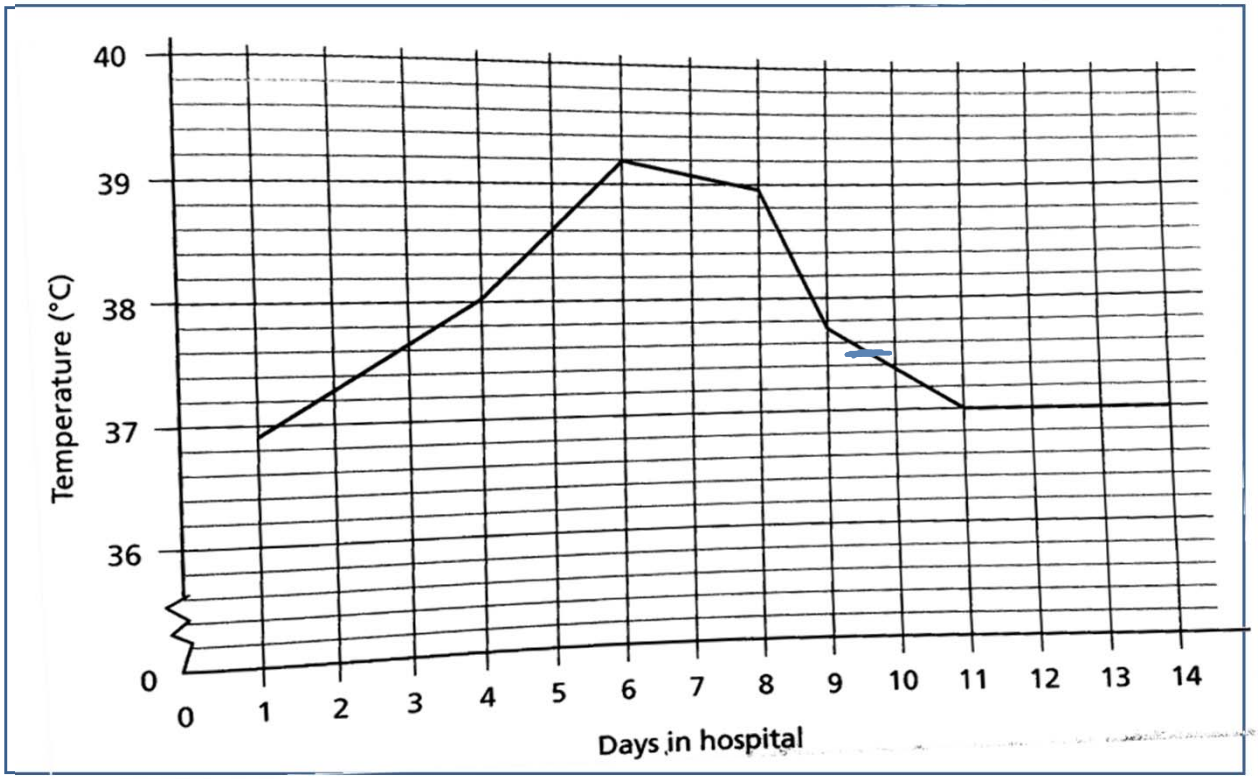
$m = \frac{1.3}{1}$

$= 1.3$

Day 9 - 11

$m = \frac{0.7}{2}$

$= 0.35$



b) Between which days did his temperature increase the most?

Day 4–6

c) What does the gradient indicate in this chart?

Rate of temperature change

d) Harry was deemed to be healthy when his temperature fell below 37.5°C. On what day was this?

During Day 9

e) Explain why the gradient of the last three days was zero.

His temperature didn't change.

STARTER

1. Lola earns £760 gross pay each week.

   Each week she pays 20% of her gross pay in tax and 6% of her gross pay in National Insurance. She also pays £8 to her company's Christmas club.

   Calculate Lola's net weekly pay.

20% of £760 = £152

10% of 760 = 76

1% of 760 = 7·60

6% = 45·60

Total deductions = 152 + 45·60 + 8 = 205·60

Net pay = 760 − 205·60

= £554·40

```
  5 9
 760.00
-205.60
 554.40
```

```
   7.60
 x    6
 45.60
 3
```

```
  152
   45.60
 1 1 8
 205.60
```

SHAPE - PERIMETER

GEOMETRY & MEASURE

PERIMETER

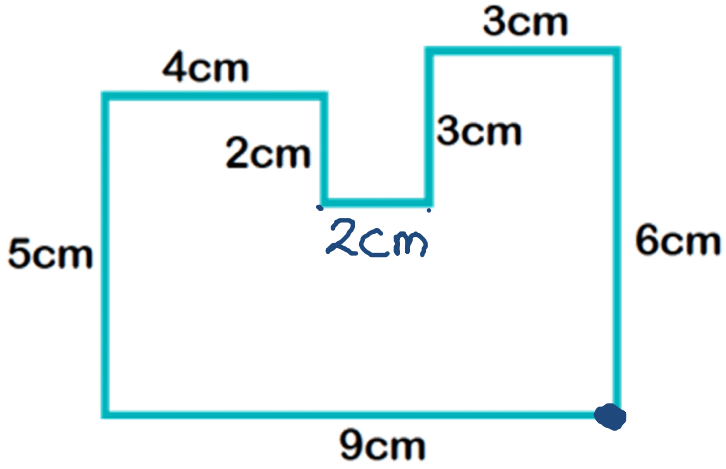
The perimeter of a shape is the total length around its sides.

Can you find the perimeter of this shape?

$9 + 5 + 4 + 2$

$+ 2 + 3 + 3 + 6$

$= 34\text{cm}$

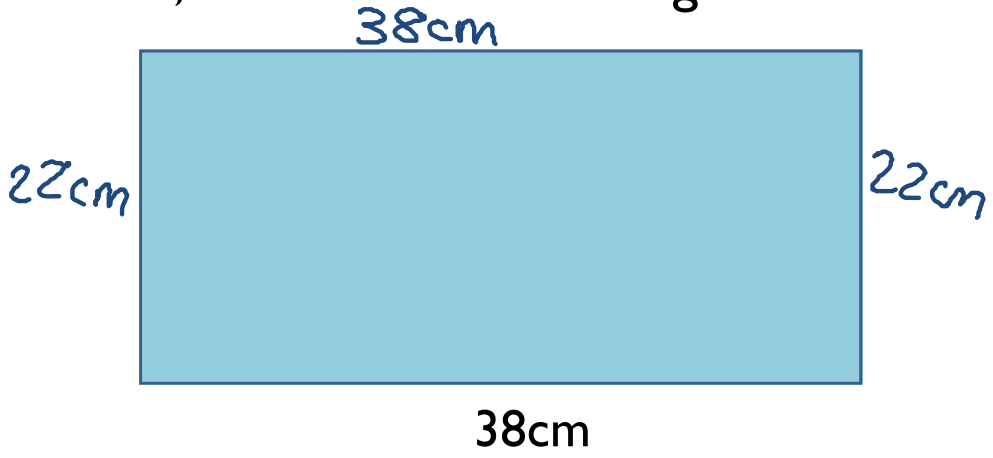


If the perimeter of this rectangle is 120cm, work out the missing side.

$38 + 38 = 76$

$120 - 76 = 44$

$44 \div 2 = 22\text{cm}$



CIRCUMFERENCE

The perimeter of a circle is known as the *circumference*

The circumference has a special formula:

$$C = \pi D$$

Circumference = π x Diameter

If you don't have a calculator $\pi = 3.14$ approximately.

(without calc)

$C = \pi D$

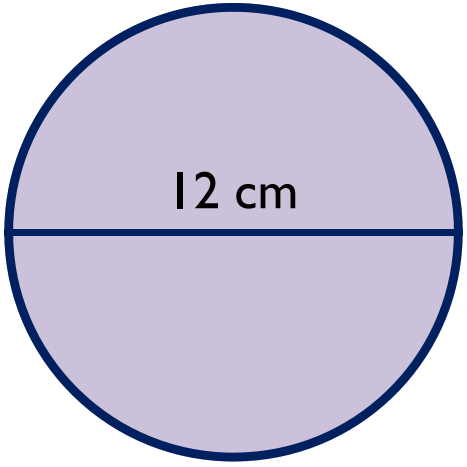
$= \pi \times 12$

$= 37.68\text{cm}$

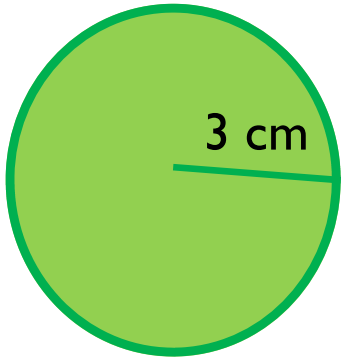
x 10
x 2
+ together.

3.14×10
$= 31.4$

$3.14 \times 2 = 6.28$



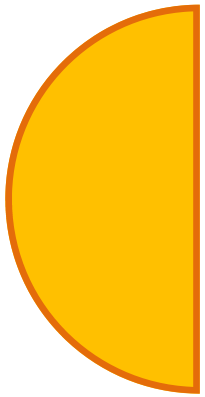
Find the perimeter of these shapes:



$C = \pi \times 6$

$= 18.84\text{cm}$

$3.14 \times 6 = 18.84$



$C = \pi \times 10$

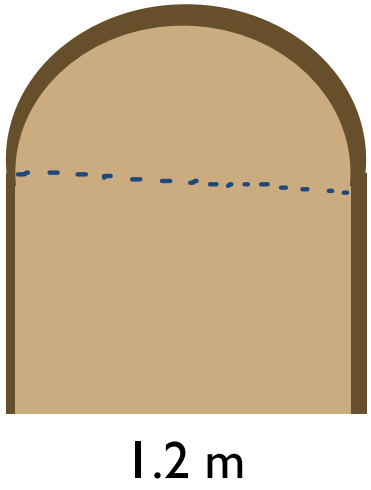
$= 31.4$

$\frac{1}{2}$ of $31.4 = 15.7\text{cm}$

$P = 15.7 + 10 = 25.7\text{cm}$

A doorway is to be lined with a wooden arch. Calculate the length of the wooden arch.

The wood costs £6.70 per metre. How much will it cost?



$C = \pi D$

$= \pi \times 1.2$

$= 3.7699...$

$\frac{1}{2}C = 3.7699... \div 2$

$= 1.88...$

$= 1.88\text{m}$

$P = 1.88 + 2.3 + 2.3$

$= 6.48\text{m}$

$\text{Cost} = 6.48 \times 6.70$

$= 43.416$

$= £43.42$

SHAPE - AREA

GEOMETRY & MEASURE

AREA

Area of rectangle = length x breadth

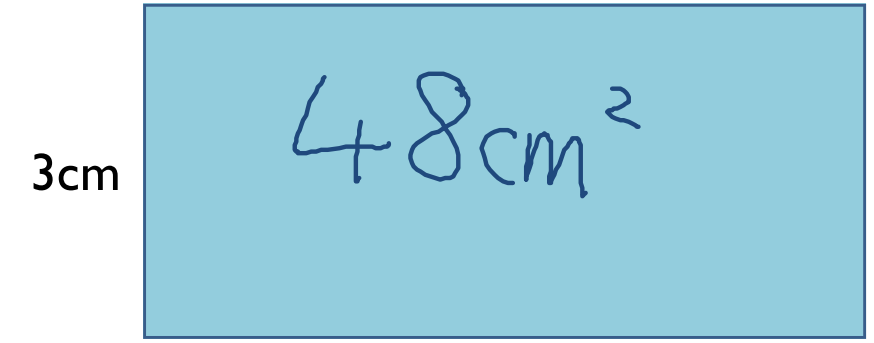
$A = 5 \times 30$

$= 150\ cm^2$

If the area of this rectangle is $48cm^2$, work out the missing side.

$3 \overline{)48} = 16$

16cm

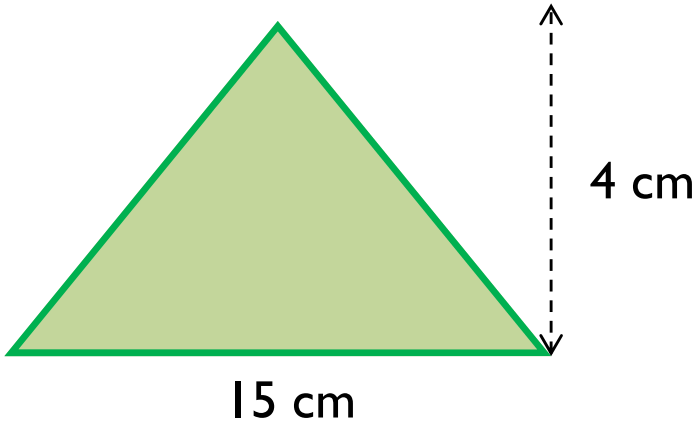


Area of triangle = $\frac{1}{2}$ x base x height

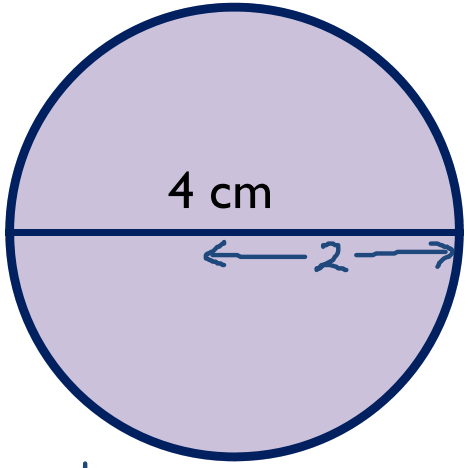
Area = $\frac{1}{2} \times 15 \times 4$

$= \frac{1}{2} \times 60$

$= \underline{30 cm^2}$



Area of circle = π x radius^2



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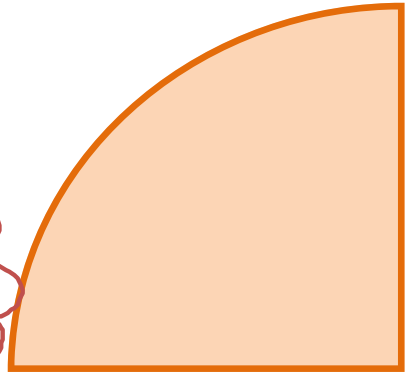


$r = 3$

$D = 4$ cm

$r = 2$ cm

$\text{Area} = \pi \times r^2$

$= 3.14 \times 2^2$

$= 3.14 \times 4$

$3.14 \times 4 = 12.56$

$= 12.56 \text{cm}^2$

Total Area $= \pi \times r^2$

$= 3.14 \times 3^2$

$= 3.14 \times 9$

$3.14 \times 9 = 28.26$

$= 28.26$

$\frac{1}{4}$ Area $= 28.26 \div 4$

$4 \overline{)28.2^{2}6^{2}0} = 7.065$

$= 7.065 \text{cm}^2$

SHAPE - COMPOUND AREA

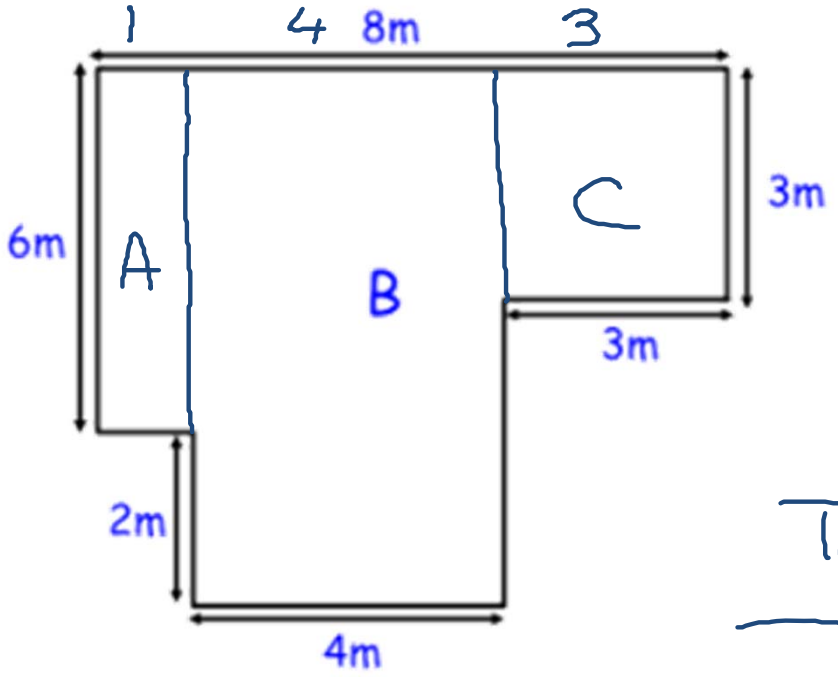
GEOMETRY & MEASURE

COMPOUND AREA

A compound (or composite) shape can be split into lots of separate shapes.

We work out the separate areas first then add them together.

This could mean you need to find out missing dimensions of a shape.



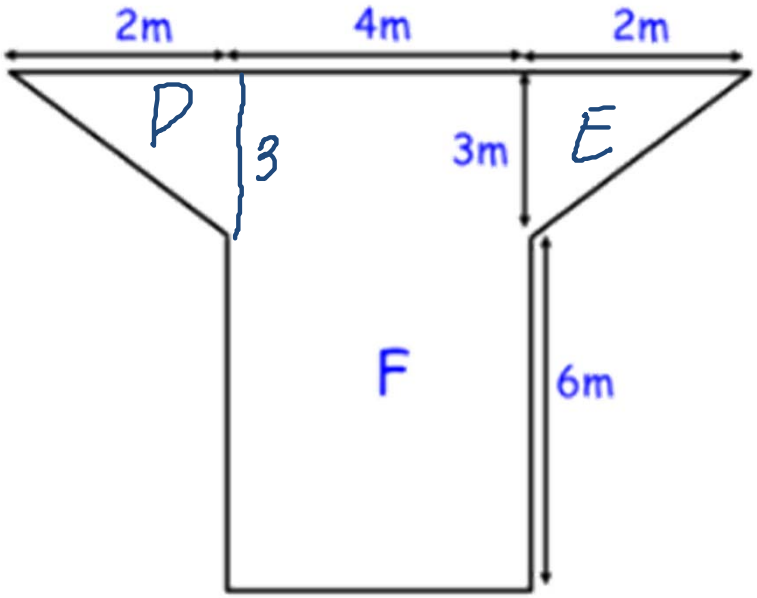
Area A = 6 x 1 = 6 m²

Area B = 4 x 8 = 32 m²

Area C = 3 x 3 = 9 m²

Total area = 47 m²

Area of triangle = $\frac{1}{2}$ x base x height

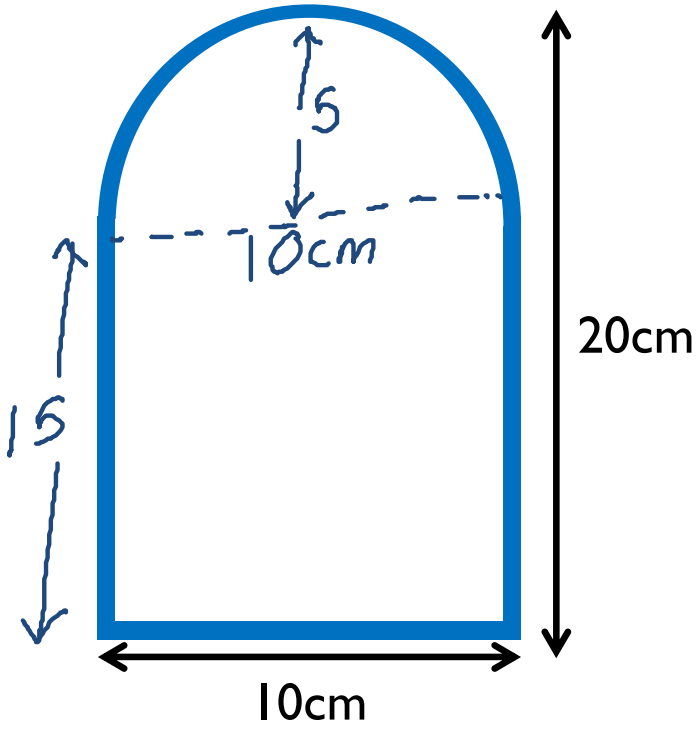


$$\text{Area } D = \frac{1}{2} \times 2 \times 3$$
$$= 3\,m^2$$

$$\text{Area } E = 3m^2$$

$$\text{Area } F = 4 \times 9$$
$$= 36m^2$$

Total area = $42m^2$



Area of circle = πx radius^2

$r = 5$

Area of full circle $= \pi \times 5^2$

$= 78.5398$

Semi-circle $= 78.5398 \div 2$

$= 39.2699 cm^2$

Area rectangle $= 10 \times 15$

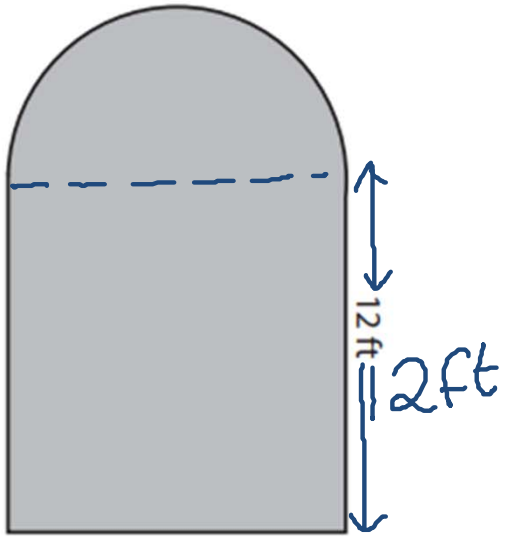
$= 150 cm^2$

Total area $= 189.2699$

$= 189.3 cm^2$

Find the area of each figure. Round the answer to 2 decimal places if necessary.

1)



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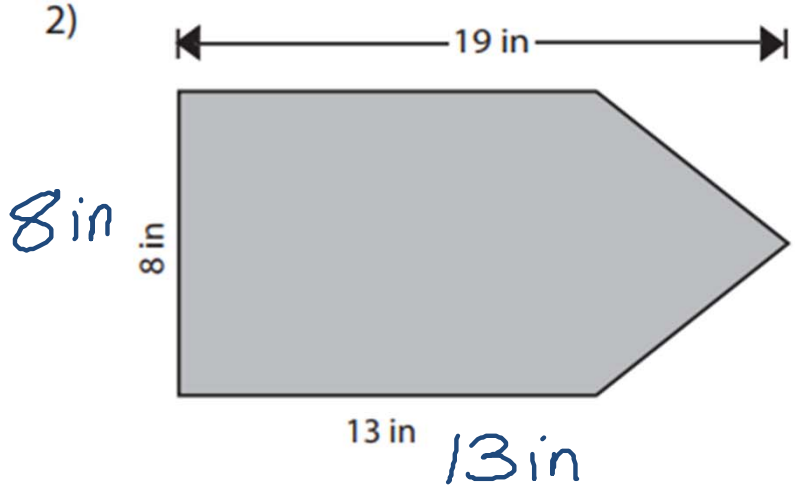

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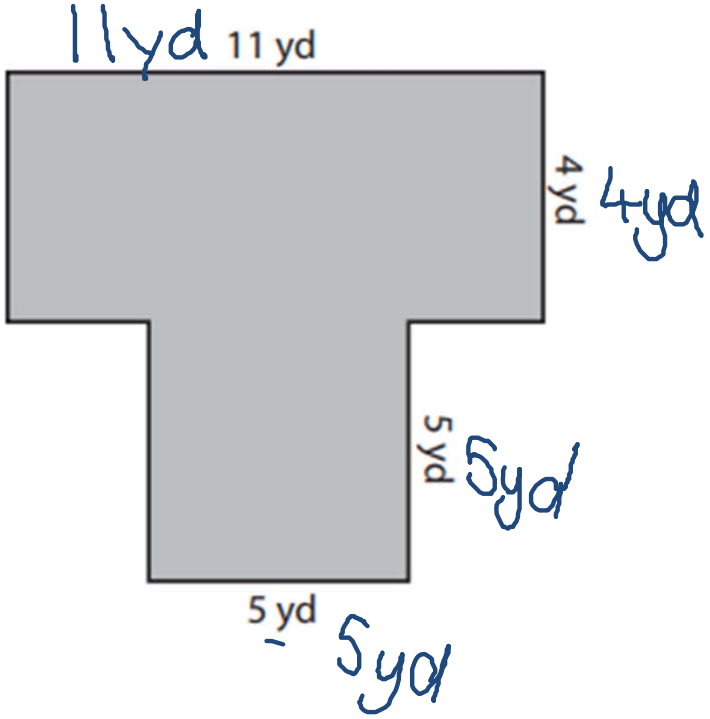

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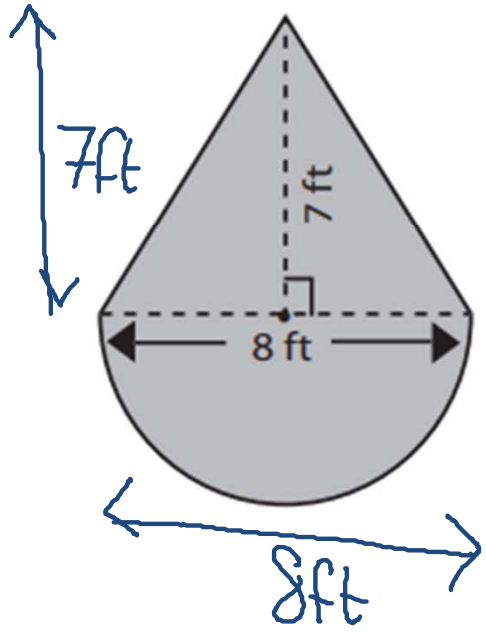
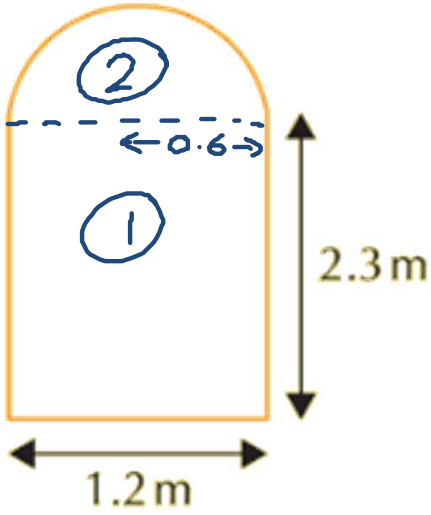


EXAMPLE 1

A window needs to be replaced. It is a composite shape made from a rectangle and a semi-circle.

a The window is to be replaced with clear double-glazing which costs £63.50 per square metre. What is the cost of replacing the window?



① $A = l \times w$
$= 1.2 \times 2.3$
$= 2.76$

② $A = \pi r^2$
$= \pi \times 0.6^2$
$= 1.1309...$
$= 1.13$

Semicircle $= 1.13 \div 2$
$= 0.565$

Total area $= 2.76 + 0.565$
$= 3.325 = 3.33$

Cost $= 3.33 \times 63.50$
$= 211.455$
$= £211.46$

b Patterned double-glazing is charged at £73.50 per square metre.
How much more expensive is this option than clear double-glazing?

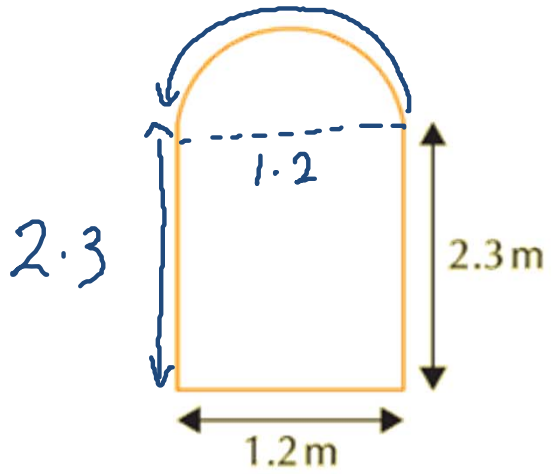
3.33×73.50

$= £244.76$

$244.76 - 211.46$

$= £33.30$ more expensive.

c The window currently has an aluminium frame. To upgrade to a steel window frame will cost £22.50 per metre. What is the cost of replacing the window frame?



$C = \pi \times D$

$= \pi \times 1.2$

$= 3.7699...$

$\frac{1}{2}C = 1.88495...$

$= 1.88\text{m}$

Cost:

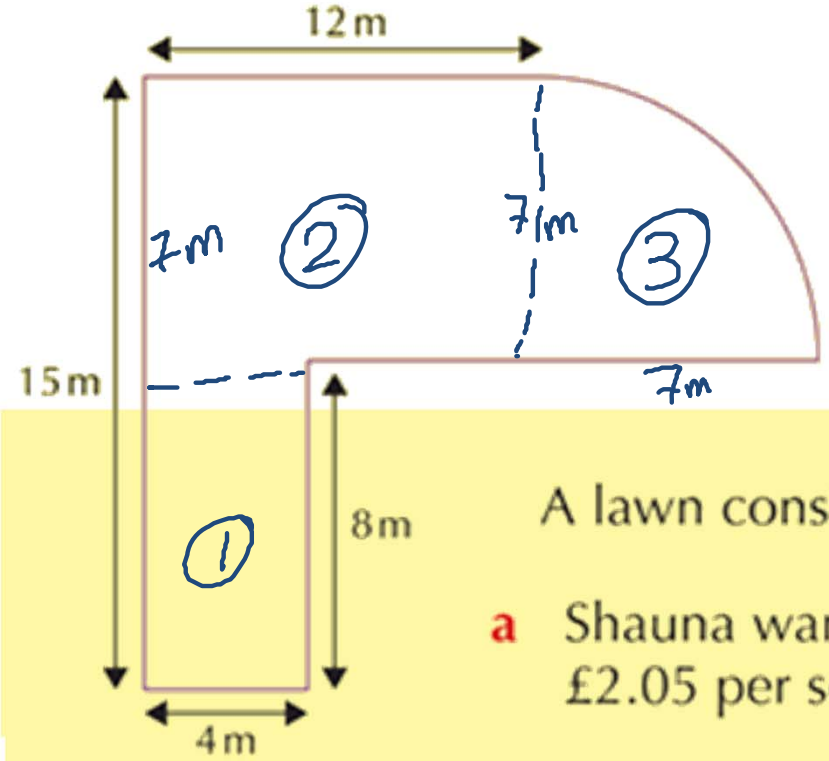
7.68×22.50

$= £172.80$

$P = 1.2 + 2.3 + 2.3 + 1.88$

$= 7.68\text{m}$

EXAMPLE 2



A lawn consists of two rectangles and a quarter circle.

a Shauna wants to re-turf her lawn. Turf is bought at £2.05 per square metre. How much will this cost her?

① $A = l \times w$
$= 4 \times 8$
$= 32 m^2$

② $A = l \times w$
$= 12 \times 7$
$= 84 m^2$

③ $A = \pi \times r^2$
$= \pi \times 7^2$
$= 153.9380...$

$\frac{1}{4} A = 153.9380 \div 4$
$= 38.4845$
$= 38.48 \ m^2$

Total area $= 32 + 84 + 38.48 = 154.48 m^2$

Cost $= 154.48 \times 2.05$
$= 316.684 \approx £316.68$

b i She has been advised to add lawn soil to the area due to be re-turfed. It is sold in bags of 25 litres and she has been advised to use 1 bag per 2.5 square metres. Only whole bags can be purchased. Bags cost £4.60. How much will this cost her?

$A = 154.48$

No. of bags $= 154.48 \div 2.5 = 61.792 \approx 62$ bags

$62 \times 4.60 =$

£285.20

ii She has been told that, in autumn, she should add a top dressing of soil using the same lawn soil, but this time at the rate of one 25 litre bag for every 12 square metres of lawn area. How much will this cost her?

No. of bags $= 154.48 \div 12 = 12.87\ldots$

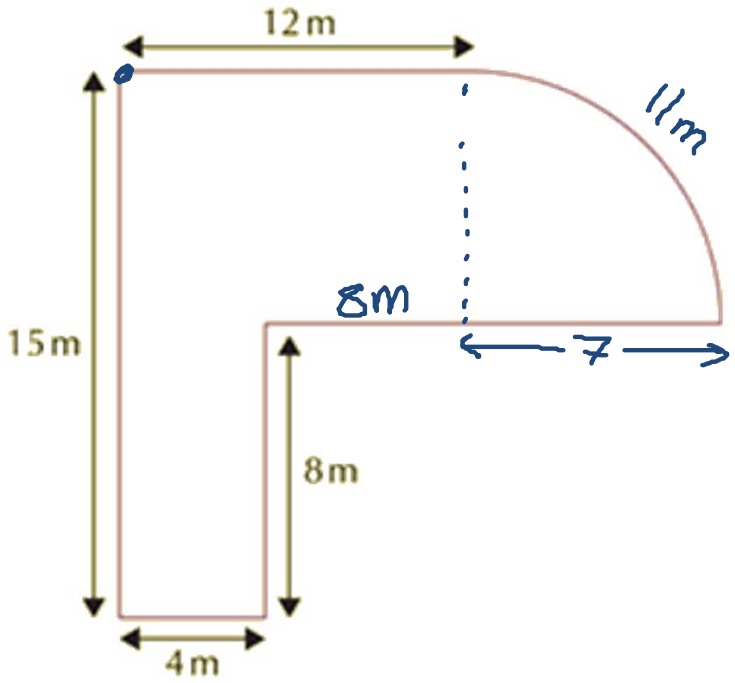
$\approx$ 13 bags

$13 \times 4.60 =$ £59.80

c What is the total cost of re-turfing her lawn?

Total cost $= 316.68 + 285.20 + 59.80 =$ £661.68

d Shauna wants to put a small fence around the edge of the garden. Fencing costs £2.99 per metre but she can only buy it in complete metres. How much will the fence cost her?



$C = \pi \times D$ $\quad r = 7$

$= \pi \times 14$ $\quad D = 14$

$= 43.9822....$

$\frac{1}{4}C = 43.9822... \div 4$

$= 10.9955....$

$= 11m$

$P = 15 + 4 + 8 + 8 + 7 + 11 + 12$

$= 65m$

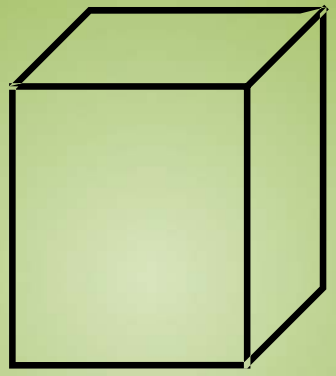
$Cost = 65 \times 2.99$

$= £194.35$

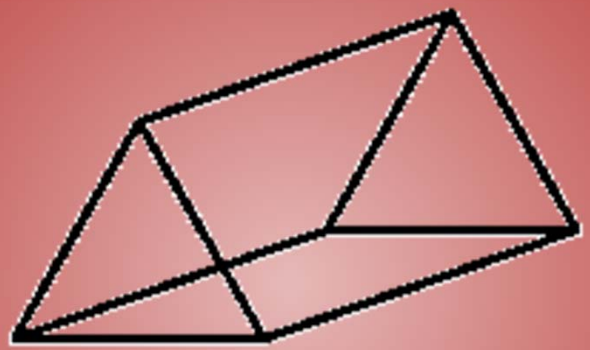
SHAPE: VOLUME

GEOMETRY & MEASURE

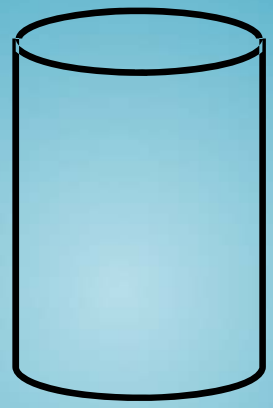
FORMULAS & FACTS



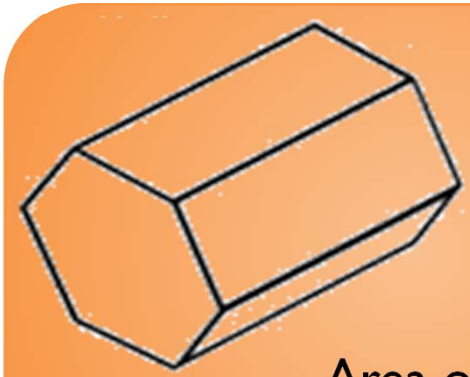
Volume =
length x base x height



Volume =
½ x length x base x height



Volume =
π x radius^2 x height



For any prism:

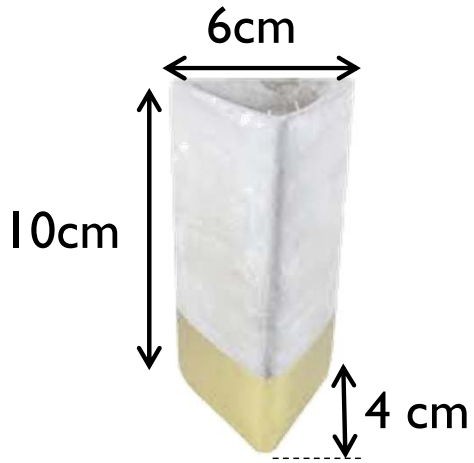
Volume =
Area of base x length

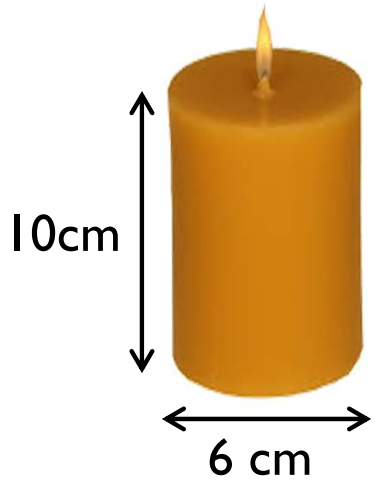
1 litre = 1000ml = 1000cm^3

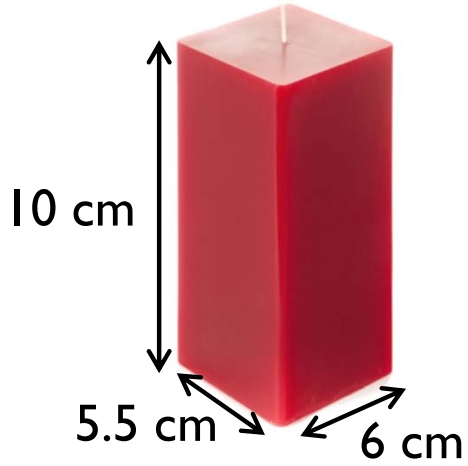
1cm^3 = 1ml

EXAMPLE

A candle company can make candles in three different shapes:







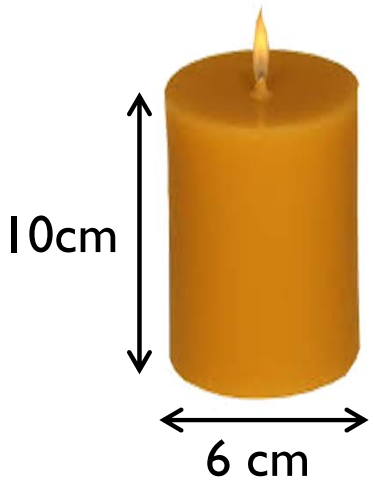
They sell their candles according to the volume of wax. 1 litre of wax is sold for £12.50.

Which candle would be the most expensive?

$V = \frac{1}{2} \times b \times h \times l$

$= \frac{1}{2} \times 6 \times 4 \times 10$

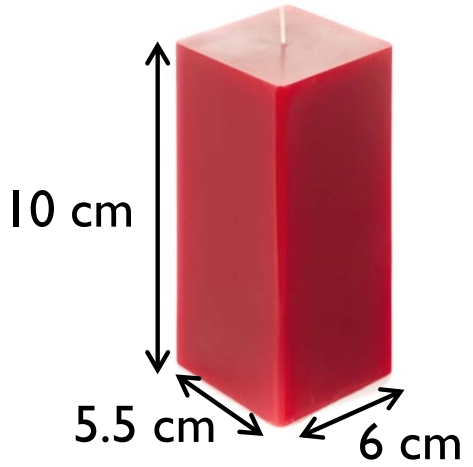
$= 120 cm^3$



$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$= 282.6 cm^3$



$V = l \times b \times h$

$= 10 \times 5.5 \times 6$

$= 330 cm^3$

1 litre = £12.50

1000ml = £12.50

1ml = £0.0125

120×0.0125

$= £1.50$

282.6×0.0125

$= 3.5325$

$= £3.53$

330×0.0125

$= 4.125$

$= £4.13$

The cuboid candle is the most expensive.

STARTER

1. The Jones' are looking to insure the contents of their home.
HomesDirect provide contents and building insurance.
The monthly premiums per £10,000 are listed in the table below.
The Jones' live in a band 3 bungalow.
They would like to ensure
their contents for £50,000.

What is the cost of their
monthly premium?

HomesDirect Insurance Monthly Premiums per £10,000		
	Contents	Building
Band 1	£4.10	£2.10
Band 2	£5.65	£3.05
Band 3	£7.15	£3.95
Band 4	£8.90	£4.40

2. Aziz runs a hairdressing salon. Each day he records the shop's income, rounded to the nearest £10.
The stem and leaf diagram shows the income over a 25-day period.

Daily Income for Hairdressers

7	1 2 6
6	0 4 5 7 8
5	2 2 2 2 4 6 6 8
4	3 5 6 7 7 8 9
3	0 4

7 | 1 = £710

$n = 25$

(a) Write down the modal daily income.

(b) Aziz took a day off at random. What is the probability that he took a day off on a day that the hairdressers took less than £500?

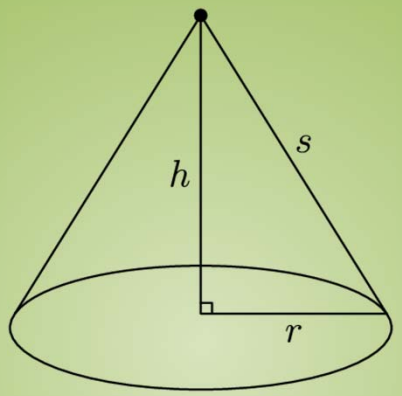
Express your answer as a percentage.

SHAPE: VOLUME

GEOMETRY & MEASURE

FORMULAS & FACTS

CONE

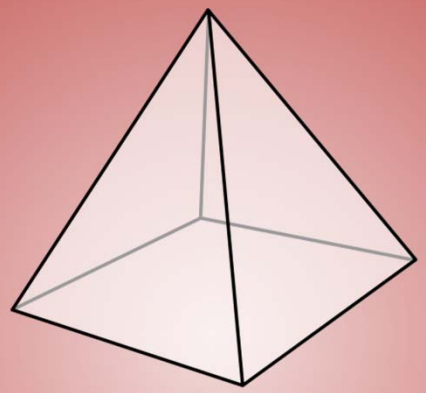


Volume =

$\frac{1}{3}$ x π x radius^2 x height

$V = \frac{1}{3}\pi r^2 h$

PYRAMID

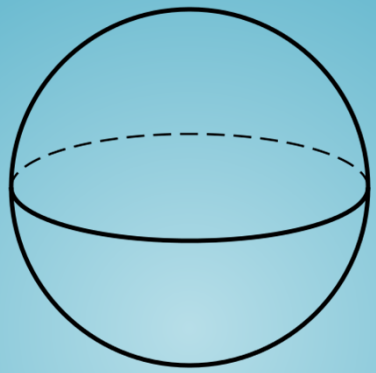


Volume =

$\frac{1}{3}$ x A x h

$V = \frac{1}{3}Ah$

SPHERE

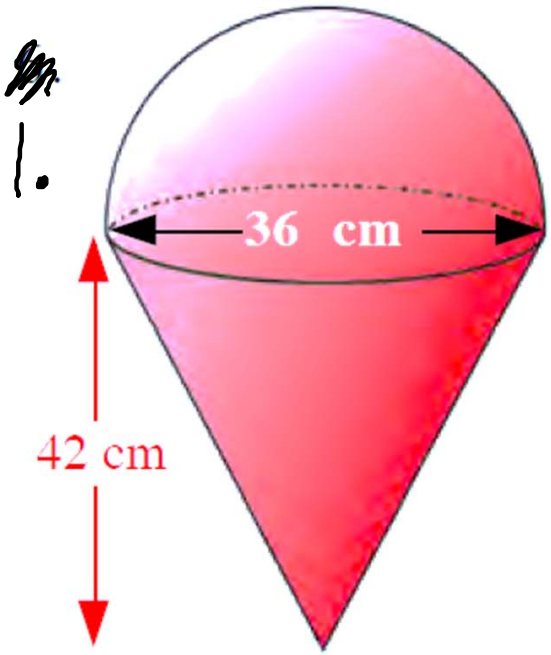


Volume =

$\frac{4}{3}$ x π x radius^3

$V = \frac{4}{3}\pi r^3$

1.



This large metal shape is used to advertise Monty's Ice-Cream Shop.

It consists of a hemi-sphere on top of a cone.

Calculate the volume of the metal shape.

Cone:

$V = \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \pi \times 18^2 \times 42$

$= 14250.264 cm^3$

Sphere:

$V = \frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 18^3$

$= \pi \times 18^3 \times 4 \div 3$

$= 24429.024$

Hemisphere $= 24429.024 \div 2$

$= 12214.512$

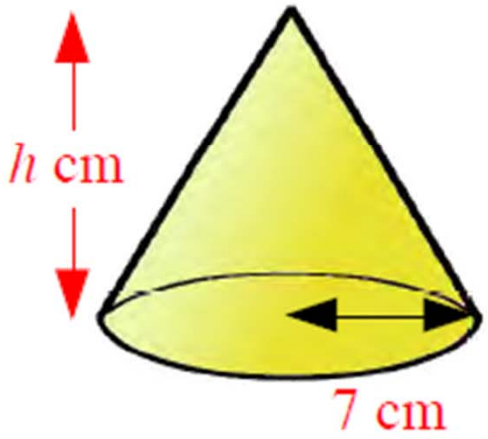
Total volume $= 14250.264 + 12214.512$

$= 26464.776 = 26464.78 cm^3$

The **volume** of this cone is 400 cm^3 .

(a) Calculate the **area** of its base.

(b) Now calculate the height (h cm) of the cone.



a) $A = \pi r^2$

$= \pi \times 7^2$

$= 153.9$

b) $V = \pi r^2 h$

$400 = 153.9 \times h$

$400 \div 153.9 = h$

$2.599... = h$

$h = 2.6cm$

$V = \pi \times r^2 \times h$

$400 = \pi \times 7^2 \times h$

$400 \div \pi \div 7^2 = h$

$2.59 = h$

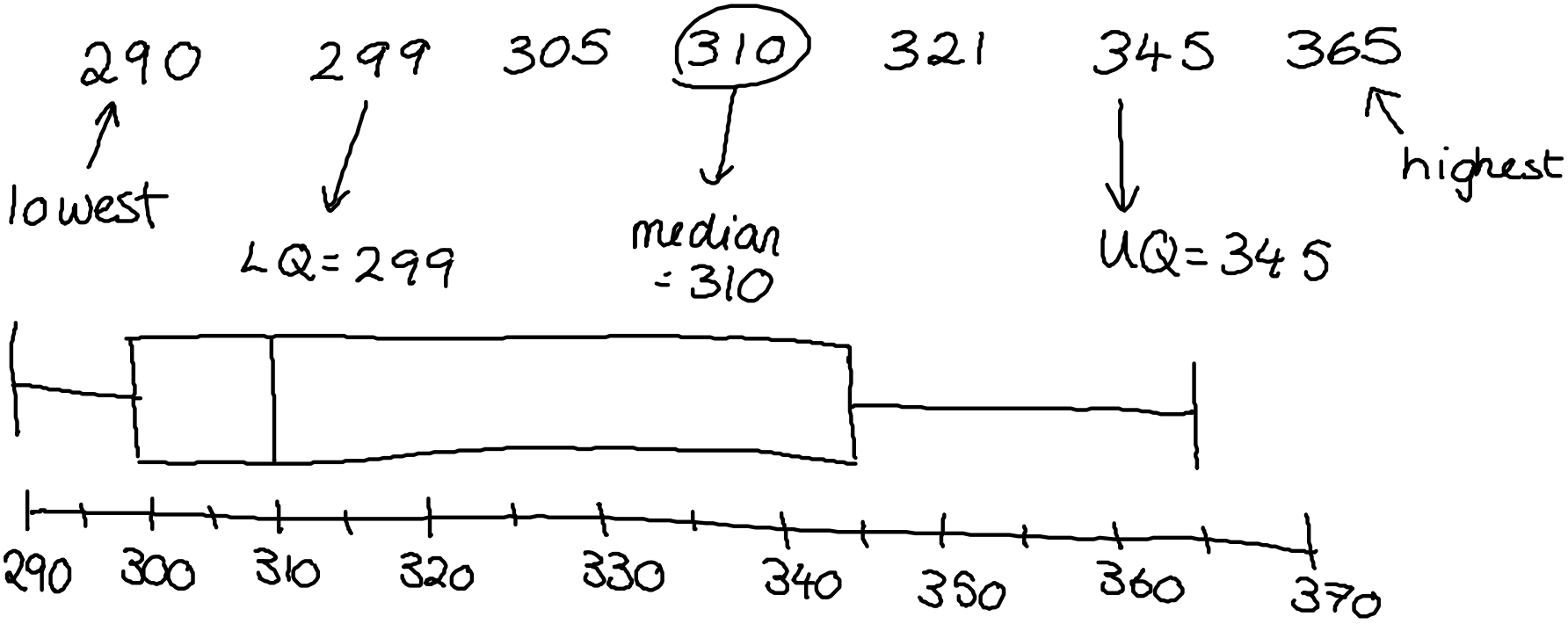
$h = 2.6$ cm

STARTER

The price of a games console was taken from 7 different stores.

Cost £	321	345	290	310	365	305	299

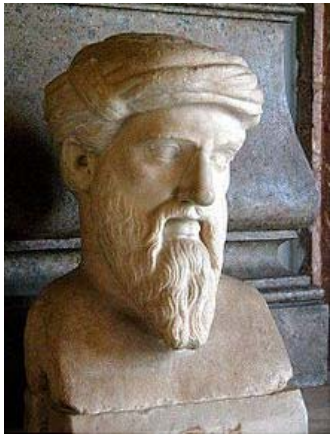
Make a five figure summary for this data and draw a box plot.



PYTHAGORAS

GEOMETRY & MEASURE

WHO IS PYTHAGORAS?



Pythagoras was a Greek philosopher and mathematician who was born in 570 BC.

He came up with a number of mathematical discoveries.

We are going to learn one of them today surrounding triangles.

SQUARES AND ROOTS

- You should know your squares and roots up to 12.

$1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$4^2 = 4 \times 4 = 16$

$5^2 = 5 \times 5 = 25$

$6^2 = 6 \times 6 = 36$

$7^2 = 7 \times 7 = 49$

$8^2 = 8 \times 8 = 64$

$9^2 = 9 \times 9 = 81$

$10^2 = 10 \times 10 = 100$

$11^2 = 11 \times 11 = 121$

$12^2 = 12 \times 12 = 144$

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

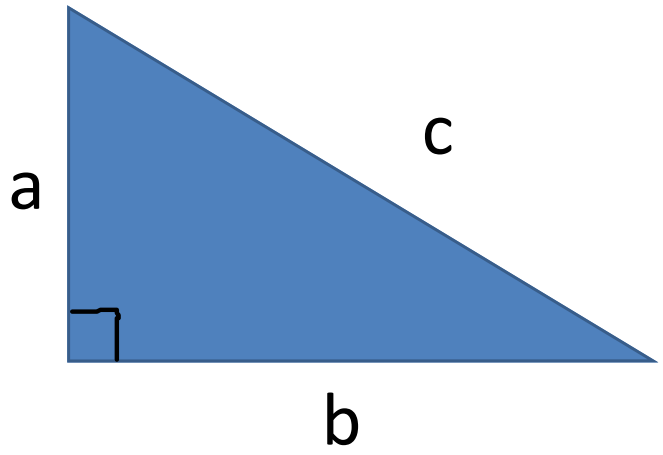
$\sqrt{16} = 4$

$\sqrt{25} = 5$

⋮

THE PYTHAGOREAN THEOREM

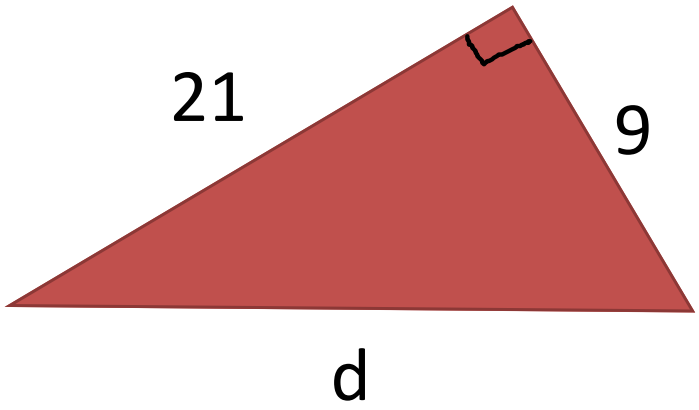
FOR A RIGHT ANGLED TRIANGLE, IF WE KNOW TWO SIDES WE CAN WORKOUT THE OTHER SIDE USING THE FORMULA:



$$a^2 + b^2 = c^2$$

EXAMPLE: LONG SIDE

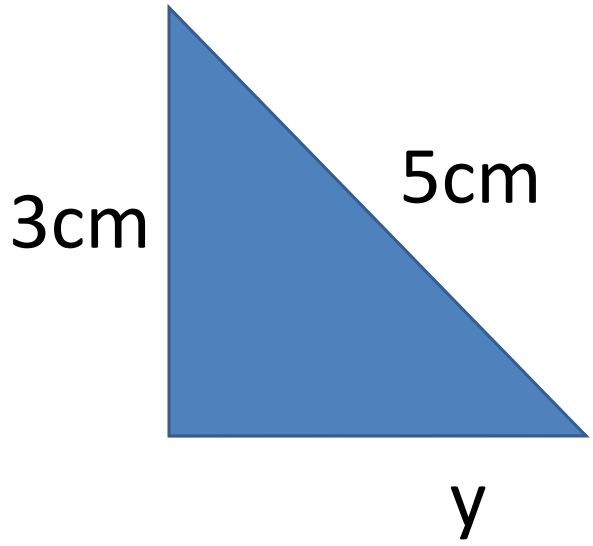
$$a^2 + b^2 = c^2$$



$$a^2 + b^2 = c^2$$
$$21^2 + 9^2 = d^2$$
$$441 + 81 = d^2$$
$$522 = d^2$$
$$\sqrt{522} = d$$
$$d = 22.8$$

SHORT SIDE

$$a^2 = c^2 - b^2$$



$a^2 = c^2 - b^2$

$a^2 = 5^2 - 3^2$

$a^2 = 25 - 9$

$a^2 = 16$

$a = \sqrt{16}$

$a = 4cm$

PYTHAGORAS

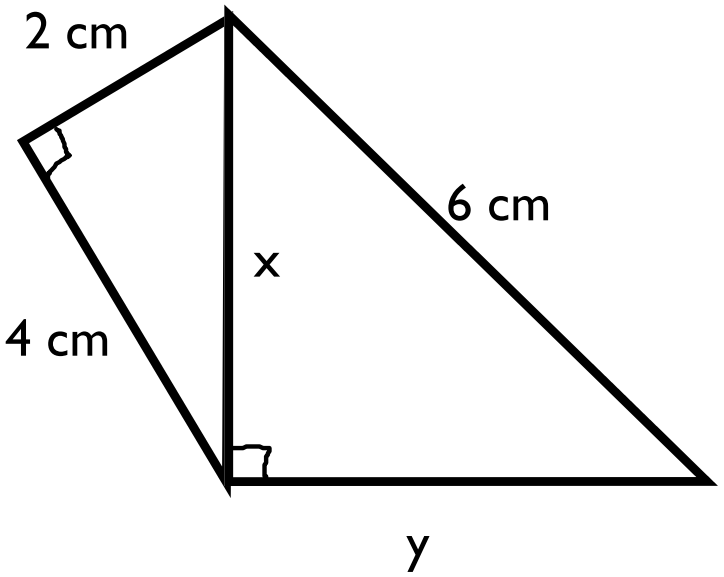
GEOMETRY & MEASURE

PROBLEM SOLVING WITH PYTHAGORAS

- You might be asked to work out:
  - More than one length
  - Area or perimeter
  - Gradient
  - To show whether it is right angled or not

EXAMPLE 1

- Find the lengths x and y



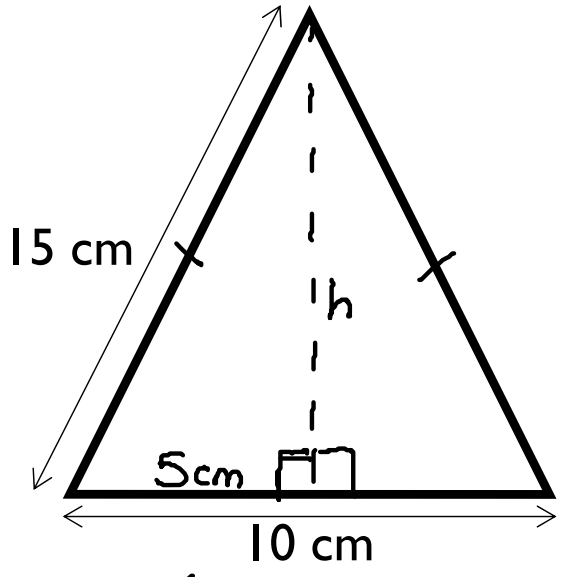
$$c^2 = a^2 + b^2$$
$$x^2 = 2^2 + 4^2$$
$$x^2 = 4 + 16$$
$$x^2 = 20$$
$$x = \sqrt{20}$$
$$x = 4.4721\,cm$$

$$a^2 = c^2 - b^2$$
$$y^2 = 6^2 - 4.4721^2$$
$$y^2 = 36 - 20$$
$$y^2 = 16$$
$$y = \sqrt{16}$$
$$y = 4\,cm$$

EXAMPLE 2

- Find the height of this isosceles triangle and then find the area.

half it down the middle



$a^2 = c^2 - b^2$

$h^2 = 15^2 - 5^2$

$h^2 = 225 - 25$

$h^2 = 200$

$h = \sqrt{200}$

$h = 14.1421$

$= 14.1\text{ cm}$

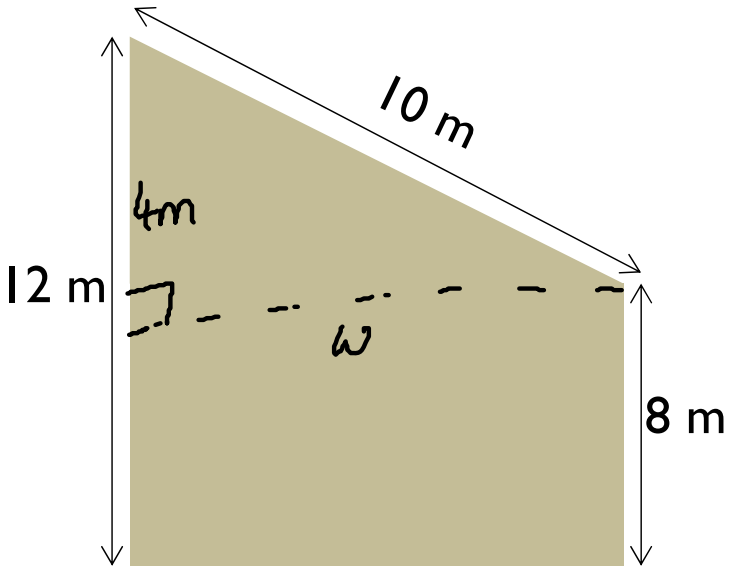
$\text{Area} = \frac{1}{2}bh$

$= \frac{1}{2} \times 10 \times 14.1421$

$= 70.706$

$= 70.7\text{cm}^2$

EXAMPLE 3



The side view of a building is shown.

a) Calculate the width of the building.

$a^2 = c^2 - b^2$

$w^2 = 10^2 - 4^2$

$w^2 = 100 - 16$

$w^2 = 84$

$w = \sqrt{84}$

$w = 9.165\ldots = 9.2\ m$

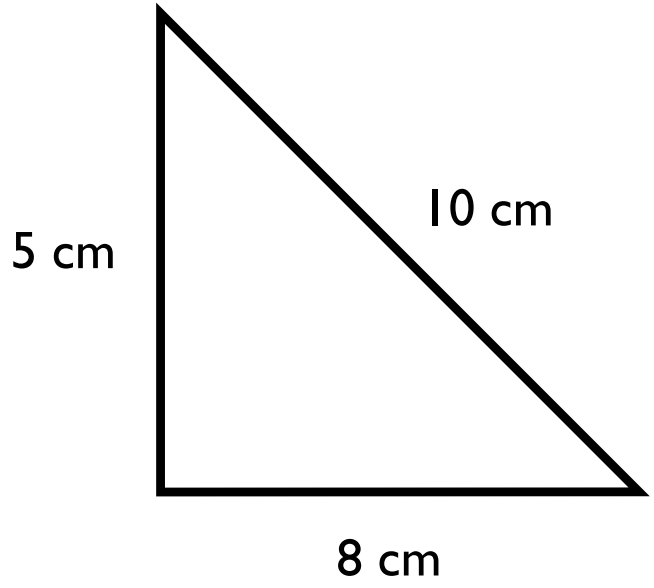
b) Find the gradient of the roof.

$\text{gradient} = \frac{\text{vertical}}{\text{horizontal}} = \frac{4}{9.165} = 0.436$

$= \underline{\underline{0.4}}$

EXAMPLE 4

- Is this triangle right-angled?



$c^2 = 10^2 = 100$

$a^2 + b^2 = 5^2 + 8^2 = 25 + 64 = 89$

$c^2 \neq a^2 + b^2$ as $100 \neq 89$

So this triangle is not right angled.

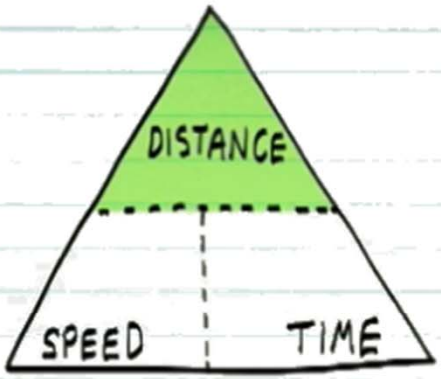
SPEED
DISTANCE
TIME

GEOMETRY & MEASURE

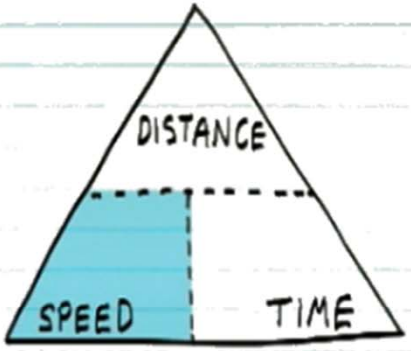
DISTANCE, SPEED & TIME

A journey consists of three things: distance (how far you travel), speed (how fast you travel) and time (how long you travel for).

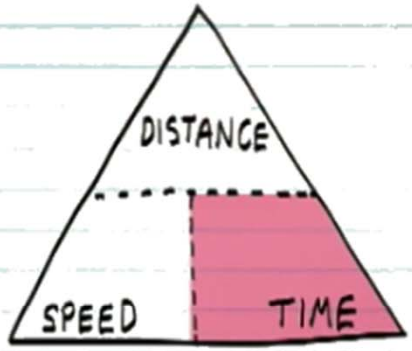
If we know two of these values we can work out the other. A handy way to remember this is the DST triangle.



SPEED × TIME = DISTANCE



$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$



$$\frac{\text{DISTANCE}}{\text{SPEED}} = \text{TIME}$$

Simple DST Examples

1. Shona cycles for 28 km. How fast did she cycle if the journey took 4 hours?

speed

$$\text{Speed} = \frac{D}{T} = \frac{28}{4} = 7 \text{ kmph}$$

2. Kelly runs from 4:20pm until 5:20pm at an average speed of 4 metres per second. How far did she run?

distance

$S = 4 \text{ m/s}$

$T = 1 \text{ hour} = 60 \text{ min} = 3600 \text{ sec}$

$D = S \times T$

$= 4 \times 3600$

$= 14\,400 \text{ m} = 14.4 \text{ km}$

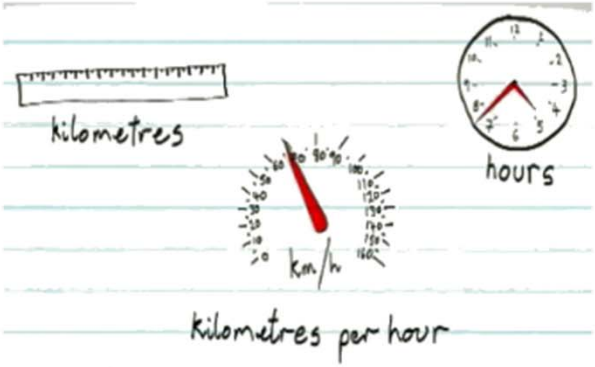
3. A train travels 420 miles at an average speed of 120mph. How long did the journey take?

D S T

$$T = \frac{D}{S}$$

$$= \frac{420}{120}$$

$= 3.5 \text{ hours} = 3 \text{ hours } 30 \text{ mins}$



When doing an exam question, make sure you double check the units used in the question.
If you're distance is km and time in hours then your speed should be given in kmph.

You may need to convert hours and minutes into decimal time to use in a calculation.

$$\text{hours} = \frac{\text{minutes}}{60}$$

$$\text{minutes} = \frac{\text{seconds}}{60}$$

$$\text{minutes} = \text{hours} \times 60$$

$$\text{seconds} = \text{minutes} \times 60$$

Examples

Change to decimal time

a) 45 min
÷60
= 0.75 hours

b) 3 min 24 sec
÷60
0.4
3.4 min

c) 1 hr 48 min
÷6
0.8
1.8 hrs

d) 40 sec
non-calc
$\frac{40}{60} = \frac{4}{6} = \frac{2}{3}$

Change to hours and minutes or minutes and seconds

a) 0.6 hours
×60
= 36 min

b) 2(95)hours
×60
= 57 min
2hr 57min

c) 0.3 min
×60
= 18 seconds

d) 5(35)hours
×60 = 21
5hrs 21min

Examples

Change to decimal time

a) 45 min

$\frac{45}{60} = 0.75$

b) 3 min 26 sec

$\frac{26}{60} = 0.433$

3.433 min

Change to hours and minutes

a) 0.6 hours

0.6×60

$= 36$ min

b) 2.95 hours

$0.95 \times 60 = 57$

2hr 57 min

1. David drives for 150 km. How fast did he drive if it took him 3 hrs and 25 min?

$S = \frac{D}{T} = \frac{150}{3.42} = 43.85\ldots$ 44km/h

$\frac{25}{60} = 0.42$

3.42 hrs

2. Bob swims from 7:15pm until 8pm at an average speed of 6 metres per second. How far did he swim?

T = 45 min (x60) = 2700sec

S = 6 m/s

$D = 2700 \times 6 = 16200$m

$= 16.2$ km

D

S

3. A train travels 126 miles at an average speed of 105mph. How long did the journey take?

$T = \frac{126}{105} = 1.2$ hrs

$0.2 \times 60 = 12$

$= 1$hr 12min

STARTER

a) Bartosz is a window cleaner and is setting up a new business. He needs to buy a van to transport equipment to his jobs. The van he has chosen costs £18,450.

He takes out a bank loan to buy the van. The bank will charge a simple interest rate of 15.6% per year on the loan. If he repays the loan over 24 months, how much will each monthly repayment be?

×2

18450 ÷ 100 × 15·6 = 2878·20

18450 + 2×2878·20 = 24206·4

24206·4 ÷ 24 = £1008·60

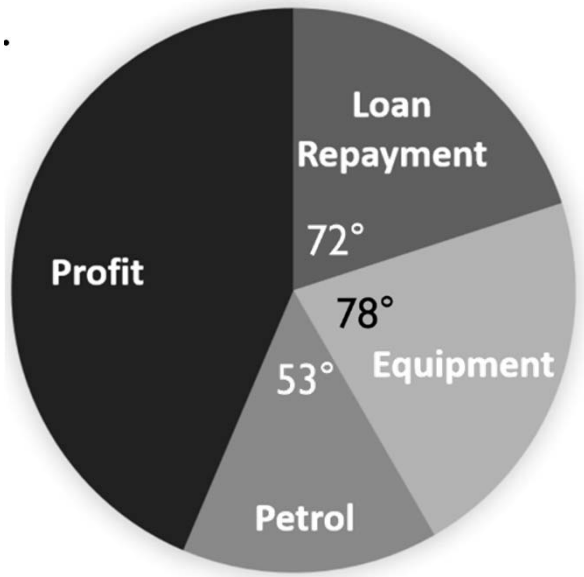
b) In one month, the window cleaning business has an income of £5043.

The pie chart shows how income is split between petrol, equipment, loan repayments and profit. Calculate Bartosz's profit that month.

Profit = 157°

$\frac{157}{360} \times 5043 = 2199.308\ldots$

£2199.31



CONTAINER PACKING

GEOMETRY & MEASURE

CONTAINER PACKING

Goods usually have three layers of packaging. These are:

- **primary packaging:** this is the wrapping or the containers that are handled by the customer.

- **secondary packaging:** this is the middle layer of packaging that uses larger wrappings, containers or boxes to group quantities of primary packaged goods. The goods might be displayed on the shelf in the secondary packaging.

- **transit packaging:** this is the outer container that uses wooden pallets (trays), plastic and board wrapping to make the transportation easier.

ORIENTATION

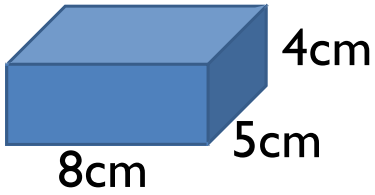
Items can be packed in different orientations depending on the dimensions of the object.

You may have to trial different orientations to see which gives you the best result.

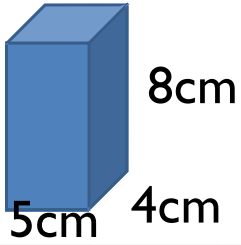
Toy cars are individually packed in small boxes measuring 8 cm by 5 cm by 4 cm.

What is the greatest number of these small boxes that can be packed into a larger container measuring 30 cm by 28 cm by 18 cm?

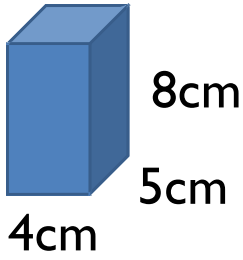
There are a number of different ways (orientations) in which the small boxes can be packed into the larger container. Each way needs to be considered to find the most efficient arrangement.



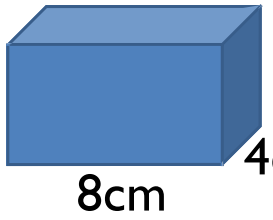
Big box	Small box	Boxes that fit
30	8	3
28	5	5
18	4	4 x
Total number of boxes = 60		



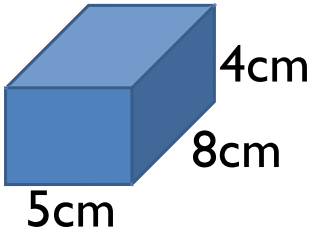
Big box	Small box	Boxes that fit
30	5	6
28	4	7
18	8	2 x
Total number of boxes = 84		



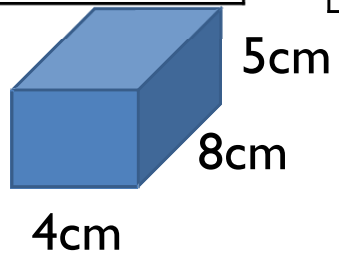
Big box	Small box	Boxes that fit
30	4	7
28	5	5
18	8	2 x
Total number of boxes = 70		



Big box	Small box	Boxes that fit
30	8	3
28	4	7
18	5	3 x
Total number of boxes = 63		

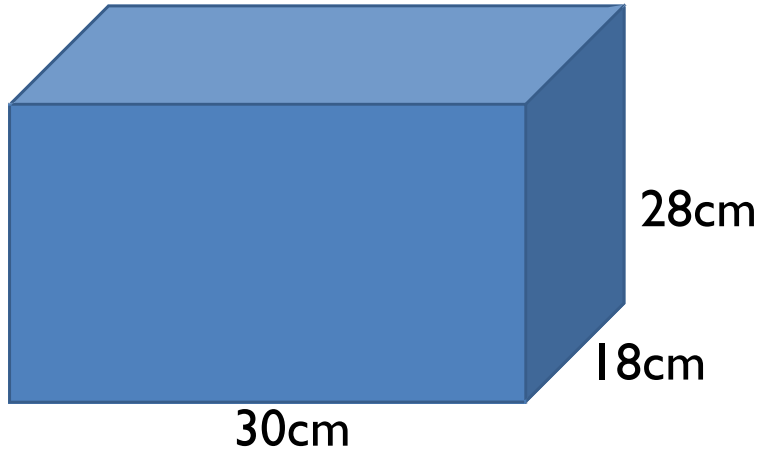


Big box	Small box	Boxes that fit
30	5	6
28	8	3
18	4	4 x
Total number of boxes = 72		



Big box	Small box	Boxes that fit
30	4	7
28	8	3
18	5	3 x
Total number of boxes = 42		

Maximum number is 84 boxes



PACKING ALGORTIHMS

An algorithm is a routine procedure.

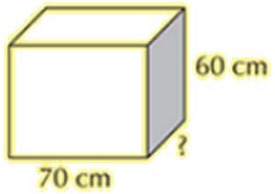
There are two container packing algorithms:

- In the **first-fit algorithm**, you take the items to be packed in the order they are given and fit each one into the first available container that will take it.
- In the **first-fit decreasing algorithm**, you sort the items to be packed into order of decreasing size, then apply the first-fit algorithm

Colin delivers goods from a factory to the company's retail outlets.

On Monday, he has 13 boxes to deliver.

Each box has a height of 60 cm and width of 70 cm, but a variable depth, as shown in the diagram.



The load compartment of Colin's van is 1.1 m high, 1 m wide and 1.6 m deep.

The depths of the boxes are shown in the table below.

Box	A	B	C	D	E	F	G	H	I	J	K	L	M
Depth (cm)	50	30	110	70	60	80	75	40	50	90	60	50	30

Colin will need to make several trips to deliver all the boxes.

The boxes can be delivered in any order.

Colin wants to make the least possible number of trips to deliver all the boxes.

a Use the first-fit algorithm to work out the best order for Colin to pack the boxes into his van.

b Use the first-fit decreasing algorithm to work out the best order for Colin to pack the boxes into his van.

Box	C	J	F	G	D	E	K	A	I	L	H	B	M
Depth (cm)	110	90	80	75	70	60	60	50	50	50	40	30	30

Load	Boxes	Depth ≤ 1.6
1	A B D	50+30+70 = 1.5
2	C H	110+40 = 1.5
3	E F	60+80 = 1.4
4	G I M	75+50+30= 1.55
5	J K	90+60 = 1.5
6	L	30 = 0.3

Load	Boxes	Depth
1	C A	110+50=1.6
2	J D	90+70=1.6
3	F G	80+75=1.55
4	E K H	60+60+40=1.6
5	I L B M	1.6 m

Freddie and Kamal work in a warehouse stacking shelves.

A section of the warehouse has 5 shelves; each shelf is 10 metres in length.

The shelves are currently stocked as shown below.

Shelf 1	Box A (7 m)
Shelf 2	Box B (5 m)
Shelf 3	Box C (6 m) Box D (3 m)
Shelf 4	Box E (4 m) Box F (3 m)
Shelf 5	Box G (2 m)

Shelf 1	
Shelf 2	
Shelf 3	
Shelf 4	
Shelf 5	

A new delivery of Box H (6 m), Box I (5 m), Box J (3 m), Box K (4 m), Box L (1 m) arrives to be stored in this section of the warehouse.

These new boxes need to be stored on different shelves from the existing stock.

The existing stock can be re-arranged to create space for the new delivery.

By writing the letters A to L in the diagram below, show how Freddie and Kamal can fit **all** the boxes onto the shelves.

PRECEDENCE TABLES

GEOMETRY & MEASURE

PRECEDENCE TABLES

To complete a task it is sometimes necessary to complete one part before another can begin. We say that one task takes **precedence** over the other.

In each case, it should be possible to decide on the best order to undertake the tasks and the shortest time it will take to complete all the tasks, using the diagram that has been constructed. This is called the **critical path**.

A precedence table can be used to plan events involving different numbers of activities.

Task	Preceded by
A	–
B	A
C	A, B

Task	Preceded by
A	–
B	A
C	–
D	C

PRECEDENCE TABLES

A plumber has to complete a number of jobs to fit a bathroom. The time taken for each item is in brackets:

- Remove old bathroom fittings (3)
- Remove old floor tiles (2)
- Install new fittings (6)
- Paint woodwork (2)
- Lay new floor (4)
- Install blinds (1)
- Paint walls (3)

The fittings and tiles can be removed at the same time. New fittings can be installed once the old floor and fittings are removed. The new floor is laid after the new fittings are complete. The woodwork and walls must not be painted until the floor is complete, and the final task is to install the blinds.

Draw a precedence table and a network diagram to work out the critical path.

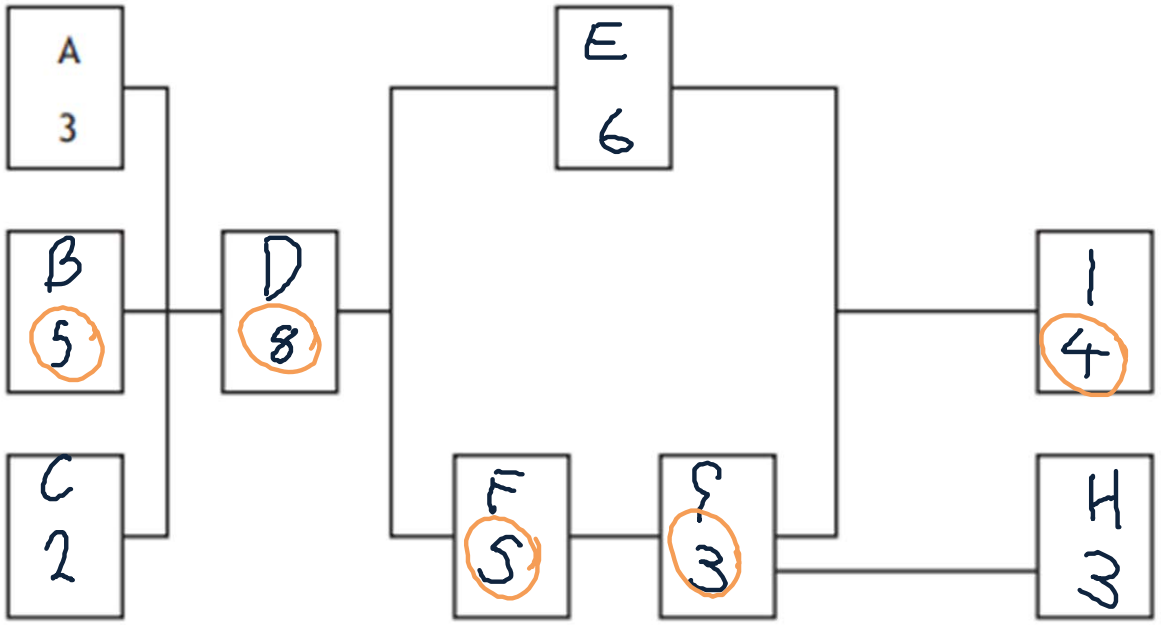
The Clarks employ Kitease to install a new kitchen for them.

Kitease provide a team of workers to install the kitchen.

The table shows the list of tasks and the time required for each.

Task	Detail	Preceding task	Time(hours)
A	Begin electrics	None	3
B	Build cupboards	None	5
C	Begin plumbing	None	2
D	Plaster walls	A,B,C	8
E	Fit wall cupboards	D	6
F	Fit floor cupboards	D	5
G	Fit worktops	F	3
H	Finish plumbing	G	3
I	Finish electrics	E,G	4

The kitchen will take 25 hours to install.

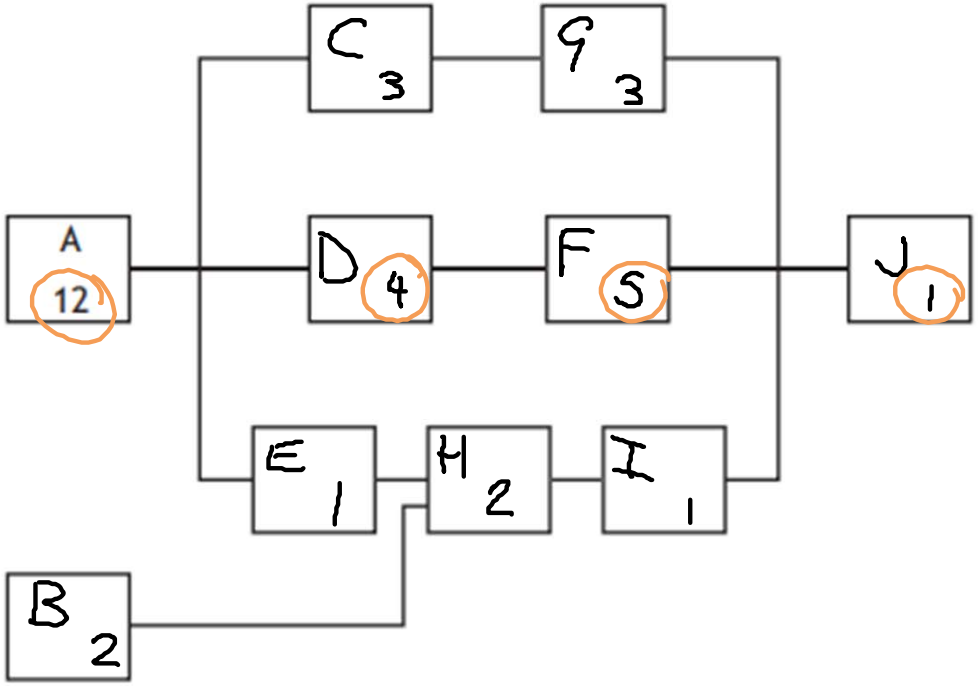


A computer company is researching how long it would take to develop a new games console and bring it to market.

The following table of necessary tasks was produced.

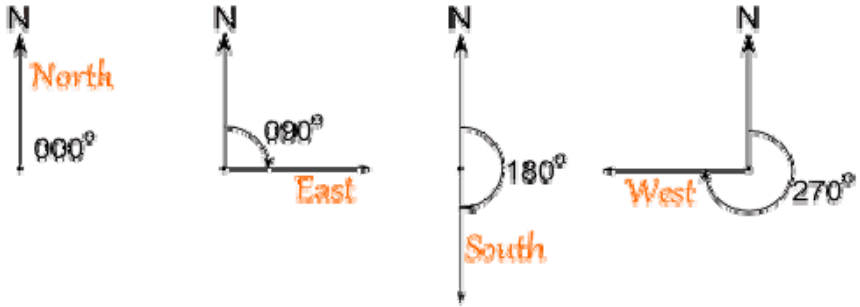
Activity	Description	Preceding Task	Time (months)
A	Product design	None	12
B	Market research	None	2
C	Production analysis	A	3
D	Product model	A	4
E	Sales brochure	A	1
F	Product testing	D	5
G	Cost analysis	C	3
H	Sales training	B,E	2
I	Pricing	H	1
J	Project report	F,G,I	1

The development takes 22 months.

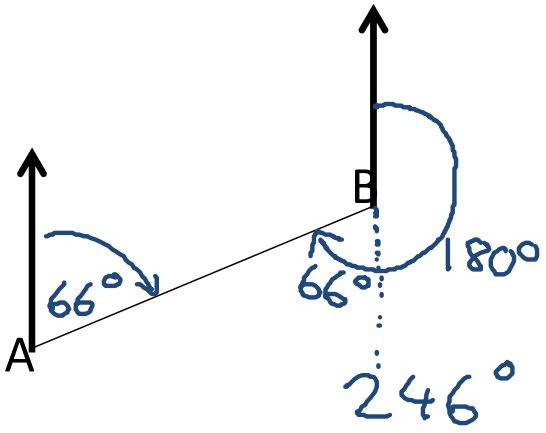


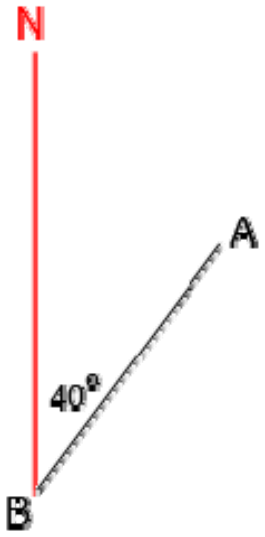
SCALE AND BEARINGS

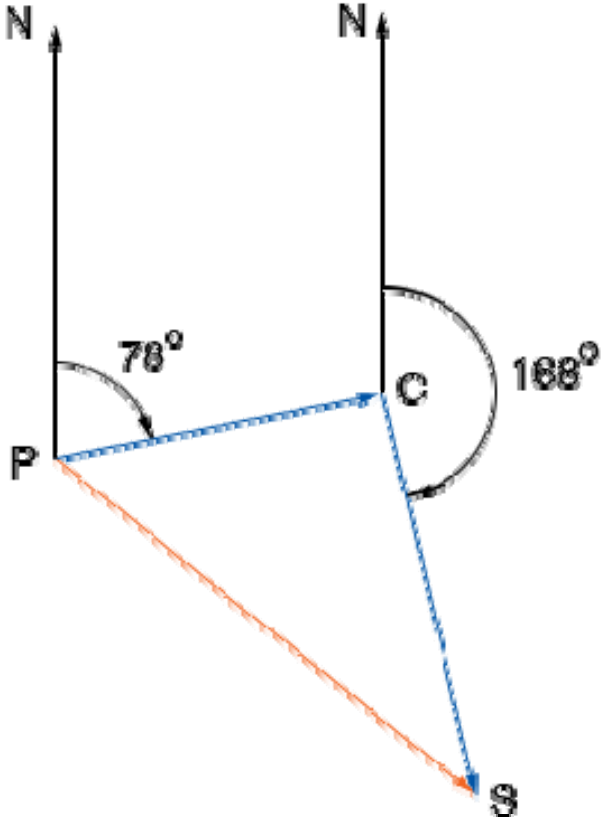
BEARINGS



- A bearing is an angle of travel
- It is measured clockwise from North (North is 000°)
- Bearings can be given as three digits

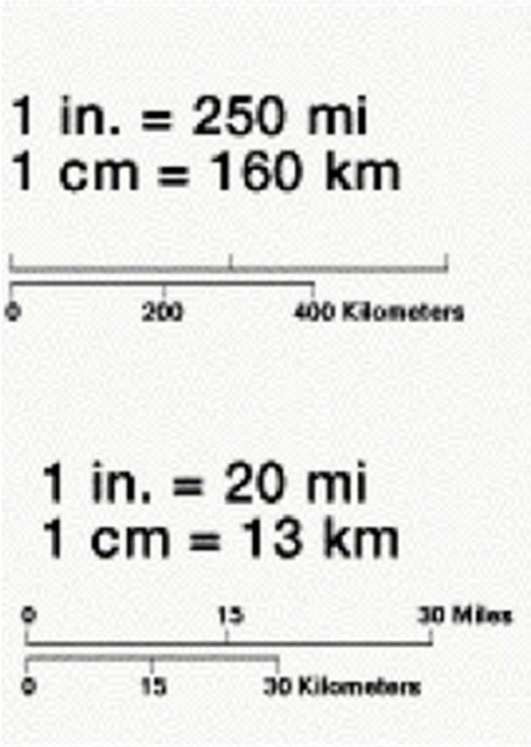


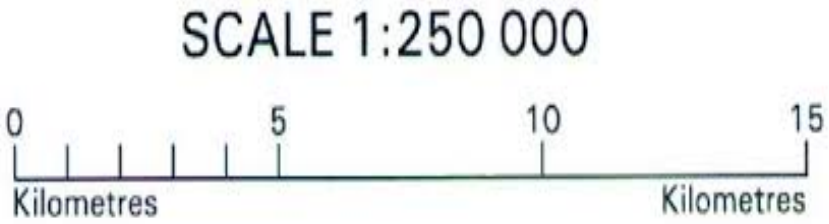


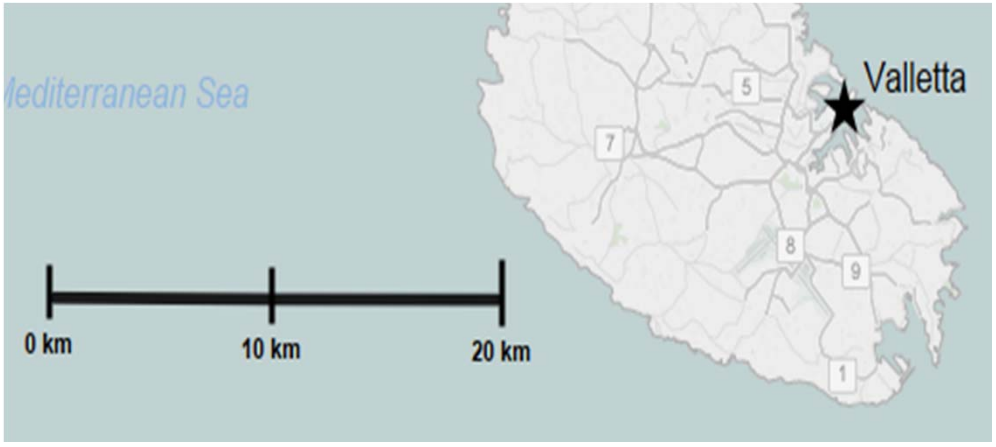


SCALE

- A scale is given so you can work out true distances and lengths based of a smaller diagram
- Sometimes scales are given as a line in the bottom corner of a map
- Scales should have the same unit so 1cm : 3km -> 1 : 300,000







1) The scale used for a map is 1 : 4000000. The distance between two towns on the map is 3.7cm. Find the true distance between the towns.

1 : 4000000

3.7 : 14800000

148 km

3.7
× 4
14.8

2) On another map, the distance between two cities is 11cm. The true distances between the cities is 4.4km. Work out the scale used on the map.

11cm : 4.4km

11cm : 440000cm ↓ change km → cm

1cm : 40000 ↓ ÷ 11

4.4km
× 1000
4400m
× 100
440000

3a) It takes 42 minutes to sail from Island A to Island B. A boat travels at 45kmph. How far apart are the islands?

$D = S \times T$

$S = 45$

$T = 42\text{min} = 0.7$

$D = 45 \times 0.7$

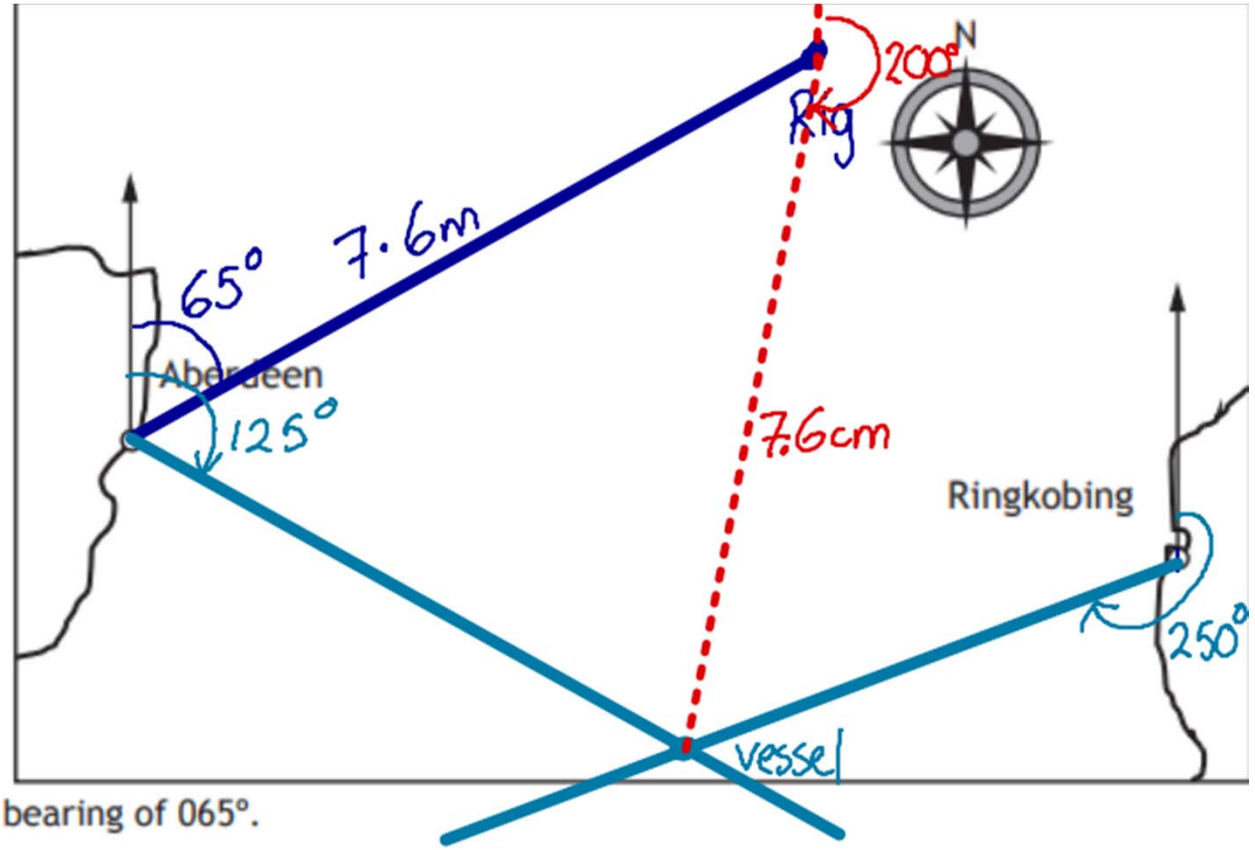
$= 31.5\text{ km}$

b) The scale of a map is 1cm : 5km.

How far apart are the islands on the map?

$31.5 \div 5 = \underline{6.3\text{cm}}$

Past Paper Questions



Scale 1 centimetre represents 50 kilometres

(a) Harkins oil rig is 380 km from Aberdeen on a bearing of 065°.

Show the position of the Harkins oil rig on the map above. **2**

(b) A fishing vessel issues an SOS call which is received by both ports.

The bearing of the fishing vessel from each port is shown in the table below.

Bearing from	Three figure bearing
Aberdeen	125°
Ringkobing	250°

250° → 360 − 250 = 110° anticlockwise

a) 380 ÷ 50 = 7.6cm

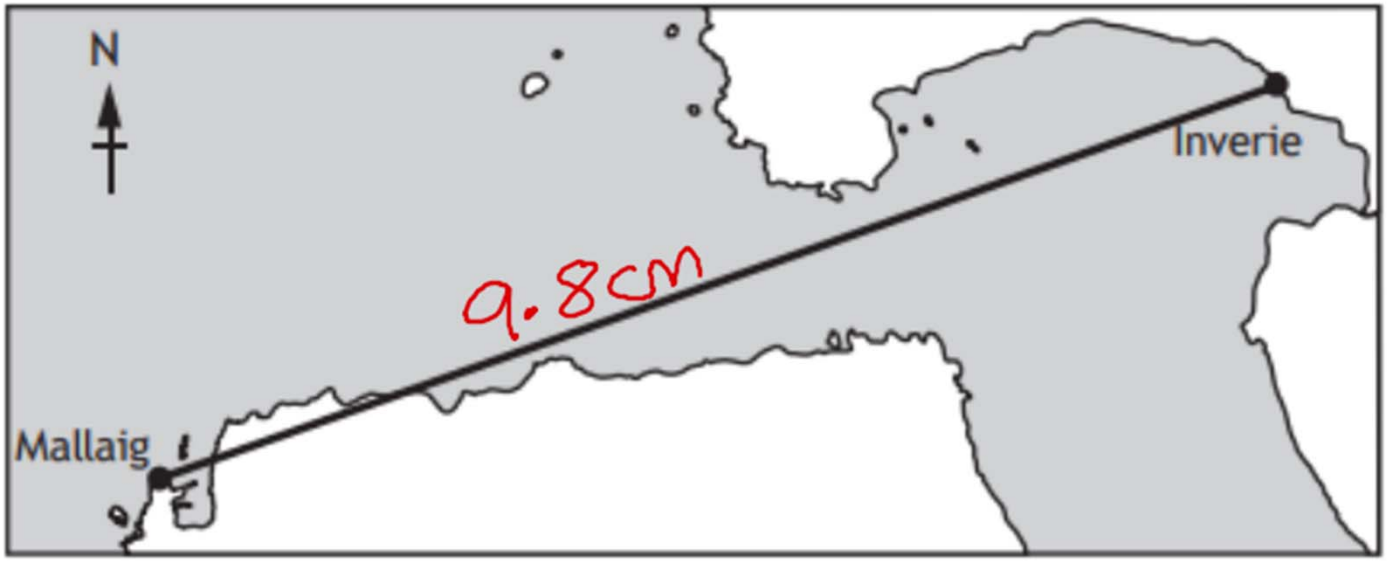
c) bearing = 200°
distance = 7cm
7 × 50 = 350km

(i) Mark the position of the fishing vessel on the map. **3**

(ii) Find the distance and bearing of the fishing vessel from the oil rig. **2**

4. Alison and Michael are travelling to Inverie on Knoydart for a holiday. They must take a ferry from Mallaig to Inverie

(a) The direct distance from Mallaig to Inverie is 9·8 kilometres.



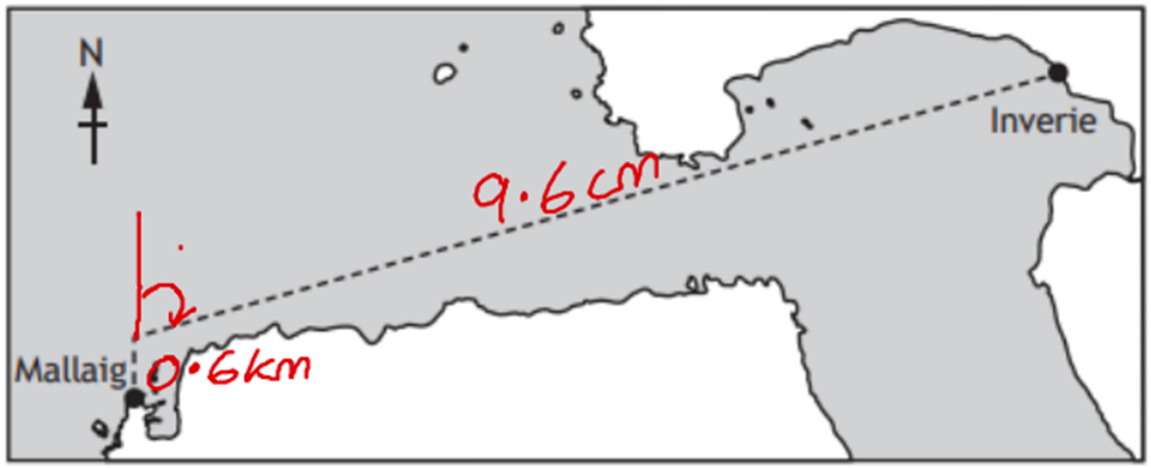
(i) Calculate the scale used in the diagram above. 1

÷9.8 ↘ 9.8cm : 9.8km ↙ ÷9.8

1cm : 1km

1cm : 1000m

1 : 100000



The ferry leaves Mallaig and travels North for 0·6 km .

The ferry then changes direction to sail directly to Inverie.

Use the second diagram to find the bearing and distance, in kilometres, that the ferry must travel on the **second part** of its journey. **2**

9·6cm = 9·6km

Bearing 73°

(b) The average speed of the ferry from Mallaig to Inverie is 24 ± 3 kilometres per hour depending on tide and weather.

What is the shortest time that the **complete** ferry journey might take?

Give your answer to the **nearest minute.** **3**

27kmph = S

10·2km = D

$T = \frac{D}{S}$

$= 10·2 \div 27$

$= 0·3\dot{7}\ldots$

$0·3\dot{7} \times 60 = 22·\dot{6}$

= 23 minutes

① A seaplane flies from an airport on a bearing of 050° at a speed of 170 mph for 36 minutes.

② It then turns onto a new bearing of 190° and flies at the same speed for a further 1 hour 12 minutes.

(a) Construct a scale drawing to illustrate this journey.

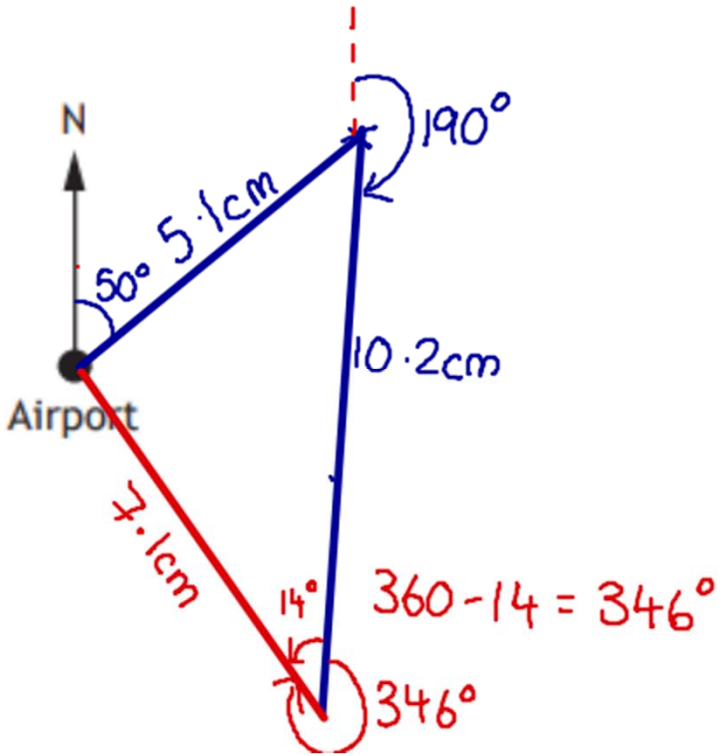
Use a scale of 1 cm:20 miles **4**

102 ÷ 20 = 5·1cm

① S = 170
T = 36min
= 0·6 hours

$D = \frac{170}{0.6} = 102$ miles

② Notice: 1hr 12min = 2x 36min
so distance = 2x102 = 204 miles
= 10·2cm



The seaplane continues at the same speed back to the airport.

(b) Use the scale drawing to determine the distance and bearing of the airport from the seaplane. **2**

$D = 7.1\text{cm} \times 20 = 142\text{km}$

Bearing of 346°

The seaplane burns fuel at 32 litres per hour.

Aviation fuel costs £2·04 per litre.

(c) Calculate the cost of the fuel for the complete journey. **4**

36min + 1hr 12min + 50mins

$= 2\text{hr } 38$

$= 2.6\dot{3}$

$2.6\dot{3} \times 32 = 84.27$ litres

$84.27 \times 2.04 = £171.91$

(3) $S = 170\text{ mph}$

$D = 142\text{miles}$

$T = \frac{142}{170} = 0.835\ldots$

$= 50\text{mins}$

The boat leaves from the harbour on a bearing of 045° for a distance of 22 miles to Puffin Island.

The boat leaves Puffin Island on a bearing of 170° and travels for a further 37 miles to Gull Isle.

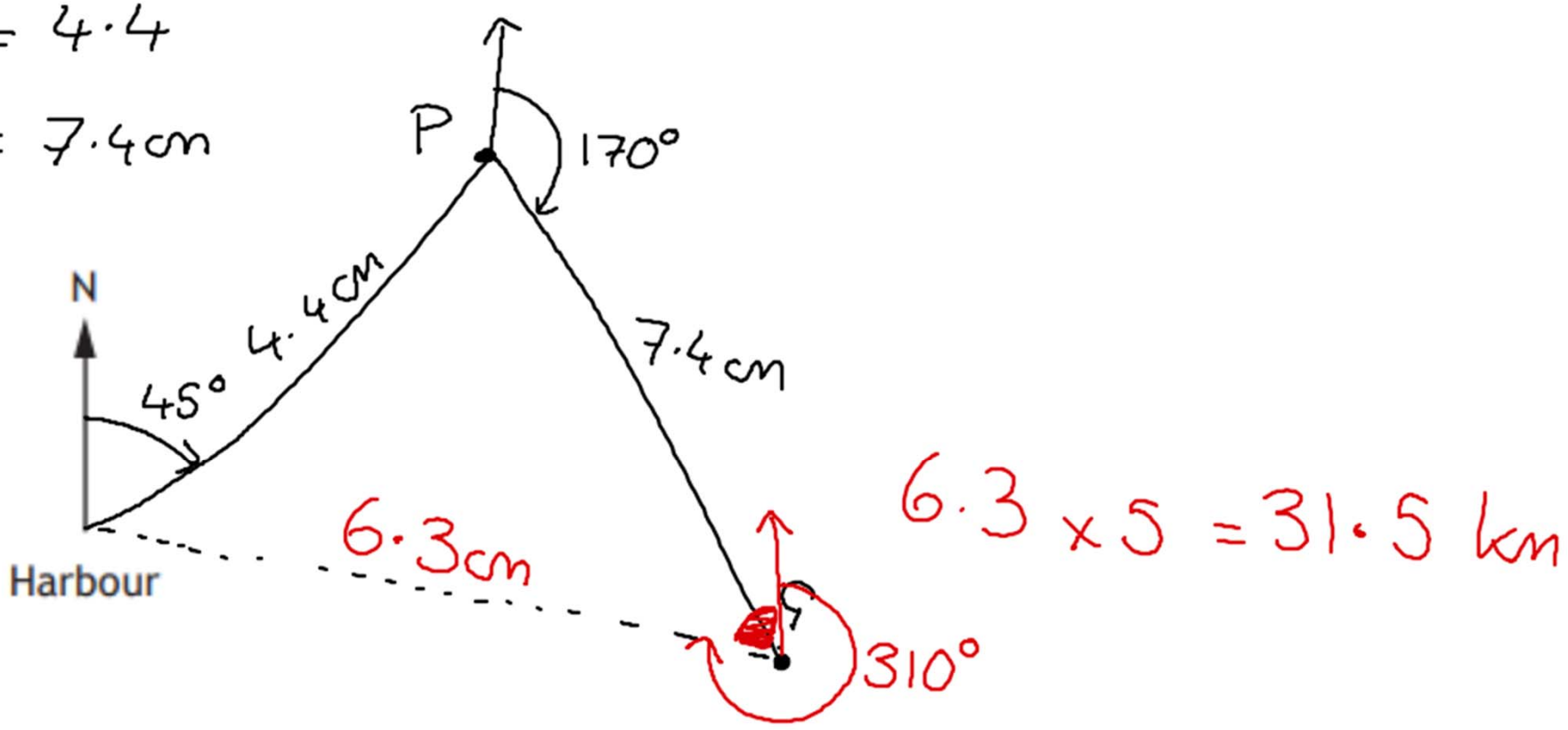
(a) Construct a scale drawing to illustrate this journey.

Use a scale of 1 cm : 5 miles. **3**

(An additional diagram, if required, can be found on *page 18*.)

$22 \div 5 = 4.4$

$37 \div 5 = 7.4\text{cm}$



$6.3 \times 5 = 31.5\ \text{km}$

The boat continues back to the harbour.

(b) Use the scale drawing to determine the bearing and distance of the harbour from the boat. **2**

$S = \frac{90.5}{4.416..} = 20.49 = \underline{\underline{20\text{mph}}}$

$S = \frac{D}{T}$

$D = 90.5\text{ km}$

$T = 4\text{hr}25$

$= 4.416.....$

$1\text{hr}15 + 2\text{h}50 = 4\text{hr}5$

$8\text{hr}30 - 4\text{hr}5 = 4\text{hr}25$

(c) The boat leaves the harbour at 0930.

It stops for 1 hour 15 minutes at Puffin Island and 2 hours 50 minutes at Gull Isle.

The boat arrives back at the harbour at 1800 the same day.

Calculate the average speed of the boat whilst it is moving. **3**