National 5 Portfolio

N5

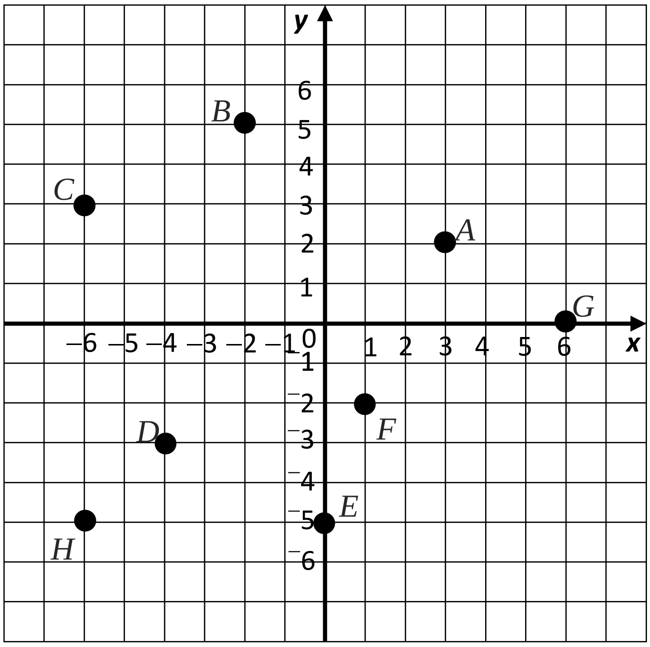
Applications 1.2 – 3D Coordinates and Vectors

Section A - Revision

This section will help you revise previous learning which is required in this topic.

R1 I can identify 2D co-ordinates.

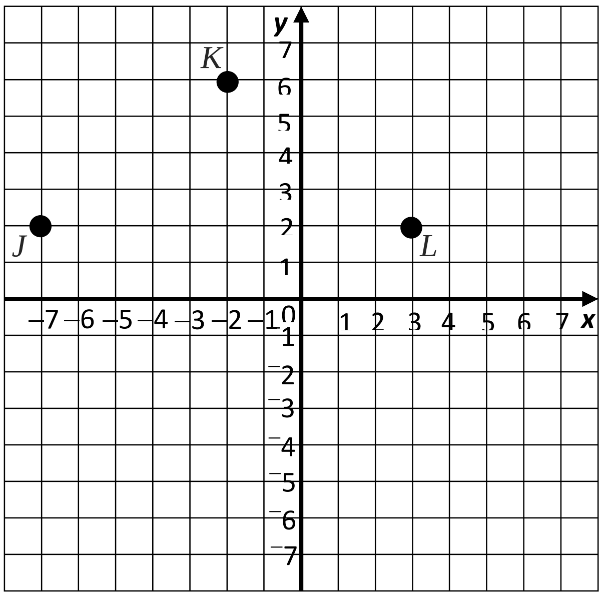
1. Write down the coordinates of the points A, B, C, D, E, F, G and H shown in the diagram below.



2. The points J, K and L have been plotted on the diagram shown below.

(a) Write down the co-ordinates of J, K and L.

(b) State the co-ordinates of M so that JKLM is a rhombus.



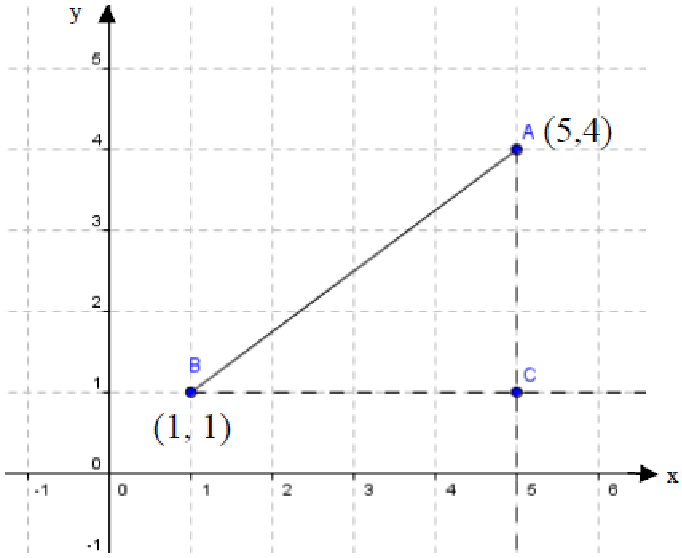
3D Coordinates and Vectors

R2 I can use Pythagoras to calculate the distance between two points without using a calculator.

1. Using the diagram opposite
   (a) What are the co-ordinates of C?

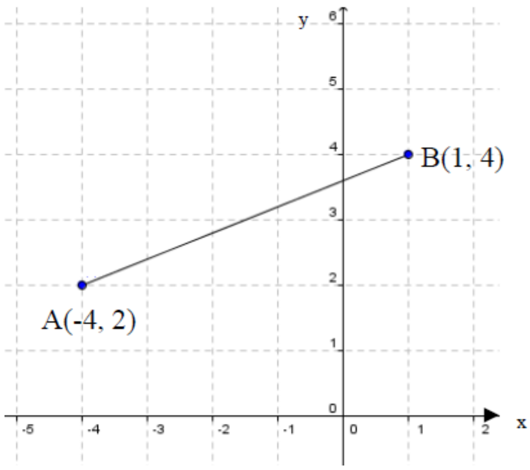
   (b) Find the length of
   (i) AC
   (ii) BC

   (c) Using Pythagoras calculate the length of AB.

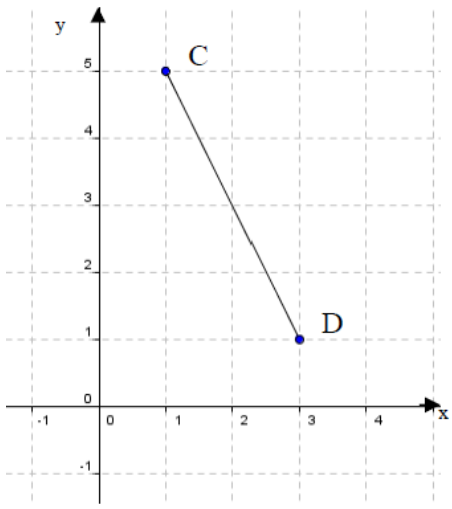


2. Determine the distance between the given points, expressing your answer as a surd in its simplest form where necessary.

(a)



(b)



(c) $E(1,1)$ and $F(7,9)$

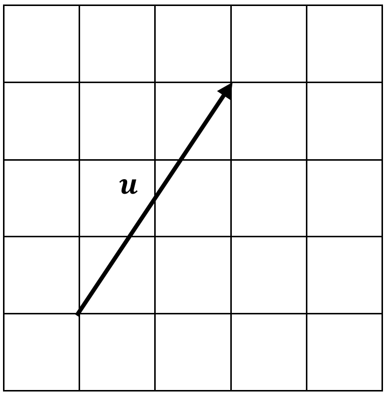
(d) $G(2,4)$ and $H(5,-2)$

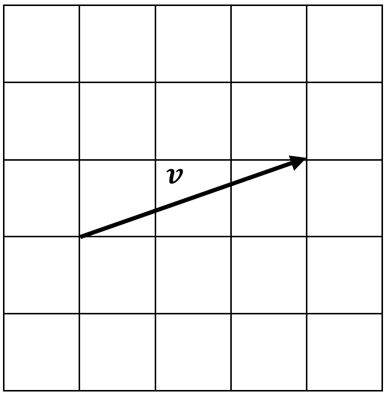
3D Coordinates and Vectors

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for 3D Coordinates and Vectors (Applications 1.2)

1. The diagrams below show 2 directed line segments $\boldsymbol{u}$ and $\boldsymbol{v}$.





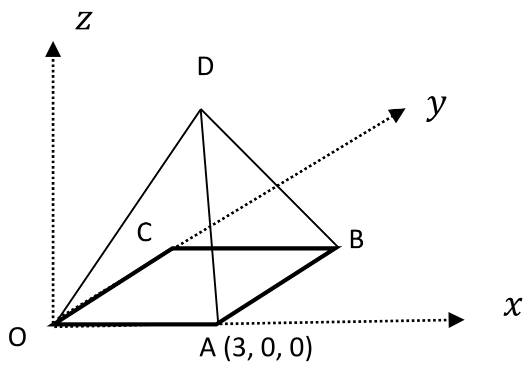
Draw the resultant of

(a) $3\boldsymbol{u} + \boldsymbol{v}$ **(b)** $2\boldsymbol{u} + 2\boldsymbol{v}$

2. The diagram below shows a square based model of a glass pyramid of height 8 cm. Square OABC has a side length of 3 cm.

The coordinates of A are (3, 0, 0).

C lies on the y-axis



Write down the coordinates of

(a) B (b) C (c) D.

3D Coordinates and Vectors

3. The forces acting on a body are represented by three vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ as given below.

$$\boldsymbol{p} = \begin{pmatrix} \mathbf{5} \\ \mathbf{4} \\ \mathbf{5 \cdot 5} \end{pmatrix} \qquad \boldsymbol{q} = \begin{pmatrix} \mathbf{2 \cdot 5} \\ \mathbf{-3} \\ \mathbf{1 \cdot 5} \end{pmatrix} \qquad \boldsymbol{r} = \begin{pmatrix} \mathbf{-7 \cdot 5} \\ \mathbf{-2} \\ \mathbf{-4} \end{pmatrix}$$

Find the resultant force.

4. The forces acting on a body are represented by three vectors **k**, **l** and **m** as given below.

$$\boldsymbol{k} = \begin{pmatrix} \mathbf{3} \\ \mathbf{2 \cdot 5} \\ \mathbf{-2} \end{pmatrix} \qquad \boldsymbol{l} = \begin{pmatrix} \mathbf{2} \\ \mathbf{3} \\ \mathbf{1 \cdot 5} \end{pmatrix} \qquad \boldsymbol{m} = \begin{pmatrix} \mathbf{-3 \cdot 5} \\ \mathbf{0} \\ \mathbf{-2} \end{pmatrix}$$

Find the resultant force.

5. Vector $\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

Calculate $|2\boldsymbol{a} + 3\boldsymbol{b}|$.

6. Vector $\boldsymbol{a} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$.

Calculate $|\boldsymbol{a} + \mathbf{2}\boldsymbol{b}|$.

3D Coordinates and Vectors

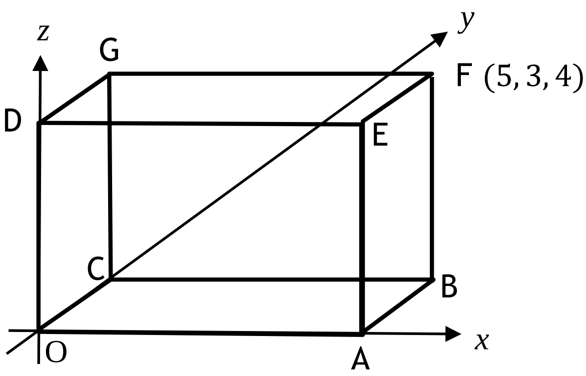
Section C – Operational Skills Section

This section provides problems with the operational skills associated with Vectors.

O1	**I can use 3D coordinates and position vectors to locate a point in 3D space.**

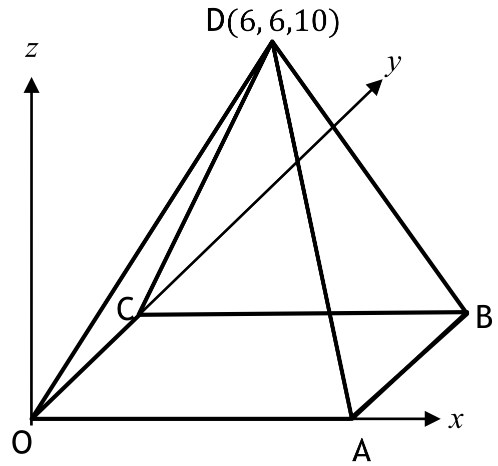
1. The diagram shows the cuboid OABCDEFG. O is the origin and OA, OC and OD are aligned with the x, y and z axes respectively.
The point F has coordinates $(5, 3, 4)$.

List the coordinates of the other six vertices.



2. The diagram shows the square based pyramid DOABC. O is the origin with OA and OC aligned with the x and y axes respectively. The point D has coordinates $(6, 6, 10)$.

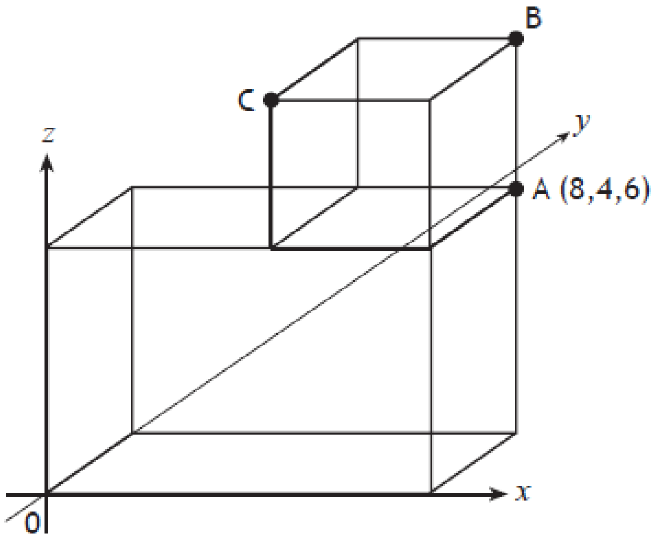
Write down the coordinates of the points A, B and C.



3. The diagram shows a cube placed on top of a cuboid, relative to the coordinate axes.

A is the point $(8, 4, 6)$.

Write down the coordinates of B and C.



3D Coordinates and Vectors

4. Three points A, B and C have the coordinates $(2,5,3)$, $(-1,3,0)$ and $(1,4,2)$ respectively. Find the vectors

(a) $\overrightarrow{OA}$ **(b)** $\overrightarrow{OB}$ **(c)** $\overrightarrow{OC}$

(d) $\overrightarrow{AB}$ **(e)** $\overrightarrow{BC}$ **(f)** $\overrightarrow{AC}$

O2	**I can add, subtract vectors and multiply a vector by a scalar to find a resultant vector.**

1. If vector $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find the resultant vector:

(a) $\boldsymbol{a}+\boldsymbol{b}$ **(b)** $\boldsymbol{a}-\boldsymbol{b}$ **(c)** $3\boldsymbol{a}+\boldsymbol{b}$

(d) $\boldsymbol{a}-2\boldsymbol{b}$ **(e)** $5\boldsymbol{a}-3\boldsymbol{b}$ **(f)** $2\boldsymbol{a}+4\boldsymbol{b}$

2. If vector $\boldsymbol{a} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$, find the resultant vector:

(a) $\boldsymbol{a}+\boldsymbol{b}$ **(b)** $\boldsymbol{a}-\boldsymbol{b}$ **(c)** $2\boldsymbol{a}+3\boldsymbol{b}$

(d) $5\boldsymbol{a}-\boldsymbol{b}$ **(e)** $3\boldsymbol{a}-2\boldsymbol{b}$ **(f)** $\boldsymbol{a}+4\boldsymbol{b}$

3. If vector $\boldsymbol{p} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ and vector $\boldsymbol{q} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$, find the resultant vector:

(a) $\boldsymbol{p}+\boldsymbol{q}$ **(b)** $\boldsymbol{p}-\boldsymbol{q}$ **(c)** $\boldsymbol{p}+2\boldsymbol{q}$

(d) $2\boldsymbol{p}-\boldsymbol{q}$ **(e)** $3\boldsymbol{p}-5\boldsymbol{q}$ **(f)** $4\boldsymbol{p}+3\boldsymbol{q}$

O3	**I can find the magnitude of vector (or resultant vector)**

1. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$, find:

(a) $|\boldsymbol{p}|$ **(b)** $|\boldsymbol{q}|$ **(c)** $|\boldsymbol{p}+\boldsymbol{q}|$

(d) $|\boldsymbol{p}-\boldsymbol{q}|$ **(e)** $|3\boldsymbol{p}-\boldsymbol{q}|$ **(f)** $|2\boldsymbol{p}+3\boldsymbol{q}|$

3D Coordinates and Vectors

2. Three vectors are defined as $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$ and $\overrightarrow{EF} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$.

Find:

(a) $|\overrightarrow{AB}|$ **(b)** $|\overrightarrow{CD}|$ **(c)** $|\overrightarrow{EF}|$

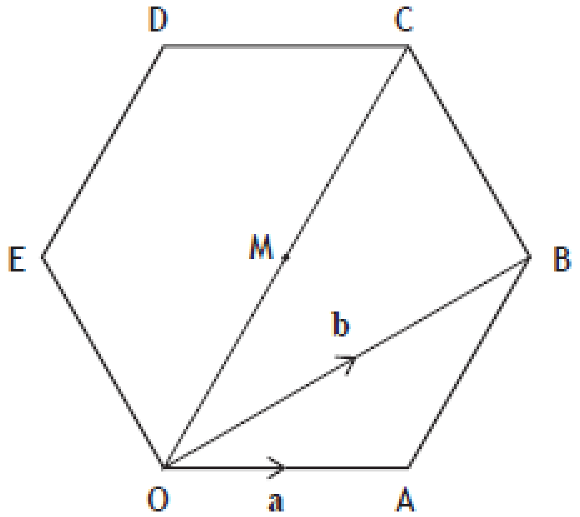
3. Three points A, B and C have the coordinates $(2, 5, 3)$, $(-1, 3, 0)$ and $(1, 4, 2)$ respectively. Find the vectors

(a) $\overrightarrow{OA}$ **(b)** $\overrightarrow{OB}$ **(c)** $\overrightarrow{OC}$

(d) $\overrightarrow{AB}$ **(e)** $\overrightarrow{BC}$ **(f)** $\overrightarrow{AC}$

O4	**I can use vectors in vector diagrams.**

1. In the diagram, OABCDE is a regular hexagon with centre M.



Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively.

(a) Express $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) Express $\overrightarrow{OC}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

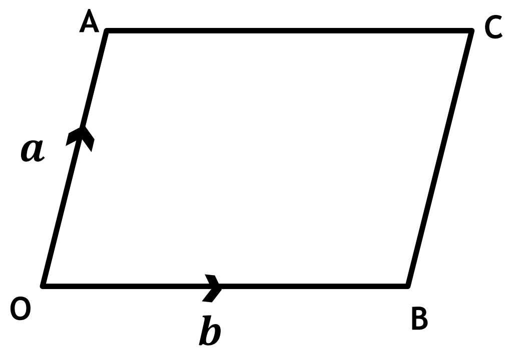
3D Coordinates and Vectors

2. In the diagram OACB is a parallelogram

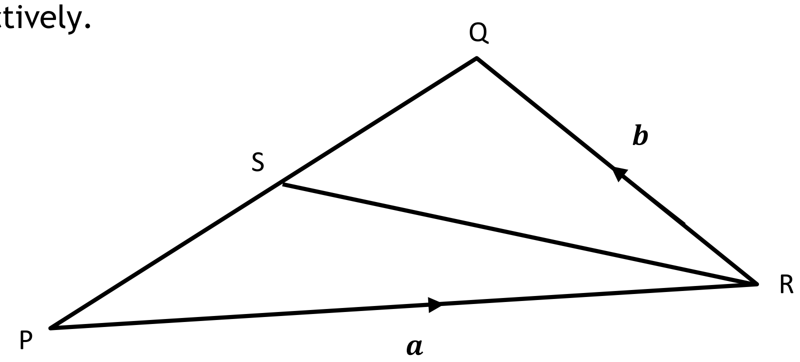
$\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$

In terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ find

(i) $\overrightarrow{OC}$ (ii) $\overrightarrow{BA}$ (iii) $\overrightarrow{CA}$



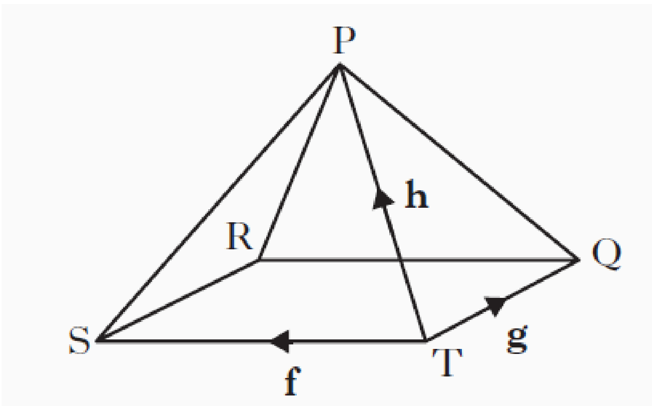
3. In the diagram below vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are represented by $\overrightarrow{PR}$ and $\overrightarrow{RQ}$ respectively.



(a) Express $\overrightarrow{PQ}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) S is the midpoint of PQ. Express $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

4. The diagram shows a square-based pyramid P, QRST.
$\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent **f**, **g** and **h** respectively.



Express $\overrightarrow{RP}$ in terms of **f**, **g** and **h**.

3D Coordinates and Vectors

Section D – Reasoning Skills Section

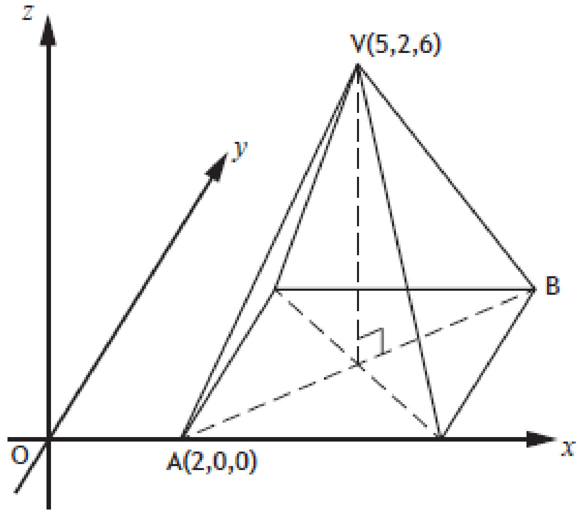
This section provides problems with the Reasoning skills associated in the context of 3D co-ordinates and Vectors.

1. The diagram shows a rectangular based pyramid, relative to the coordinate axes.

- A is the point (2, 0, 0).
- V is the point (5, 2, 6).

(a) Write down the coordinates of B.

(b) Calculate the length of edge AV of the pyramid.



2. Two forces acting on a rocket are represented by vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

$$\boldsymbol{u} = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} \text{ and } \boldsymbol{v} = \begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix}.$$

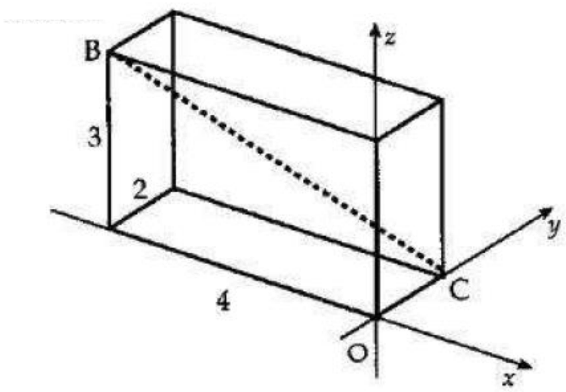
Calculate $|\boldsymbol{u} + \boldsymbol{v}|$, the magnitude of the resultant force.

Express your answer as a surd in its simplest form.

3. A cuboid crystal is placed relative to the coordinate axes as shown.

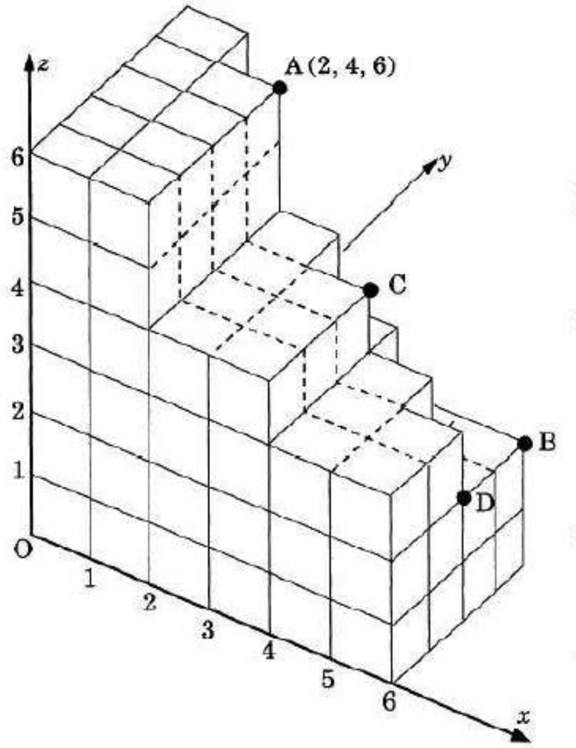
(a) Write down $\overrightarrow{BC}$ in component form.

(b) Calculate $|\overrightarrow{BC}|$.



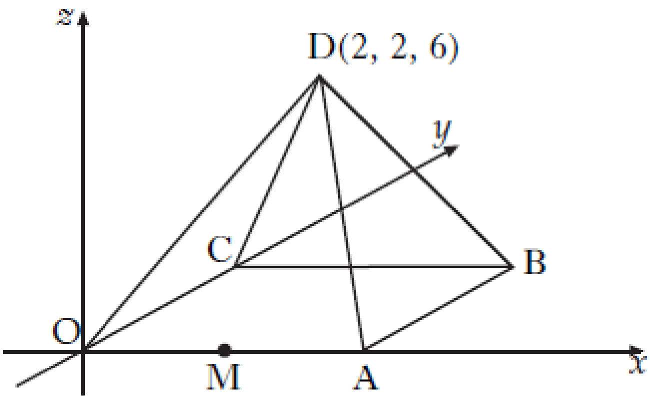
3D Coordinates and Vectors

4. With coordinate axes as shown, the point A is (2, 4, 6).



Write down the coordinates of B, C and D.

5. DOABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(a) State the coordinates of B.

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form.

3D Coordinates and Vectors

Answers

Section A

R1

1. A(3, 2) B(-2, 5) C(-6, 3) D(-4, -3) E(0, -5) F(1, -2) G(6, 0) H(-6, -5)
2. (a) J(-7, 2) K(-2, 6) L(3, 2)
   (b) M(-2, -2)

R2

1. **(a)** C (5, 1)
   (b) (i) AC = 3 (ii) BC = 4
   (c) AB = 5
2. **(a)** AB = $\sqrt{29}$ **(b)** CD = $2\sqrt{5}$ **(c)** EF = 10 **(d)** GH = $3\sqrt{5}$

Section B

1. **(a)&(b)** See Diagram
2. **(a)**B(3, 3, 0) **(b)** C(0, 3, 0) **(c)** D(1·5, 1·5, 8)
3. $\begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$
4. $\begin{pmatrix} 1 \cdot 5 \\ 5 \cdot 5 \\ -2 \cdot 5 \end{pmatrix}$
5. $\sqrt{337}$
6. $2\sqrt{13}$

Section C

O1

1. A(5, 0, 0) B(5, 3, 0) C(0, 3, 0) D(0, 0, 4) E(5, 0, 4) F(0, 3, 4)

2. A(12, 0, 0) B(12, 12, 0) C(0, 12, 0)

3. B(8, 4, 10) C(4, 0, 10)

4. **(a)** $\begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ **(d)** $\begin{pmatrix} -3 \\ -2 \\ -3 \end{pmatrix}$ **(e)** $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ **(f)** $\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

O2

1. **(a)** $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$ **(b)** $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ **(c)** $\begin{pmatrix} 9 \\ 7 \end{pmatrix}$ **(d)** $\begin{pmatrix} -4 \\ -7 \end{pmatrix}$ **(e)** $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$ **(f)** $\begin{pmatrix} 16 \\ 18 \end{pmatrix}$

3D Coordinates and Vectors

2. **(a)** $\begin{pmatrix}5\\4\\5\end{pmatrix}$ **(b)** $\begin{pmatrix}1\\-4\\-1\end{pmatrix}$ **(c)** $\begin{pmatrix}0\\-12\\-4\end{pmatrix}$ **(d)** $\begin{pmatrix}13\\-4\\3\end{pmatrix}$ **(e)** $\begin{pmatrix}5\\-8\\-1\end{pmatrix}$ **(f)** $\begin{pmatrix}-5\\-16\\-7\end{pmatrix}$

3. **(a)** $\begin{pmatrix}-2\\6\\0\end{pmatrix}$ **(b)** $\begin{pmatrix}-4\\2\\4\end{pmatrix}$ **(c)** $\begin{pmatrix}5\\8\\-2\end{pmatrix}$ **(d)** $\begin{pmatrix}-5\\6\\6\end{pmatrix}$ **(e)** $\begin{pmatrix}-18\\2\\16\end{pmatrix}$ **(f)** $\begin{pmatrix}5\\22\\2\end{pmatrix}$

O3

1. **(a)** $\sqrt{14}$ **(b)** $\sqrt{10}$ **(c)** $\sqrt{26}$ **(d)** $\sqrt{14}$ **(e)** $\sqrt{130}$

(f) $\sqrt{158}$

2. **(a)** $\sqrt{13}$ **(b)** 3 **(c)** $\sqrt{27} = 3\sqrt{3}$

3. **(a)** $\begin{pmatrix}2\\5\\3\end{pmatrix}$ **(b)** $\begin{pmatrix}-1\\3\\0\end{pmatrix}$ **(c)** $\begin{pmatrix}1\\4\\2\end{pmatrix}$ **(d)** $\begin{pmatrix}-3\\-2\\-3\end{pmatrix}$ **(e)** $\begin{pmatrix}2\\1\\2\end{pmatrix}$

(f) $\begin{pmatrix}-1\\-1\\-1\end{pmatrix}$

O4

1. **(a)** $\boldsymbol{b} - \boldsymbol{a}$ **(b)** $2(\boldsymbol{b} - \boldsymbol{a})$

2. **(i)** $\boldsymbol{b} + \boldsymbol{a}$ **(ii)** $-\boldsymbol{b} + \boldsymbol{a}$ **(iii)** $-\boldsymbol{b}$

3. **(a)** $\boldsymbol{a} + \boldsymbol{b}$ **(b)** $\frac{1}{2}(-\boldsymbol{a} - \boldsymbol{b})$

4. $-\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

Section D

1. **(a)** B (8, 4, 0) **(b)** AV = 7
2. $7\sqrt{2}$
3. **(a)** $\overrightarrow{BC} = \begin{pmatrix}4\\2\\-3\end{pmatrix}$ **(b)** $\sqrt{29}$
4. B(6, 4, 2) C(4, 3, 4) D(6, 2, 2)
5. **(a)** B (4, 4, 0) **(b)** $\overrightarrow{DB} = \begin{pmatrix}2\\2\\-6\end{pmatrix}$ $\overrightarrow{DM} = \begin{pmatrix}0\\-2\\-6\end{pmatrix}$