National 5 Portfolio

N5

Relationships 1.2 and 1.3 – Quadratics

Section A - Revision

This section will help you revise previous learning which is required in this topic.

R1   I can multiply out brackets and gather like terms

1.   Multiply out the brackets and simplify

(a) $(4x + 1)(5x + 2)$ (b) $(2x - 1)(x + 3)$ (c) $(2x - 3)^2$

(d) $(5x + 1)^2$ (e) $(5x - 3)(2x - 5)$ (f) $(7x + 2)^2$

(g) $(x - 6)(3x - 1)$ (h) $(4x + 5)(x - 2)$ (i) $3(x - 2)^2$

(j) $(6x - 5)(-3x + 4)$ (k) $(3x - 2)(x + 4)$ (l) $5(2x + 1)^2$

2.   Multiply out the brackets and simplify

(a) $(x + 4)(x^2 + 2x - 5)$ (b) $(x - 2)(3x^2 - x + 8)$

(c) $(x + 1)(3x^2 - 7x + 2)$ (d) $(-2x + 3)(x^2 - 2x + 1)$

R2   I can factorise expressions

1.   Factorise fully

(a) $25x^2 - 81$ (b) $36p^2 - 9q^2$ (c) $4x^2 - 81$

(d) $121 - 36x^2$ (e) $9x^2 - 400y^2$ (f) $64k^2 - l^2$

2.   Factorise fully

(a) $x^2 + 5x + 6$ (b) $x^2 + 11x + 10$ (c) $x^2 + 10x + 21$

(d) $x^2 + 8x + 16$ (e) $x^2 + 7x + 6$ (f) $x^2 + 8x + 15$

Quadratics

3. Factorise fully

(a) $2x^2 - 7x + 3$ (b) $2x^2 + 11x + 12$ (c) $3x^2 + 10x + 8$

(d) $x^2 + x - 6$ (e) $6x^2 + 7x + 2$ (f) $x^2 - 3x + 2$

(g) $5x^2 + 4x - 1$ (h) $7x^2 + 16x + 4$ (i) $2x^2 + 7x - 15$

4. Factorise fully

(a) $3x^2 + 6x - 24$ (b) $15x^2y + 5x$ (c) $2x^2 - 32$

(d) $5x^3 - 45x$ (e) $18x^2 - 6x - 12$ (f) $12x^2y + 8xy^3$

(g) $10x^2 + 25x - 15$ (h) $6x^3 + 30x^2 + 36x$ (i) $7x^2 - 28$

(j) $2x^2 - 10x + 12$ (k) $3x^3 - 21x^2 - 54x$ (l) $6x^3 - 63x$

R3 I can express a quadratic in the form $a(x - b)^2 + c$ (complete the square)

1. Express in the form $(x - b)^2 + c$.

(a) $x^2 + 8x + 3$ (b) $x^2 + 4x - 2$ (c) $x^2 - 6x + 4$

(d) $x^2 + 4x + 2$ (e) $x^2 - 2x - 7$ (f) $x^2 + 8x - 5$

(g) $x^2 + 5x + 2$ (h) $x^2 + 7x - 1$ (i) $x^2 + 3x + 1$

2. Express in the form $a(x - b)^2 + c$. (Higher Extension)

(a) $2x^2 + 4x + 5$ (b) $5x^2 + 10x + 1$ (c) $3x^2 - 12x + 2$

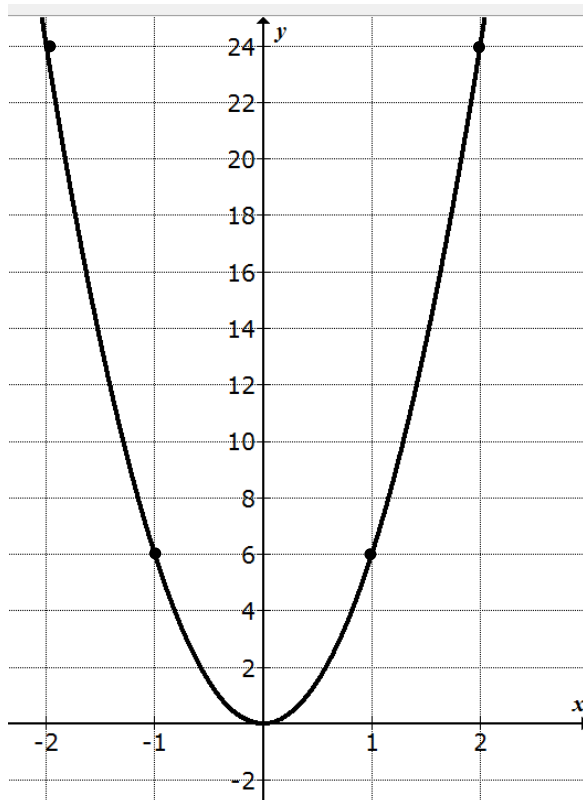(d) $4x^2 + 8x - 7$ (e) $2x^2 - 12x + 3$ (f) $5x^2 + 40x - 7$

Quadratics

Section B - Assessment Standard Section

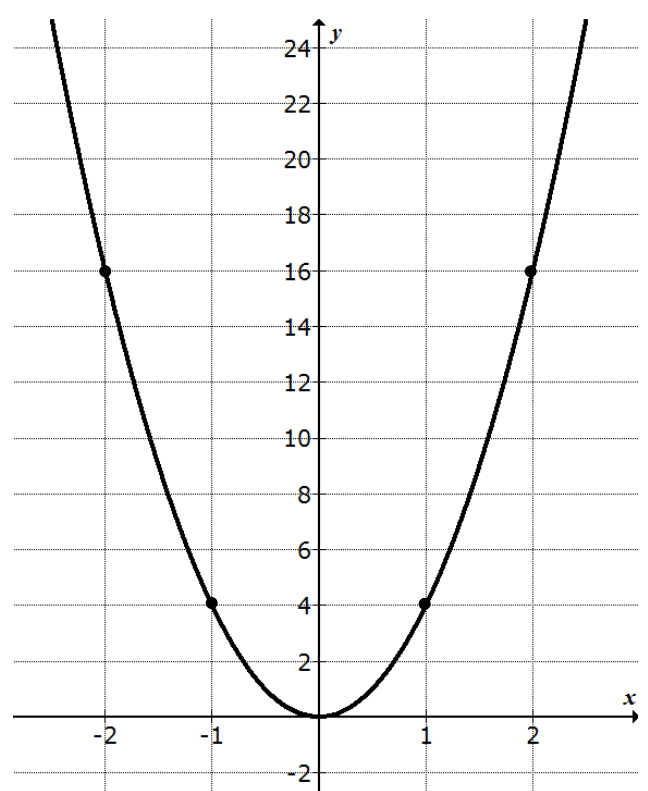This section will help you practise for your Assessment Standard Test for Quadratics (Relationships 1.2 and 1.3)

Practice Assessment Standard Questions – Relationships 1.2

1. The diagrams show parabolas with equation $y = kx^2$.

   (a)

   (b)

   In each case state the value of k?

2. Sketch the graph $y = (x + 2)(x - 4)$ and $y = (x + 3)(x - 5)$ on separate pieces of plain paper.

   For each graph mark clearly where the graph crosses the axes, the axis of symmetry and indicate clearly the turning point, stating the coordinates of the turning point.
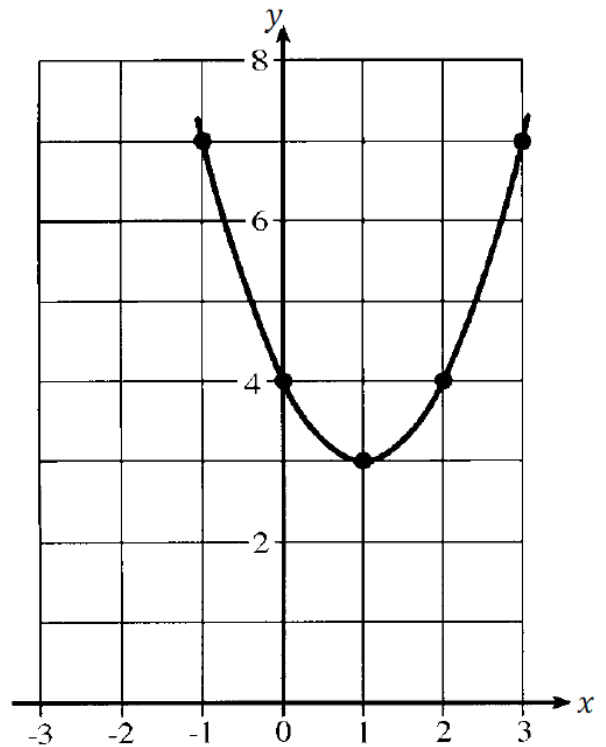

Quadratics

3. The equations of each quadratic function, whose graph are shown below, are of the form $y = (x + a)^2 + b$, where a and b are integers.
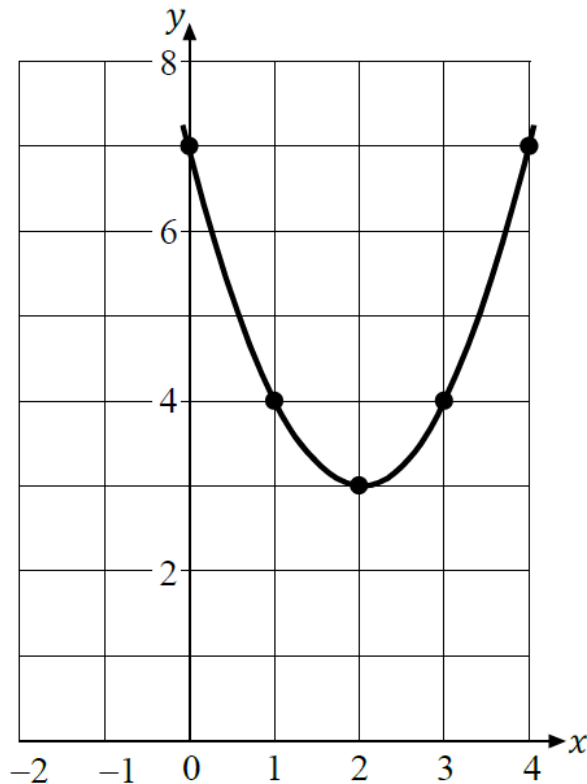
(a)   **(b)**

Write down the values of a and b.

4. For each of the parabolas below,

    (i)   Write down the equation of its axis of symmetry.

    (ii)   Write down the coordinates of the turning point on the parabola and state whether it is a maximum or minimum.

    (a)   A parabola with equation $y = (x - 2)^2 - 3$.

    (b)   A parabola with equation $y = (x + 7)^2 + 9$.

Quadratics

Practice Assessment Standard Questions – Relationships 1.3

1. Solve the equation $(x + 4)(x - 2) = 0$

2. Solve the equation $(x - 3)(x + 5) = 0$

3. Solve the equation $x^2 + 4x - 6 = 0$ using the quadratic formula.

4. Solve the equation $x^2 + 6x + 4 = 0$ using the quadratic formula.

5. Find the discriminant of the equation $x^2 - 4x + 5 = 0$.

6. Show that the equation $x^2 - 4x + 3 = 0$ has real and distinct roots.

7. Show that the equation $x^2 + 2x + 7 = 0$ has no real roots.

Quadratics

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Quadratics.

01   I can sketch and interpret Quadratic functions and Graphs

1. Sketch the graph of $y = (x + 3)(x - 1)$ showing clearly where the graph cuts the axes, the axis of symmetry and the coordinates of the turning point.

2. Sketch the graph of $y = (x - 5)(x + 3)$ showing clearly where the graph cuts the axes, the axis of symmetry and the coordinates of the turning point.

3. Sketch the graph of $y = (x + 3)^2 + 6$ showing clearly the coordinates of the turning point and the axis of symmetry.

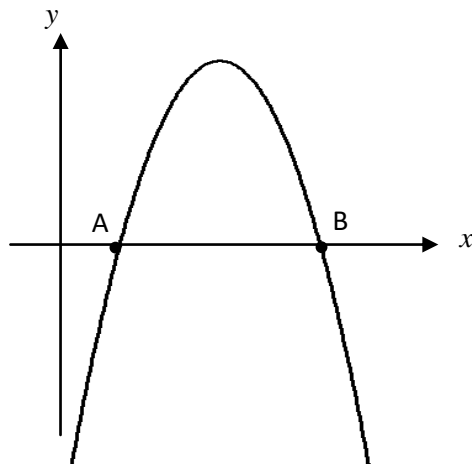4. The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.

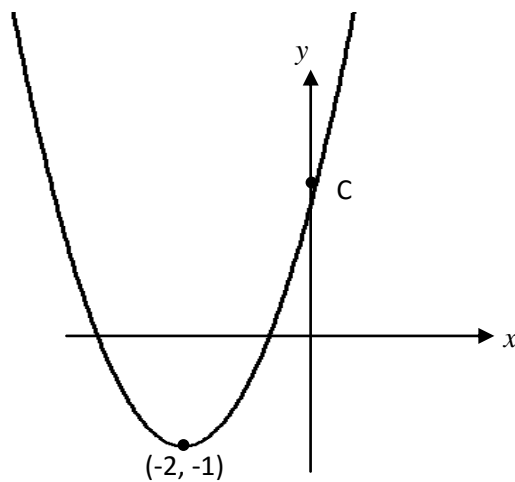   The equation of the axis of symmetry of the parabola is $x = 3$.
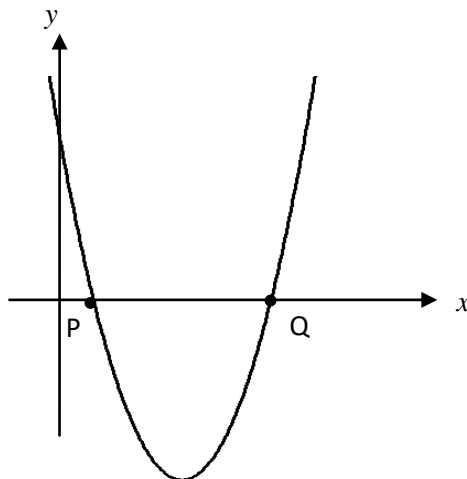
    (a) State the value of a.

    (b) P is the point (1, 0). State the coordinates of Q.

Quadratics

5. The graph below shows part of a parabola with equation of the form $y = -(x - 6)(2 - x)$.
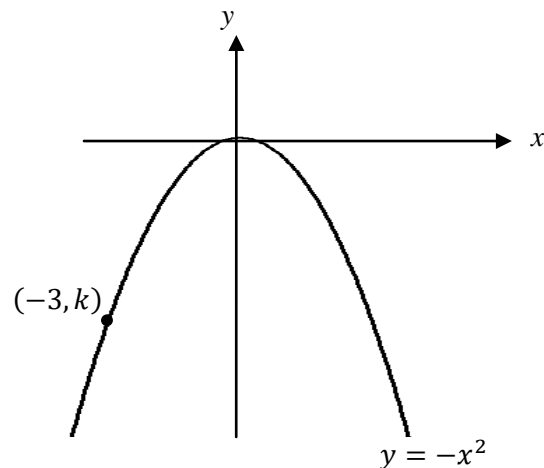

(a) State the coordinates of A and B.

(b) State the equation of the axis of symmetry.

6. The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.

(-2, -1)

(a) Write down the equation of the axis of symmetry.

(b) Write down the equation of the parabola.

(c) Find the coordinates of C.

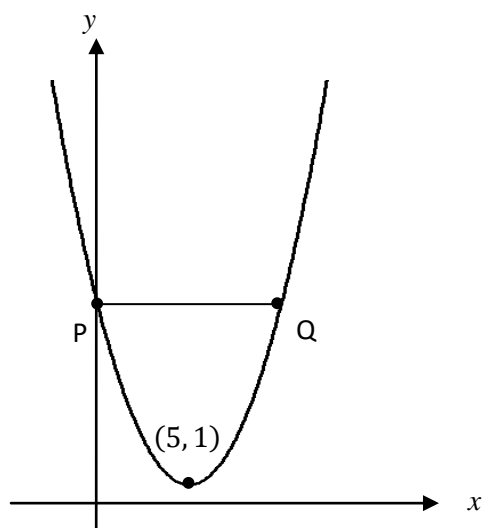# Quadratics

7. The diagram below shows the graph of $y = -x^2$.

The point $(-3, k)$ lies on the graph.

Find the value of k.

8. The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.

(a) State the values of a and b.

(b) State the equation of the axis of symmetry.

(c) The line PQ is parallel to the x-axis.

Find the coordinates of P and Q.

Quadratics

1.   The graph shows a parabola with equation $y = x^2 + 2x - 3$.

Solve the equation $x^2 + 2x - 3 = 0$.
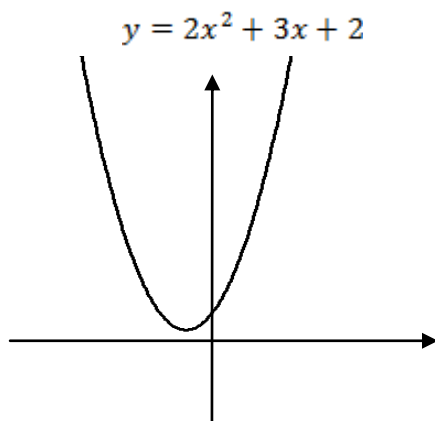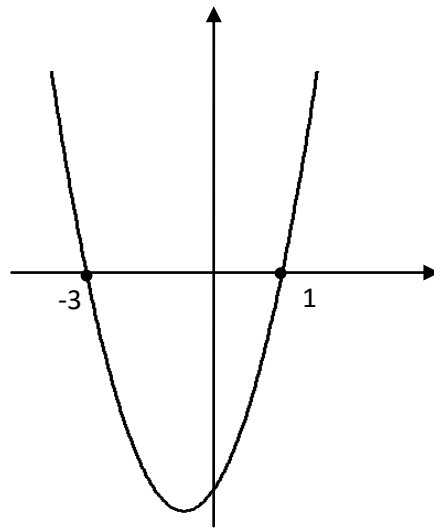
$y = 2x^2 + 3x + 2$

2.   The graph shows a parabola with equation $y = 2x^2 + 3x + 2$.

From the graph, what is clear about the roots of the equation $2x^2 + 3x + 2 = 0$

3.   The graph shows a parabola with equation $y = 8 - 2x^2$.

Solve the equation $8 - 2x^2 = 0$.

$y = 8 - 2x^2$

Quadratics

O3 I can solve Quadratic Equations (by factorisation)

1. Solve these quadratic equations

   (a) $6x^2 - 18x = 0$
   (b) $x^2 - 81 = 0$

   (c) $5x^2 - 45 = 0$
   (d) $9 - 4x^2 = 0$

   (e) $x^2 + 9x + 20 = 0$
   (f) $x^2 + 9x + 14 = 0$

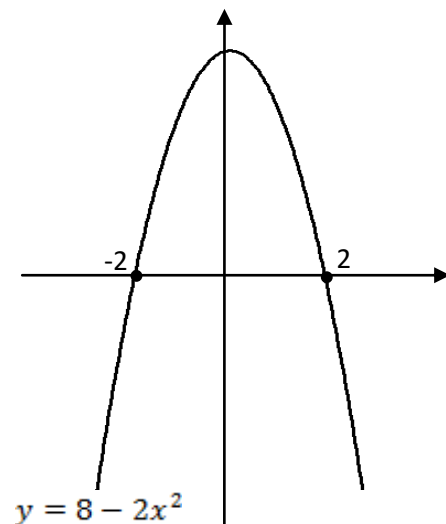   (g) $x^2 + x - 6 = 0$
   (h) $x^2 - 9x + 18 = 0$

   (i) $x^2 - 3x - 28 = 0$
   (j) $x^2 + 4x + 4 = 0$

   (k) $x^2 - 2x - 8 = 0$
   (l) $x^2 - 8x + 15 = 0$

2. Solve these quadratic equations

   (a) $x^2 - 4x = 45$
   (b) $2x^2 + 5x + 2 = 0$

   (c) $4x^2 + 14x + 6 = 0$
   (d) $6 - 5x - 6x^2 = 0$

   (e) $(x - 2)^2 = 16$
   (f) $3x^2 = 10x - 8$

   (g) $3(x + 1)^2 = 3$
   (h) $2x(4 - x) + 3(4 - x) = 0$

O4 I can solve Quadratic Equations (using the Quadratic Formula)

1. Solve these quadratic equations, giving your answers correct to 2 decimal places

   (a) $x^2 + 3x + 1 = 0$
   (b) $x^2 + 7x + 2 = 0$

   (c) $x^2 - 7x + 11 = 0$
   (d) $x^2 + x - 3 = 0$

   (e) $2x^2 + 5x + 1 = 0$
   (f) $3x^2 - 5x - 1 = 0$

   (g) $4x^2 - 13x + 2 = 0$
   (h) $2x^2 + 7x + 2 = 0$

Quadratics

2. Solve these quadratic equations, giving your answers correct to 2 decimal places

(a) $1 + 6x - x^2 = 0$

(b) $8 - x - x^2 = 0$

(c) $-2x^2 = 8x - 4$

(d) $6 - x^2 = 9x$

(e) $2 - x^2 = x(x + 1)$

(f) $2(x^2 - 2) = 3$

(g) $x - 3 + \dfrac{1}{x} = 0$

(h) $3 - 4x^2 = 10x$

O5	I can use The Discriminant to determine the nature of the roots of a Quadratic Equation

1. Determine the nature of the roots of each quadratic equation using the discriminant.

(a) $x^2 + 8x + 16 = 0$

(b) $x^2 - 3x + 4 = 0$

(c) $2x^2 + 3x - 4 = 0$

(d) $3x^2 - 7x + 2 = 0$

(e) $x^2 - 5x + 3 = 0$

(f) $2x^2 - 4x + 2 = 0$

(g) $4 - 2x - x^2 = 0$

(h) $3 - 3x + 7x^2 = 0$

2. Calculate the discriminant.

(a) $x^2 - 7x - 2 = 0$

(b) $2x^2 - 3x + 2 = 0$

(c) $4 - 7x + x^2 = 0$

(d) $3x + 7 = -5x^2 + 4$

(e) $2 - x^2 = x(x + 1)$

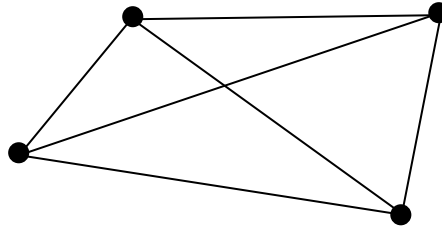(f) $x - 3 + \dfrac{1}{x} = 0$

(g) $-x^2 - 4 = 0$

(h) $\dfrac{4}{x} + x = 5$

Quadratics

Section D - Reasoning Skills Section

This section provides problems with Reasoning Skills in the context of Quadratics.

1. The minimum number of roads joining 4 towns to each other is 6 as shown.

    The minimum number of roads, r, joining n towns to each other is given by the formula $r = \frac{1}{2}n(n-1)$.

    (a) State the minimum number of roads needed to join 7 towns to each other.

    (b) When $r = 55$, show that $n^2 - n - 110 = 0$.

    (c) Hence find algebraically the value of n.

2. A rectangular garden has a length of $(x + 7)$ metres and a breadth of $(x + 3)$ metres.

    (a) Show that the area, A square metres, of the garden is given by $A = x^2 + 10x + 21$.

    (b) If the area of the garden is 45 square metres, find x.

3. A right angled triangle has dimensions, in centimetres, as shown.
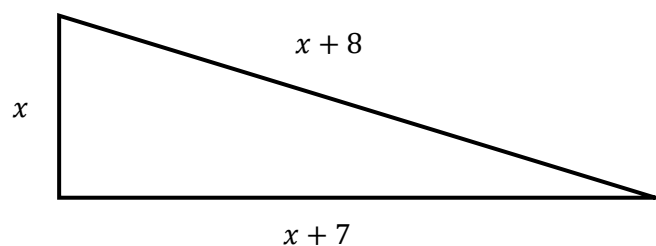
    Calculate the value of x.

Quadratics

4. The weight, W kilograms, of a giraffe is related to its age, M months, by the formula

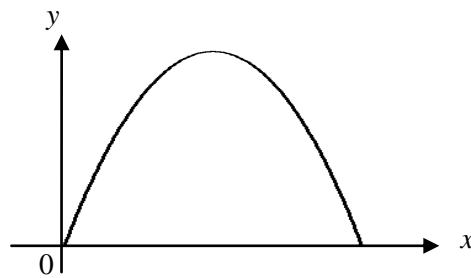$$W = \frac{1}{4}(M^2 - 4M + 272).$$

At what age will the giraffe weigh 83 kilograms?

5. The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x)$$

where y is the profit (in pounds) and x is selling price (in pence) of the magazine.

The graph below represents the profit y against the selling price x.

Find the maximum profit the company can make from the sale of the magazine.
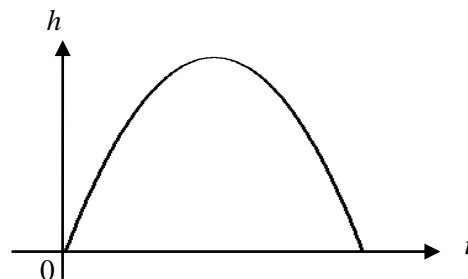
6. The diagram below shows the path of a rocket which is fired into the air.

The height, h metres, of the rocket after t seconds is given by

$$h(t) = -2t(t - 14).$$

(a) For how many seconds is the rocket in flight?
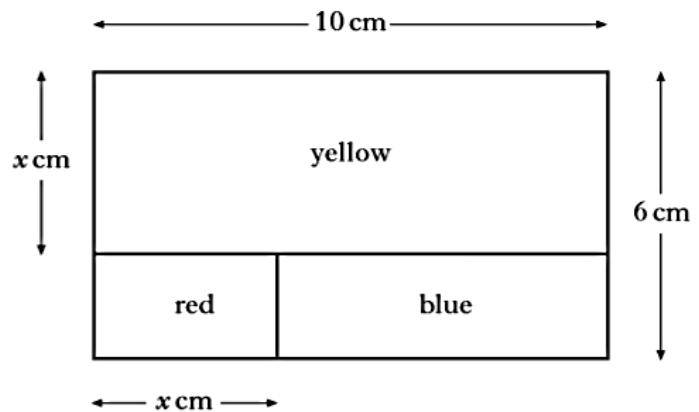
(b) What is the maximum height reached by the rocket?

Quadratics

7. A decorator's logo is rectangular and measures 10 centimetres by 6 centimetres.

It consists of three rectangles: one red, one yellow and one blue.

The yellow rectangle measures 10 centimetres by x centimetres. The width of the red rectangle is x centimetres.

(a) Show that the area, A, of the blue rectangle is given by the expression

$$A = x^2 - 16x + 60.$$

(b) The area of the blue rectangle is equal to $\frac{1}{5}$ of the total area of the logo.

Calculate the value of x.

8. $ax^2 + 4x - 2 = 0$ has equal roots.

Find a.

9. $x^2 + bx + 25 = 0$ has 1 root.

Find 2 values for b.

10. $px^2 + 8x - 2 = 0$ has 2 real roots.

Set up an inequality in p, and solve for p.

11. $x^2 + x - t = 0$ has no real roots.

Solve for t.

Quadratics

Answers

Section A - Revision

R1

Q1 (a) $20x^2 + 13x + 2$
    (b) $2x^2 + 5x - 3$

(c) $4x^2 - 12x + 9$
(d) $25x^2 + 10x + 1$

(e) $10x^2 - 31x + 15$
(f) $49x^2 + 28x + 4$

(g) $3x^2 - 19x + 6$
(h) $4x^2 - 3x - 10$

(i) $3x^2 - 12x + 12$
(j) $-18x^2 + 39x - 20$

(k) $3x^2 + 10x - 8$
(l) $20x^2 + 20x + 5$

Q2 (a) $x^3 + 6x^2 + 3x - 20$
    (b) $3x^3 - 7x^2 + 10x - 16$

(c) $3x^3 - 4x^2 - 5x + 2$
(d) $-2x^3 + 7x^2 - 8x + 3$

R2

Q1 (a) $(5x - 9)(5x + 9)$
    (b) $9(2 - q)(2 + q)$

(c) $(2x - 9)(2x + 9)$
(d) $(11 - 6x)(11 + 6x)$

(e) $(3x - 20y)(3x + 20y)$
(f) $(8k - l)(8k + l)$

Q2 (a) $(x + 2)(x + 3)$
    (b) $(x + 1)(x + 10)$

(c) $(x + 3)(x + 7)$
(d) $(x + 4)(x + 4)$

(e) $(x + 1)(x + 6)$
(f) $(x + 3)(x + 5)$

Q3 (a) $(2x - 1)(x - 3)$
    (b) $(2x + 3)(x + 4)$

(c) $(3x + 4)(x + 2)$
(d) $(x - 2)(x + 3)$

(e) $(3x + 2)(2x + 1)$
(f) $(x - 1)(x - 2)$

(g) $(5x - 1)(x + 1)$
(h) $(7x + 2)(x + 2)$

(i) $(2x - 3)(x + 5)$

Q4 (a) $3(x - 2)(x + 4)$
    (b) $5x(3xy + 1)$

(c) $2(x - 4)(x + 4)$
(d) $5x(x - 3)(x + 3)$

(e) $6(3x + 2)(x - 1)$
(f) $4xy(3x + 2y^2)$

Quadratics

    (g) $5(2x-1)(x+3)$ (h) $6x(x+2)(x+3)$

    (i) $7(x-2)(x+2)$ (j) $2(x-2)(x-3)$

    (k) $3x(x-9)(x+2)$ (l) $3x(2x^2-21)$

R3

Q1   (a) $(x+4)^2-13$ (b) $(x+2)^2-6$

      (c) $(x-3)^2-5$ (d) $(x+2)^2-2$

      (e) $(x-1)^2-8$ (f) $(x+4)^2-21$

      (g) $\left(x+\frac{5}{2}\right)^2-\frac{17}{4}$ (h) $\left(x+\frac{7}{2}\right)^2-\frac{53}{4}$

      (i) $\left(x+\frac{3}{2}\right)^2-\frac{5}{4}$

Q2   (a) $2(x+1)^2+3$ (b) $5(x+1)^2-4$

      (c) $3(x-2)^2-10$ (d) $4(x+1)^2-11$

      (e) $2(x-3)^2-15$ (f) $5(x+4)^2-87$

Section B - Practice Assessment Standard Questions
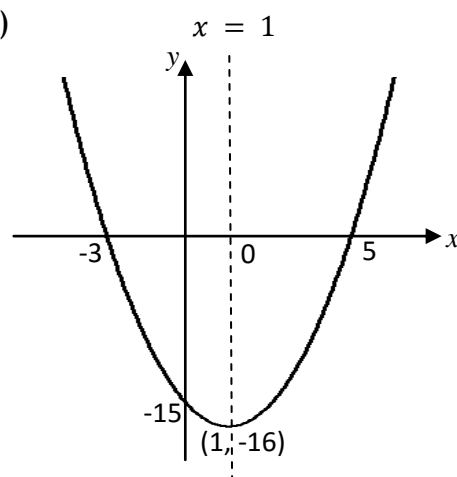
Relationships 1.2

Q1   (a) $k=6$ (b) $k=4$

Q2   (a) $x=1$ (b) $x=1$
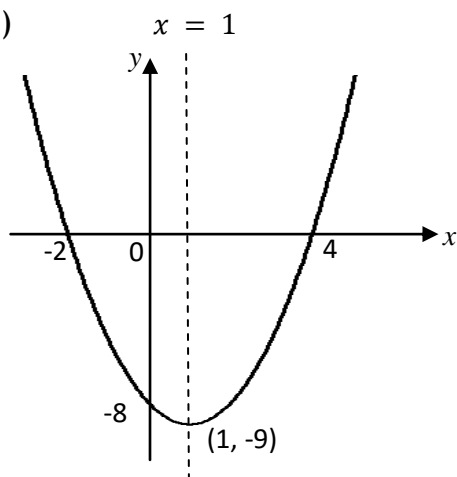
Q3   (a) $a=-2$ $b=3$ (b) $a=-1$ $b=3$

Quadratics

Q4   (a) $y = (x-2)^2 - 3$ (i) $x = 2$ (ii) minimum turning point $(2, -3)$

       (b) $y = (x+7)^2 + 9$ (i) $x = -7$ (ii) minimum turning point $(-7, 9)$

Relationships 1.2

Q1 $x = -4, \; x = 2$ Q2 $x = 3, \; x = -5$

Q3 $x = -5\cdot16, \; x = 1\cdot16$ Q4 $x = -5\cdot24, \; x = -0\cdot76$
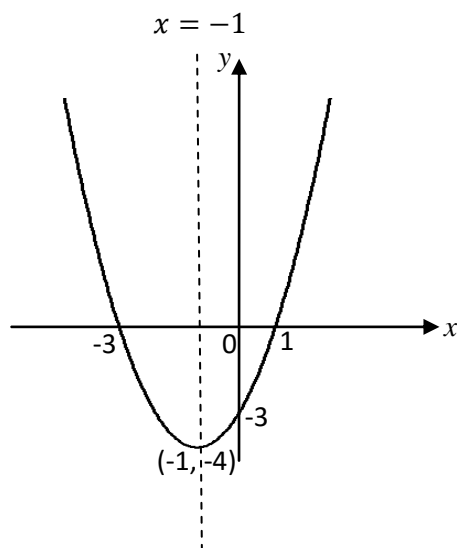
Q5 $b^2 - 4ac = -4.$

Q6 $b^2 - 4ac = 4$, $\therefore$ since 4 > 0, the quadratic equation has 2 real and distinct roots.

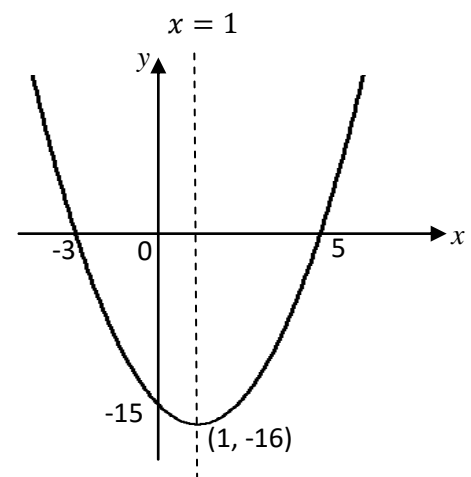Q7 $b^2 - 4ac = -24$, $\therefore$ since -24 < 0, the quadratic equation has no real roots.

Section C – Operational skills

O1

Q1

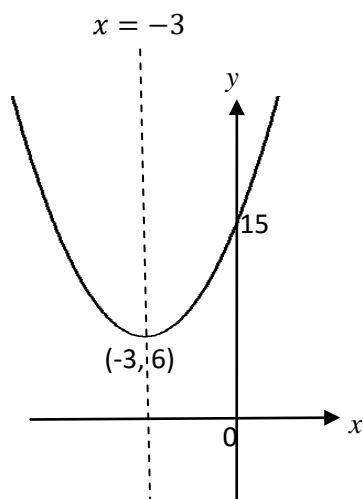Q2

Q3

Quadratics

Q4 (a) $a = -3$ (b) $Q\,(5,0)$

Q5 (a) $A\,(2,0)\ B\,(6,0)$ (b) $x = 4$

Q6 (a) $x = -2$ (b) $y = (x+2)^2 - 1\ \ or\ \ y = x^2 + 4x + 3$

(c) $C\,(0,3)$

Q7 $k = -9$

Q8 (a) $a = -5\ b = 1$ (b) $x = 5$ (c) $P\,(0,26)\ Q\,(10,26)$

O2

Q1 $x = -3\ \ x = 1$

Q2 *There are no real roots*

Q3 $x = -2\ \ x = 2$

O3

Q1 (a) $x = 0\ x = 3$ (b) $x = -9\ x = 9$ (c) $x = -3\ x = 3$

(d) $x = -\frac{3}{2}\ x = \frac{3}{2}$ (e) $x = -5\ x = -4$ (f) $x = -7\ x = -2$

(g) $x = -3\ x = 2$ (h) $x = 3\ x = 6$ (i) $x = -4\ x = 7$

(j) $x = -2\ x = -2$ (k) $x = -2\ x = 4$ (l) $x = 3\ x = 5$

Q2 (a) $x = -5\ x = 9$ (b) $x = -2\ x = -\frac{1}{2}$ (c) $x = -3\ x = -\frac{1}{2}$

(d) $x = -\frac{3}{2}\ x = \frac{2}{3}$ (e) $x = -2\ x = 6$ (f) $x = \frac{4}{3}\ x = 2$

(g) $x = 0\ x = -2$ (h) $x = -\frac{3}{2}\ x = 4$

O4

Q1 (a) $x = -2 \cdot 62\ x = -0 \cdot 38$ (b) $x = -6 \cdot 70\ x = -0 \cdot 30$

(c) $x = 2 \cdot 38\ x = 4 \cdot 62$ (d) $x = -2 \cdot 30\ x = 1 \cdot 30$

(e) $x = -2 \cdot 28\ x = -0 \cdot 22$ (f) $x = -0 \cdot 18\ x = 1 \cdot 85$

(g) $x = 0 \cdot 16\ x = 3 \cdot 09$ (h) $x = -3 \cdot 19\ x = -0 \cdot 31$

Quadratics

Q2 (a) $x = -0\cdot16$ $x = 6\cdot16$ (b) $x = -3\cdot37$ $x = 2\cdot37$

(c) $x = -4\cdot45$ $x = 0\cdot45$ (d) $x = -9\cdot62$ $x = 0\cdot62$

(e) $x = -1\cdot28$ $x = 0\cdot78$ (f) $x = -1\cdot87$ $x = 1\cdot87$

(g) $x = 0\cdot38$ $x = 2\cdot62$ (h) $x = -2\cdot77$ $x = 0\cdot27$

O5

Q1 (a) *real and equal roots*   (b) *no real roots*

(c) *real and distinct roots*   (d) *real and distinct roots*

(e) *real and distinct roots*   (f) *real and equal roots*

(g) *real and distinct roots*   (h) *no real roots*

Q2 (a) 57   (b) -7 (c) 33   (d) -51 (e) 17

(f) 5   (g) -16 (h) 9

Section D - Reasoning Skills Section

Q1 (a) $r = 21$ (b) $n^2 - n - 110 = 0$ *as required*   (c) $n = 11$

Q2 (a) $A = x^2 + 10x + 21$ *as required*   (b) $x = 2$

Q3 $x = 5$

Q4 10 *months*

Q5 £19 600

Q6 (a) 14 *seconds*   (b) 98 *metres*

Q7 (a) $A = x^2 - 16x + 60$ *as required*   (b) $x = 4$

Q8 $a = -2$

Q9 $b = -10$ and $b = 10$

Q10 $p > -8$

Q11 $t < -\frac{1}{4}$