National 5 Portfolio

N5

Relationships 1.1 – Linear Relationships

Section A - Revision

This section will help you revise previous learning which is required in this topic.

R1   I can solve simple linear equations and inequalities.

1.   Solve the following equations

(a) $5x + 3 = 2x + 24$ (b) $6a - 3 = 2a + 17$

(c) $3b + 2 = 7b - 22$ (d) $p + 9 = 5p + 1$

(e) $15 - 2r = 3r$ (f) $21 - 3s = s + 5$

2.   Solve these equations

(a) $4(x + 3) = 28$ (b) $3(x + 5) = 9$

(c) $9(x + 3) = -27$ (d) $14 - x = 2 - 7x$

(e) $4(4 - x) = 3(5 - 2x)$ (f) $3(x + 2) + 5(x + 1) = x + 4$

3.   Solve the following

(a) $6x + 4 < 3x + 7$ (b) $7t + 5 \geq 2t + 30$

(c) $7x + 11 > 5x - 9$ (d) $10m - 3 \leq m + 6$

(e) $3y - 11 > 2y + 5$ (f) $5p - 9 < 2p$

(g) $14 + 8c \geq c$ (h) $3a + 2 \leq a + 2$

(i) $9q - 1 > 2q - 1$

Linear Relationships

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Linear Relationships (Relationships 1.1)

Practice Assessment Standard Questions

1. A straight line with gradient –1 passes through the point (4, -2). Determine the equation of this straight line.

2. A straight line with gradient –3 passes through the point (– 2, 5). Determine the equation of this straight line.

3. Solve the inequations   (a) $4p - 8 < p - 3.$

    (b) $5p - 12 < 2p - 17.$

4. (a)   Solve the following system of equations algebraically:

    $$x - 8y = 2$$
    $$2x + 4y = 24$$

    (b)   Solve the following system of equations algebraically:

    $$5a + 4b = 11$$
    $$3a + 2b = 7$$

5. Andrew and Margaret each book in at the Sleepwell Lodge.

    (a) Andrew stays for 3 nights and has breakfast on 2 mornings. His bill is £145. Write down an algebraic equation to illustrate this.

    (b) Margaret stays for 5 nights and has breakfast on 3 mornings. Her bill is £240. Write down an equation to illustrate this.

    (c) Find the cost of one breakfast.

6. In this formula change the subject to C.

    (a) $F = \dfrac{6C}{7} - 12$ (b) $X = \dfrac{8C}{3} + 16$

Linear Relationships

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Linear Relationships

O1	I understand and can use function notation

1. Given that $f(x) = x^2 + 6$, find

    (a) $f(3)$ (b) $f(-3)$ (c) $f(0)$

2. Given that $g(x) = 3x - x^2$, find

    (a) $g(2)$ (b) $g(-1)$ (c) $g(0)$

3. Given that $f(x) = 5x + 6$,

    (a)   find x when $f(x) = 21$

    (b)   find t when $f(t) = -14$

    (c)   find p when $f(2p) = 46$

4. Given that $f(x) = 7 - 4x$

    (a) find $f(-2)$.

    (b) given that $f(t) = 9$, find t.

5. Given that $g(x) = \frac{2x+1}{3}$ and that $g(n) = -3$, find n.

6. Function f is defined by $(x) = \frac{8}{1-x}$, $x \neq 1$.

    (a) Why can x not be equal to 1?

    (b) find   (i) $f(0)$ (ii) $f(-1)$ (iii) $f(\frac{1}{2})$

    (c) given that $f(x) = 2$, find x.

Linear Relationships

O2	I can solve problem which involve the equation of a Straight Line

1. The equations below represent six different straight lines. For each straight line state the gradient and the y-intercept.

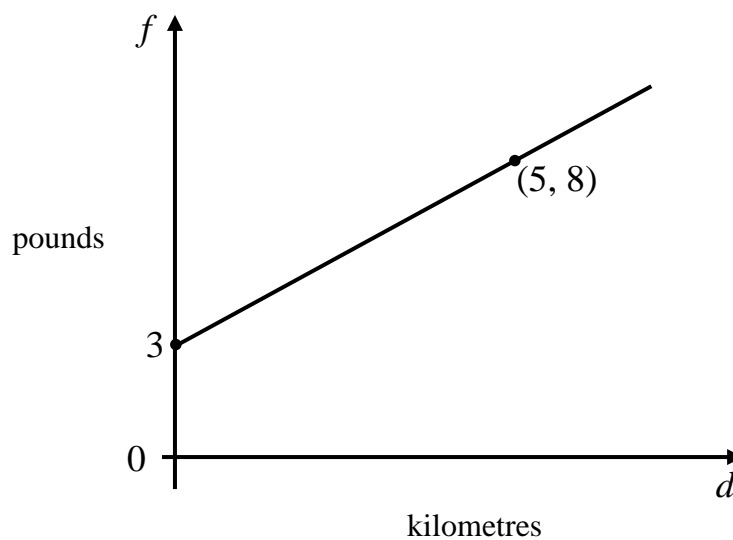   (a) $y = 3x + 2$ (b) $y = 5 - 2x$ (c) $2x + y = 7$

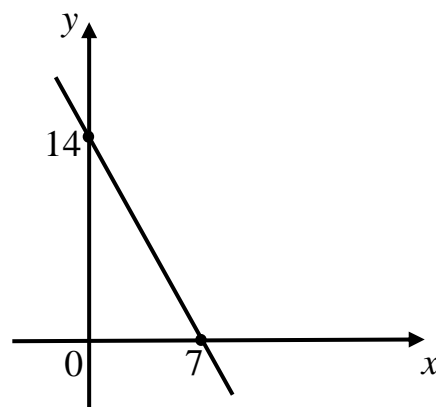   (d) $x + 3y = -1$ (e) $4x - y = 3$ (f) $2x - 3y + 1 = 0$

2. The graph shows the fare, f pounds for a journey of d kilometres.

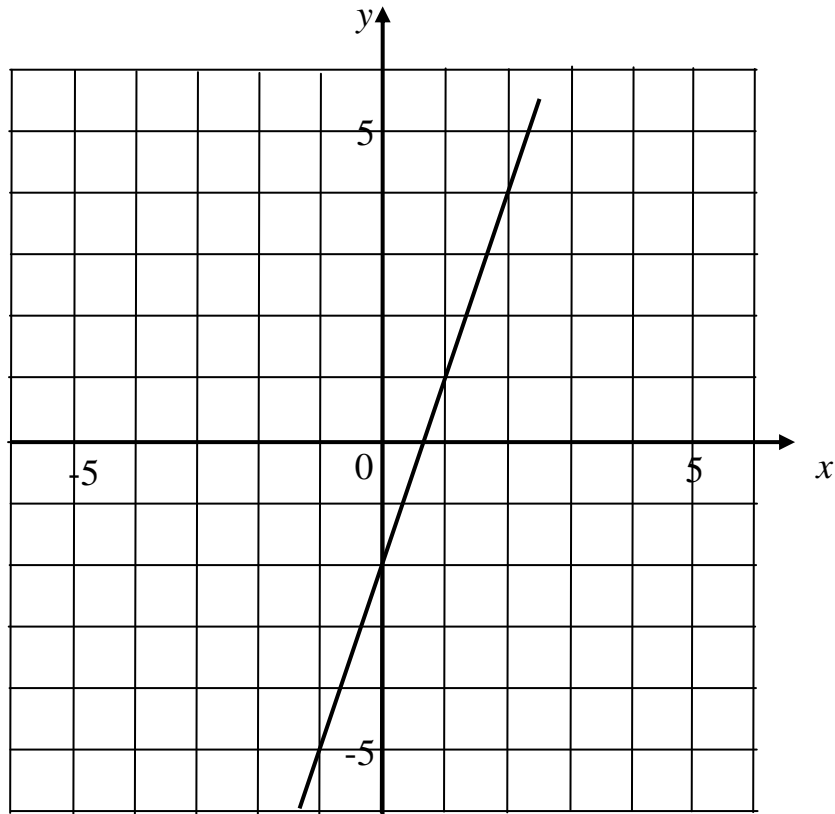Write down the equation of the straight line.

3.

The straight line shown cuts the axes at $(0, 14)$ and $(7, 0)$
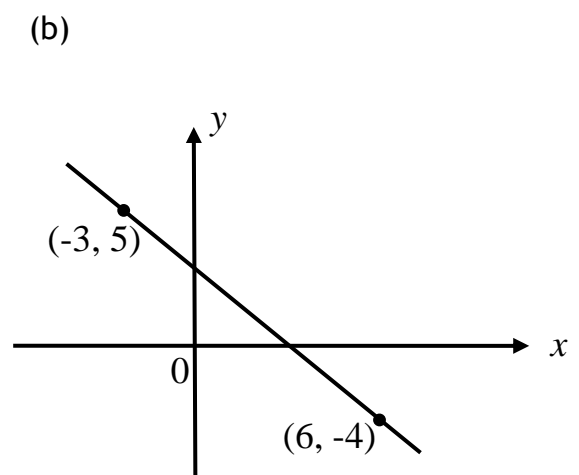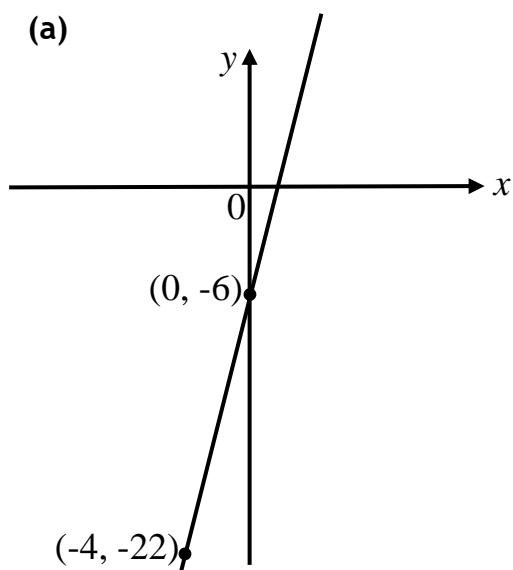
Write down the equation of the straight line.

Linear Relationships

4.

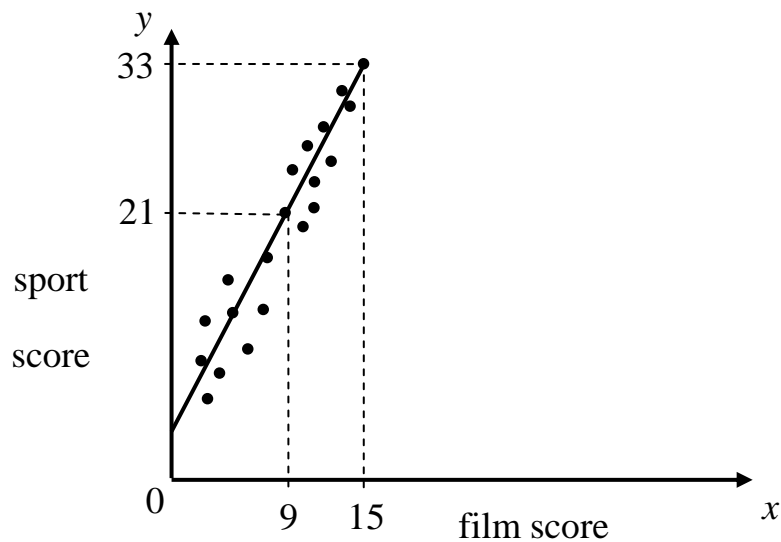
Write down the equation of the straight line.

5. Write down the equation of the straight lines below.

(a)

(b)

Linear Relationships

6. A quiz has a sports round and a film round. The scores for each team taking part are plotted on the scattergraph shown and a line of best fit is drawn.

Find the equation of this line of best fit.

7. A straight line graph is represented by $y = mx + c$.

Sketch possible straight line graphs to illustrate this equation when

(a) $m > 0$ and $c < 0$.

(b) $m < 0$ and $c > 0$.

(c) $m > 0$ and $c = 0$.

Linear Relationships

1.   Solve these equations

(a) $14 + 8x = x$ (b) $10 - 3(2x + 1) = x$

(c) $-3(x + 5) = -27$ (d) $-4(3x - 2) = -16$

(e) $8 - 2(x - 1) = 4$ (f) $x - 2(x + 1) = 8$

(g) $2x - 3(3x + 1) = 18$ (h) $8 - 3(x - 2) = 15 - 5(x + 1)$

2.   Solve these equations

(a) $\frac{1}{4}x = 3$ (b) $\frac{2}{7}x = 2$

(c) $\frac{2}{3}x = -4$ (d) $\frac{x}{2} - \frac{x}{5} = 6$

(e) $\frac{x}{3} = 1 + \frac{x}{4}$ (f) $2x - \frac{(3x-1)}{4} = 4$

(g) $\frac{1}{3}(x + 2) - \frac{1}{4}x = 1$ (h) $10 - \frac{1}{3}(x + 2) = \frac{1}{4}(x + 21)$

3.   Solve these Inequalities

(a) $3 - 2x < 4x - 3$ (b) $3x > 2 - (x - 6)$

(c) $-2(1 + 3x) \leq 3x + 1$ (d) $4 - 2x \leq 6$

(e) $1 - 3x \leq 2(x + 5) - 2x$ (f) $\frac{x}{4} - \frac{1}{2} < 5$

(g) $2x < 3 - (x + 6)$ (h) $12 + 3(2 - 3x) \leq 4 - 5x$

Linear Relationships

O4	I can solve systems of simultaneous equation

1.   Solve these simultaneous equations

(a) $2x + y = 15$

   $x - y = 6$

(b) $3x + y = 9$

   $x + y = 5$

(c) $2x - 5y = -21$

   $3x + 10y = 56$

(d) $3x + 4y = 10$

   $6x + 5y = 17$

(e) $3x - 5y = 8$

   $x - 7y = 8$

(f) $7x + 3y = -13$

   $3x + y = -5$

(g) $4x - y = 16$

   $5x + 3y = 3$

(h) $3x - 2y = 10$

   $x + y = 5$

2.   Solve these simultaneous equations

(a) $5x + 2y = 9$

   $2x + 3y = 8$

(b) $3x + 5y = 22$

   $5x - 2y = 16$

(c) $5x - 3y = 12$

   $7x - 2y = 19$

(d) $4x - 5y = -22$

   $3x + 2y = -5$

(e) $7x + 8y = -5$

   $9x + 10y = -7$

(f) $6x + 5y - 9 = 0$

   $2y - 9x + 42 = 0$

(g) $y = 2x + 8$

   $x - y = -5$

(h) $y = 3x - 14$

   $x - 2y = 13$

Linear Relationships

O5   I can Change the subject of a formula

1.   Change the subject of each formula to the given variable.

(a) $y = 2x + 3$ (x) (b) $p = sr^2 + t$ (s)

(c) $k = 5m^2 - n$ (m) (d) $y = sr^2 + 3\sqrt{x}$ (x)

(e) $h = 7 - g$ (g) (f) $3p = 5 - 2t$ (t)

(g) $x^2 = \dfrac{2yr}{t}$ (y) (h) $a = \dfrac{3c^2}{2b}$ (c)

(i) $h = \sqrt{\dfrac{k}{g}}$ (k) (j) $y = \dfrac{x}{2} - 3$ (x)

2.   Change the subject of each formula to the given variable.

(a) $p = \dfrac{8}{t}$ (t) (b) $d = \dfrac{a+1}{x}$ (x)

(c) $b = \dfrac{3 - y}{4}$ (y) (d) $h = \dfrac{m}{x} + k$ (x)

(e) $k = \dfrac{5m + 3n}{2}$ (m) (f) $p = \dfrac{3a - 5b}{c}$ (b)

(g) $y = \dfrac{1}{3}(a - b)$ (a) (h) $y = \sqrt{\dfrac{x}{z}}$ (z)

3.   Change the subject of each formula to the given variable.

(a) $L = \dfrac{1}{2}(h - t)$ (t) (b) $a = \dfrac{b}{c - x}$ (x)

(c) $d = \dfrac{t(n-1)}{n}$ (n) (d) $y = \sqrt{\dfrac{x-1}{x+2}}$ (x)

(e) $na = \sqrt{1 - n^2}$ (n) (f) $k = \dfrac{b(d+2)}{d}$ (d)

Linear Relationships

Section D - Reasoning Skills Section

This section provides problems with Reasoning Skills in the context of Linear Relationships.

1. Suzie has a new mobile phone. She is charged x pence per minute for calls and y pence for each text she sends. During the first month her calls last a total of 280 minutes and she sends 70 texts. Her bill is £52·50.

   (a) Write down an equation in x and y which satisfies the above condition.

   The next month she reduces her bill. She restricts her calls to 210 minutes and sends 40 texts. Her bill is £38·00.

   (b) Write down a second equation in x and y which satisfies this condition.

   (c) Calculate the price per minute for a call and the price for each text.

2. Alan is taking part in a quiz. He is awarded x points for each correct answer and y points for each wrong answer. During the quiz, Alan gets 24 questions correct and 6 wrong. He scores 60 points.

   (a) Write down an equation in x and y which satisfies the above condition.

   Helen also takes part in the quiz. She gets 20 questions correct and 10 wrong. She scores 40 points.

   (b) Write down a second equation in x and y which satisfies this condition.

   (c) Calculate the score for David who gets 17 correct and 13 wrong.

3. Two straight lines, $y = 2x - 3$ and $x + 2y = 14$ intersect and the point P.

   Find, algebraically, the coordinates of the point P.

Linear Relationships

4. A straight line has equation $y = mx + c$, where m and c are constants.

    (a) The point $(2, 7)$ lies on this line.

        Write down an equation in m and c to illustrate this information.

    (b) A second point $(4, 17)$ also lies on this line.

        Write down another equation in m and c to illustrate this information.

    (c) Hence calculate the values of m and c.

    (d) Write down the gradient and the coordinates of the y intercept of this line.
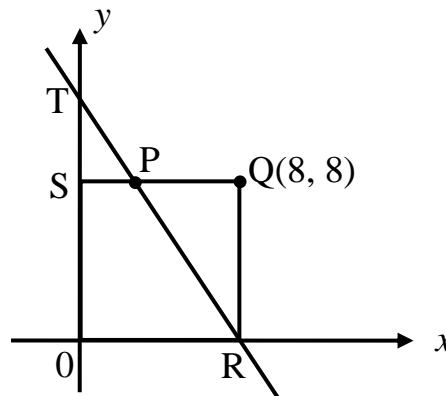
5. A straight line with equation $y = \frac{1}{3}x + 2$ cuts the x-axis at B.

    (a) Find the coordinates of B.

    (b) For what values of x is $y < 0$.

6. A square, OSQR, is shown below.

    Q is the point $(8, 8)$

The straight line TR cuts the y-axis at T$(0, 12)$ and the x-axis at R.

    (a) Find the equation of the line TR.

The line TR also cuts SQ at P.

    (b) Find the coordinates of P.

Linear Relationships

7. Jane enters a two-part race.

(a) She cycles for 2 hours at a speed of $(x + 8)$ kilometres per hour.

Write down an expression in x for the distance cycled.

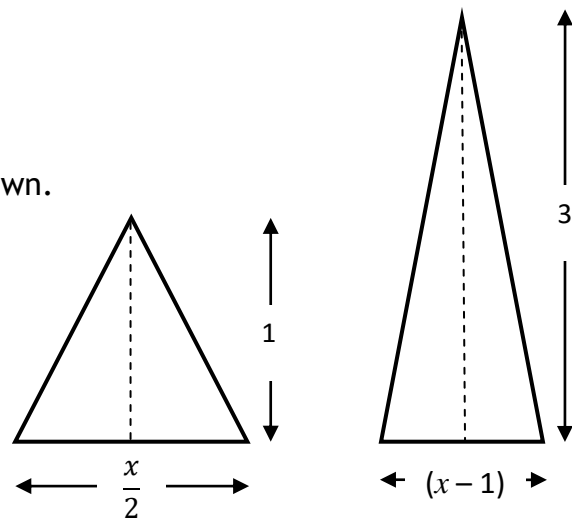(b) She then runs for 30 minutes at a speed of x kilometres per hour.

Write down an expression in x for the distance run.

(c) The total distance of the race is 46 kilometres.
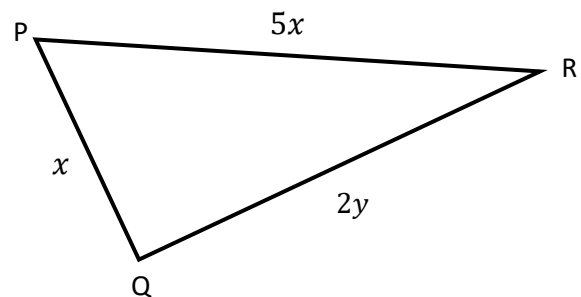
Calculate Jane's running speed.

8. Two triangles have dimensions as shown.

The triangles are equal in area.

Calculate the value of x.

9. The diagram show triangle PQR.

(a) The perimeter of PQR is 42 centimetres.

Write down an equation in x and y to illustrate this information.

(b) PR is 2 centimetres longer than QR.

Write down a second equation in x and y to illustrate this information.

(c) Hence calculate the values of x and y.

Linear Relationships

Answers

Section A – Revision

R1

Q1 (a) $x = 7$ (b) $a = 5$ (c) $b = 6$ (d) $p = 2$ (e) $r = 3$

(f) $s = 4$

Q2 (a) $x = 4$ (b) $x = -2$ (c) $x = -6$ (d) $x = -2$ (e) $x = -\frac{1}{2}$

(f) $x = -1$

Q3 (a) $x < 1$ (b) $t \geq 5$ (c) $x > -10$ (d) $m \leq 1$ (e) $y > 16$

(f) $p < 3$ (g) $c \geq -2$ (h) $a \leq 0$ (i) $q > 0$

Section B - Practice Assessment Standard Questions

Q1 $y = -x + 2$ Q2 $y = -3x - 1$

Q3(a) $p < \frac{5}{3}$ (b) $p < -\frac{5}{3}$

Q4(a) $x = 10$ $y = 1$ (b) $a = 3$ $b = -1$

Q5(a) $3n + 2b = 145$ (b) $5n + 3b = 240$ (c)   One breakfast = £5

Q6(a) $c = \frac{7(F + 12)}{6}$ (b) $c = \frac{3(X - 16)}{8}$

Section C – Operational Skills

O1

Q1 (a) 15   (b) 15   (c) 6

Q2 (a) 2   (b) -4 (c) 0

Q3 (a) $x = 3$ (b) $t = -4$ (c) $p = 4$

Q4 (a) 15   (b) $t = -\frac{1}{2}$

Q5 $n = -5$

Q6 (a) The divisor cannot be equal to zero. [$If\ x = 1\ \rightarrow 1 - x = 1 - 1 = 0$]

(b) (i) 8   (ii) 4   (iii) 16

(c) $x = -3$

Linear Relationships

O2

Q1 (a) $m = 3 \quad c = 2$ (b) $m = -2 \quad c = 5$

     (c) $m = -2 \quad c = 7$ (d) $m = -\frac{1}{3} \quad c = -\frac{1}{3}$

     (e) $m = 4 \quad c = -3$ (f) $m = \frac{2}{3} \quad c = \frac{1}{3}$

Q2 $f = d + 3$

Q3 $y = -2x + 14$

Q4 $y = 3x - 2$

Q5 (a) $y = 4x - 6$ (b) $y = -x + 2$

Q6 $y = 2x + 3$

Q7 (a)   (b)   (c)
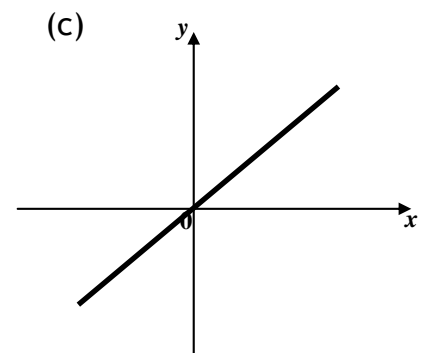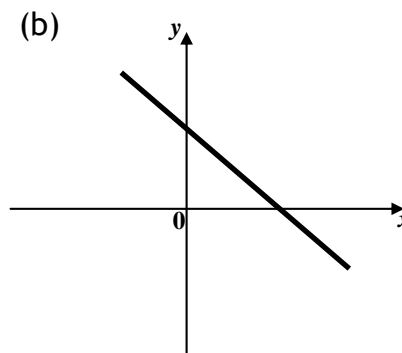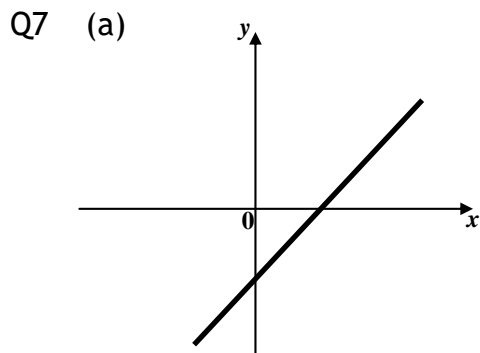
O3

Q1 (a) $x = -2$ (b) $x = 1$ (c) $x = 4$ (d) $x = 2$ (e) $x = 3$

     (f) $x = -10$ (g) $x = -3$ (h) $x = -2$

Q2 (a) $x = 12$ (b) $x = 7$ (c) $x = -6$ (d) $x = 20$ (e) $x = 12$

     (f) $x = 3$ (g) $x = 4$ (h) $x = 7$

Q3 (a) $x > 1$ (b) $x > 2$ (c) $x \geq -\frac{1}{3}$ (d) $x \geq -1$ (e) $x \geq -3$

     (f) $x < 22$ (g) $x < -1$ (h) $x \geq \frac{7}{2}$

Linear Relationships

O4

Q1 (a) $x = 7 \quad y = 1$ (b) $x = 2 \quad y = 3$

(c) $x = 2 \quad y = 5$ (d) $x = 2 \quad y = 1$

(e) $x = 1 \quad y = -1$ (f) $x = -1 \quad y = -2$

(g) $x = 3 \quad y = -4$ (h) $x = 4 \quad y = 1$

Q2 (a) $x = 1 \quad y = 2$ (b) $x = 4 \quad y = 2$

(c) $x = 3 \quad y = 1$ (d) $x = -3 \quad y = 2$

(e) $x = -3 \quad y = 2$ (f) $x = 4 \quad y = -3$

(g) $x = -3 \quad y = 2$ (h) $x = 3 \quad y = -5$

O5

Q1 (a) $x = \frac{y-3}{2}$ (b) $s = \frac{p-t}{r^2}$ (c) $m = \sqrt{\frac{k+n}{5}}$ (d) $x = \left(\frac{y-sr^2}{3}\right)^2$

(e) $g = 7 - h$ (f) $t = \frac{5-3p}{2}$ (g) $y = \frac{x^2 t}{2r}$ (h) $c = \sqrt{\frac{2ab}{3}}$

(i) $k = h^2 g$ (j) $x = 2(y + 3)$

Q2 (a) $t = \frac{8}{p}$ (b) $x = \frac{a+1}{d}$ (c) $y = 3 - 4b$ (d) $x = \frac{m}{h-k}$

(e) $m = \frac{2k-3n}{5}$ (f) $b = \frac{3a-pc}{5}$ (g) $a = 3y + b$ (h) $z = \frac{x}{y^2}$

Q3 (a) $t = h - 2L$ (b) $x = \frac{ac-b}{a}$ (c) $n = \frac{t}{t-d}$

(d) $x = \frac{1+2y^2}{1-y^2}$ (e) $n = \sqrt{\frac{1}{a^2+1}}$ (f) $d = \frac{2b}{k-b}$

Section D - Reasoning Skills Section

Q1 (a) $280x + 70y = 52 \cdot 50$ (b) $210x + 40y = 38 \cdot 00$

(c) The cost of a call = £0·16 and the cost of a text = £0·11.

Q2 (a) $24x + 6y = 60$ (b) $20x + 10y = 40$

(c) David's score would be 25 points $(17 \times 3 + 13 \times (-2))$.

Linear Relationships

Q3   (4, 5)

Q4   (a) $2m + c = 7$ (b) $4m + c = 17$

        (c) $m = 5, c = -3$

        (d) $gradient = 5, \; y - intercept \; at \; (0, -3)$.

Q5   (a) $B(-6, 0)$ (b) $x < -6$

Q6   (a) $y = -\frac{3}{2}x + 12$ (b) $P\left(\frac{8}{3}, 8\right)$

Q7   (a) $D_1 = 2x + 16$ (b) $D_2 = \frac{1}{2}x$

        (c) $Jane's \; running \; speed = 12 \; km/h$.

Q8 $x = \frac{6}{5}$

Q9   (a) $6x + 2y = 42$ (b) $5x = 2y + 2$

        (c) $x = 4, y = 9$