

## EF 1.3 - Algebraic Fractions

### Section A - Revision

This section will help you revise previous learning which is required in this topic.

**R1 I have revised how to use the four operations applied to vulgar fractions and mixed numbers.**

1. Carry out the following calculations, showing clearly all working, leaving your answer as a vulgar fraction.

(a)  $\frac{2}{3} + \frac{1}{4}$

(b)  $\frac{4}{5} \times \frac{1}{6}$

(c)  $\frac{7}{8} - \frac{3}{5}$

(d)  $\frac{9}{14} \div \frac{3}{7}$

(e)  $\frac{3}{8} + \frac{1}{6}$

(f)  $\frac{1}{12} \div \frac{3}{4}$

(g)  $\frac{2}{15} \times \frac{5}{12}$

(h)  $\frac{5}{12} + \frac{1}{3}$

(i)  $\frac{4}{5} - \frac{3}{11}$

2. Carry out the following calculations, showing clearly all working, leaving your answer as a mixed.

(a)  $\frac{4}{5} + \frac{3}{4}$

(b)  $\frac{7}{11} + \frac{3}{5}$

(c)  $\frac{3}{4} \div \frac{7}{8}$

(d)  $\frac{5}{7} \div \frac{10}{21}$

(e)  $\frac{7}{9} + \frac{4}{5}$

(f)  $\frac{3}{4} \div \frac{1}{20}$

(g)  $\frac{2}{5} + \frac{7}{8}$

(h)  $\frac{7}{10} + \frac{5}{6}$

(i)  $\frac{4}{5} \div \frac{3}{11}$

3. Carry out the following calculations, showing clearly all working, leaving your answer as a mixed number where appropriate.

(a)  $3\frac{1}{2} + \frac{5}{6}$

(b)  $2\frac{3}{10} \div \frac{1}{15}$

(c)  $5\frac{1}{8} - \frac{4}{5}$

(d)  $3\frac{1}{8} \times 1\frac{2}{5}$

(e)  $4\frac{3}{4} - 2\frac{1}{6}$

(f)  $2\frac{3}{10} \div 1\frac{3}{5}$

(g)  $3\frac{4}{15} \times 1\frac{3}{7}$

(h)  $4\frac{1}{15} + 3\frac{3}{5}$

(i)  $7\frac{1}{5} - 1\frac{7}{11}$

# Algebraic Fractions

**R2 I have revised multiplying out brackets, gathering like terms and factorisation.**

1. Multiply out the brackets and simplify

- |                             |                            |                     |
|-----------------------------|----------------------------|---------------------|
| (a) $3(2x + 5) + 6$         | (b) $5(x + 9) + 2x$        | (c) $7(3x - 2) + 4$ |
| (d) $4(2x + 7) + 5x$        | (e) $6(2x - 9) + 7x$       | (f) $6(2x - 9) + 7$ |
| (g) $2(3x + 1) + 3(x - 5)$  | (h) $5(x + 4) - 3(x - 7)$  | (i) $4 - 5(2x + 1)$ |
| (j) $4(5x + 2) - 7(2x + 3)$ | (k) $9(2x + 3) + 2(x + 4)$ | (l) $8 - 3(x - 9)$  |

2. Multiply out the brackets and simplify

- |                              |                              |                      |
|------------------------------|------------------------------|----------------------|
| (a) $(x + 3)(x + 4)$         | (b) $(x + 2)(x - 5)$         | (c) $(x - 7)(x + 3)$ |
| (d) $(x - 9)(x - 2)$         | (e) $(2x + 5)(x + 3)$        | (f) $(x - 1)(x + 4)$ |
| (g) $(3x - 2)(x - 9)$        | (h) $(5x + 1)(3x - 7)$       | (i) $(x - 3)(x - 5)$ |
| (j) $(x + 3)(x^2 - 3x + 4)$  | (k) $(x - 5)(2x^2 + 4x - 3)$ | (l) $(x - 2)(3 - x)$ |
| (m) $(2x + 5)(3x^2 - x - 8)$ | (n) $(3x - 1)(x^2 + x - 5)$  | (o) $(3 - x)(5 + x)$ |

3. Factorise completely

- |                     |                       |                      |
|---------------------|-----------------------|----------------------|
| (a) $8a + 12b$      | (b) $49 - p^2$        | (c) $x^2 + 5x + 6$   |
| (d) $2x^2 - 7x + 3$ | (e) $2xy + xz$        | (f) $x^2 - 25$       |
| (g) $9mn - 12mp$    | (h) $a^2 - 8a + 7$    | (i) $25g^2 - 49$     |
| (j) $9x^2 - 16$     | (k) $5x^2 + 4x - 1$   | (l) $15x^2 + 10x^2y$ |
| (m) $x^2 - 2x - 15$ | (n) $3x^2 + 2xy + 6x$ | (o) $6r^2 - 24q^2$   |
| (p) $8r^2 + 2r - 3$ | (q) $x^2 + 6x + 9$    | (r) $15 - 7r - 2r^2$ |

# Algebraic Fractions

## Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Algebraic Fractions (Expressions and Formulae 1.3)

### Practice Assessment Standard Questions

1. Write each algebraic fraction in its simplest form.

(a)  $\frac{(2x-3)(x+1)}{(x+1)^2}, x \neq -1$

(b)  $\frac{(3x-1)(x+2)}{(x+2)^2}, x \neq -2$

(c)  $\frac{(x+3)(2x+1)}{(x+3)^2}, x \neq -3$

(d)  $\frac{(x-1)(x+1)}{(x-1)^2}, x \neq 1$

(e)  $\frac{(5x-2)(x+1)}{(x-3)(x+1)}, x \neq -1 \text{ or } 3$

(f)  $\frac{(x-1)^2}{(x-1)(x+2)}, x \neq -2 \text{ or } 1$

(g)  $\frac{(x+3)^2}{(x-2)(x+3)}, x \neq -3 \text{ or } 2$

(h)  $\frac{(x-1)(x+8)}{(x-1)(x+1)}, x \neq -1 \text{ or } 1$

2. Write each of the following as a single fraction

(a)  $\frac{2}{x} - \frac{3}{y}, x, y \neq 0$

(b)  $\frac{x}{6} \times \frac{t}{y}, t, x, y \neq 0$

(c)  $\frac{3}{x} + \frac{4}{y}, x, y \neq 0$

(d)  $\frac{x}{8} \div \frac{t}{y}, t, x, y \neq 0$

(e)  $\frac{3}{x} + \frac{5}{y}, x, y \neq 0$

(f)  $\frac{1}{x} - \frac{1}{y}, x, y \neq 0$

(g)  $\frac{7}{x} + \frac{3}{y}, x, y \neq 0$

(h)  $\frac{x}{10} \times \frac{t}{y}, t, x, y \neq 0$

(i)  $\frac{9}{x} - \frac{2}{y}, x, y \neq 0$

(j)  $\frac{x}{5} \div \frac{t}{y}, t, x, y \neq 0$

(k)  $\frac{x}{100} \times \frac{t}{y}, t, x, y \neq 0$

(l)  $\frac{5}{x} - \frac{3}{y}, x, y \neq 0$

# Algebraic Fractions

## Section C - Operational Skills

This section provides problems with the operational skills associated with Algebraic fractions

### N1 Simplifying Algebraic fractions

#### 1. Simplify

(a)  $\frac{6b^5}{6b^2}$

(b)  $\frac{15x^6}{5x^4}$

(c)  $\frac{14p^7}{21p^2}$

(d)  $\frac{a^4b^7}{a^2b^3}$

(e)  $\frac{x^4y^8}{xy^3}$

(f)  $\frac{p^9q^8}{p^5q^4}$

(g)  $\frac{5b^3}{5b^7}$

(h)  $\frac{12x^3}{6x^8}$

(i)  $\frac{9p}{15p^2}$

(j)  $\frac{8a^4b}{4a^2b^3}$

(k)  $\frac{6x^4y^5}{10xy^7}$

(l)  $\frac{5p^3q^3}{10p^5q}$

#### 2. Simplify

(a)  $\frac{3(x-5)}{(x-5)^2}$

(b)  $\frac{5(x+1)}{(x+1)(x-2)}$

(c)  $\frac{(x+2)(x+3)}{x+2}$

(d)  $\frac{(2x+5)^2}{(2x-1)(2x+5)}$

(e)  $\frac{2x+8}{(x+4)^2}$

(f)  $\frac{3x^2+6x}{x(3x-1)}$

(g)  $\frac{2x^2+x-3}{3(2x+3)}$

(h)  $\frac{x^2+4x+4}{(x-1)(x+2)}$

(i)  $\frac{10x-5}{4x^2-4x+1}$

(j)  $\frac{14x+21}{4x^2+12x+9}$

(k)  $\frac{9x^2-1}{3x^2-7x+2}$

(l)  $\frac{5x^2+4x-1}{25x^2-1}$

# Algebraic Fractions

## N2 Adding and subtracting algebraic Fractions

1. Write each of the following as a single fraction

(a)  $\frac{a}{b} - \frac{b}{a}$   $a, b \neq 0$       (b)  $\frac{x}{y} + \frac{y}{x}$   $x, y \neq 0$       (c)  $\frac{3a}{3} + \frac{2}{a}$   $a \neq 0$

(d)  $\frac{2a}{3b} - \frac{b}{2a}$   $a, b \neq 0$       (e)  $\frac{x}{5y} + \frac{3y}{2x}$   $x, y \neq 0$       (f)  $\frac{a}{4} + \frac{1}{a}$   $a \neq 0$

(g)  $\frac{1}{b} - \frac{2}{ab}$   $a, b \neq 0$       (h)  $\frac{3}{xy} + \frac{1}{x^2}$   $x, y \neq 0$       (i)  $\frac{3}{a^2} + \frac{1}{a}$   $a \neq 0$

2. Express each of the following as a single fraction

(a)  $\frac{1}{p} + \frac{2}{(p+5)}$       (b)  $\frac{3}{x} - \frac{4}{x+1}$       (c)  $\frac{2}{a} - \frac{3}{(a+4)}$

(d)  $\frac{3}{(p+1)} - \frac{1}{(p+5)}$       (e)  $\frac{2}{x-1} + \frac{4}{x+2}$       (f)  $\frac{1}{(a-3)} + \frac{4}{(a+4)}$

(g)  $\frac{p}{p-1} + \frac{1}{p}$       (h)  $\frac{2}{x+2} - \frac{4x}{x+1}$       (i)  $\frac{2a}{(a+2)} - \frac{5}{(a+4)}$

(j)  $\frac{p+5}{2} + \frac{p}{3}$       (k)  $\frac{x+1}{2} - \frac{x+3}{6}$       (l)  $\frac{(a+3)}{2} - \frac{(2a-1)}{5}$

## N3 Multiplying and dividing Algebraic fractions

1. Write each of the following as a single fraction

(a)  $\frac{5p^2}{8} \div \frac{p}{2}$       (b)  $\frac{s^2}{t} \times \frac{3t}{2s}$       (c)  $\frac{x^5}{y} \times \frac{4y^3}{3x}$

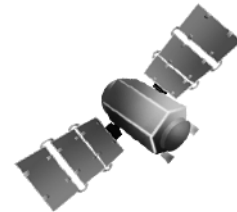
(d)  $\frac{4p^2}{15} \div \frac{p}{3r^5}$       (e)  $\frac{3s^2}{t^4} \times \frac{7t}{9s^5}$       (f)  $\frac{3p^4}{49} \div \frac{12p}{14}$

# Algebraic Fractions

## Section D - Course Assessment Section

This section provides problems which you can use to practise course assessment style questions.

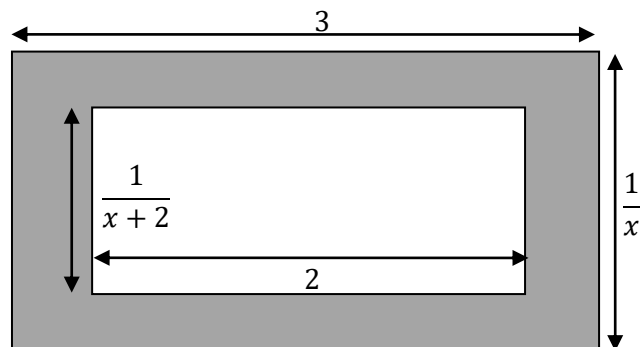
1. A satellite travels a distance of  $3xy^3$  kilometres in  $12x^2y$  hours. Calculate the speed of the satellite in kilometres per hour. **Give your answer as a fraction in its simplest form.**



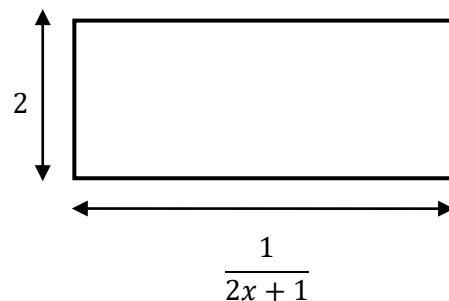
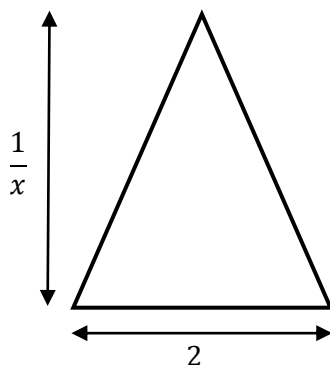
2. A particle travels at a speed of  $\frac{5x^2}{2}$  metres per second for  $\frac{4t^3}{3x}$  seconds. Calculate the distance travelled by the particle.

3. Look at the diagram.  
Show that the shade area can be represented by

$$A = \frac{x+6}{x(x+2)}$$



4. In the diagram below, the triangle has an area 3 square units greater than the area of the rectangle.



Show that  $\frac{1}{x(2x+1)} = 3$ .

# Algebraic Fractions

## Answers

### Section A

#### R1

- Q1 (a)  $\frac{11}{12}$  (b)  $\frac{2}{15}$  (c)  $\frac{11}{40}$  (d)  $\frac{3}{2}$  (e)  $\frac{13}{24}$   
(f)  $\frac{1}{9}$  (g)  $\frac{1}{18}$  (h)  $\frac{3}{4}$  (i)  $\frac{29}{55}$
- Q2 (a)  $1\frac{11}{20}$  (b)  $1\frac{13}{55}$  (c)  $\frac{6}{7}$  (d)  $1\frac{1}{2}$  (e)  $1\frac{26}{45}$   
(f) 15 (g)  $1\frac{11}{40}$  (h)  $1\frac{8}{15}$  (i)  $2\frac{14}{15}$
- Q3 (a)  $4\frac{1}{3}$  (b)  $34\frac{1}{2}$  (c)  $4\frac{13}{40}$  (d)  $4\frac{3}{8}$  (e)  $2\frac{7}{12}$   
(f)  $1\frac{7}{16}$  (g)  $4\frac{2}{3}$  (h)  $7\frac{2}{3}$  (i)  $5\frac{31}{55}$

#### R2

- Q1 (a)  $6x + 21$  (b)  $7x + 45$  (c)  $21x - 10$  (d)  $13x + 28$  (e)  $19x - 54$   
(f)  $12x - 47$  (g)  $9x - 13$  (h)  $2x + 41$  (i)  $-1 - 10x$  (j)  $6x - 13$   
(k)  $20x + 35$  (l)  $35 - 3x$
- Q2 (a)  $x^2 + 7x + 12$  (b)  $x^2 - 3x - 10$   
(c)  $x^2 - 4x - 21$  (d)  $x^2 - 11x + 18$   
(e)  $2x^2 + 11x + 15$  (f)  $x^2 + 3x - 4$   
(g)  $3x^2 - 29x + 18$  (h)  $15x^2 - 32x - 7$   
(i)  $x^2 - 8x + 15$  (j)  $x^3 - 5x + 12$   
(k)  $2x^3 - 6x^2 - 23x + 15$  (l)  $-x^2 + 5x - 6$   
(m)  $6x^3 + 13x^2 - 21x - 40$  (n)  $3x^3 + 2x^2 - 16x + 5$   
(o)  $-x^2 - 2x + 15$
- Q3 (a)  $4(2a + 3b)$  (b)  $(7 - p)(7 + p)$   
(c)  $(x + 2)(x + 3)$  (d)  $(2x - 1)(x - 3)$   
(e)  $x(2y + z)$  (f)  $(x - 5)(x + 5)$

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(g)  $3m(3n - 4p)$

(h)  $(a - 7)(a - 1)$

(i)  $(5g - 7)(5g + 7)$

(j)  $(3x - 4)(3x + 4)$

(k)  $(5x - 1)(x + 1)$

(l)  $5x^2(3 + 2y)$

(m)  $(x - 5)(x + 3)$

(n)  $x(3x + 2y + 6)$

(o)  $6(r - 2q)(r + 2q)$

(p)  $(2r - 1)(4r + 3)$

(q)  $(x + 3)^2$

(r)  $(5 + r)(3 - 2r)$

## Section B

Q1 (a)  $\frac{2x-3}{x+1}$  (b)  $\frac{3x-1}{x+2}$  (c)  $\frac{2x+1}{x+3}$  (d)  $\frac{x+1}{x-1}$  (e)  $\frac{5x-2}{x-3}$

(f)  $\frac{x-1}{x+2}$  (g)  $\frac{x+3}{x-2}$  (h)  $\frac{x+8}{x+1}$

Q2 (a)  $\frac{2y-3x}{xy}$  (b)  $\frac{tx}{6y}$  (c)  $\frac{3y+4x}{xy}$  (d)  $\frac{xy}{8t}$  (e)  $\frac{3y+5x}{xy}$

(f)  $\frac{y-x}{xy}$  (g)  $\frac{7y+3x}{xy}$  (h)  $\frac{xt}{10y}$  (i)  $\frac{9y-2x}{xy}$  (j)  $\frac{xy}{5t}$

(k)  $\frac{tx}{100y}$  (l)  $\frac{5y-3x}{xy}$

## Section C

### N1

Q1 (a)  $b^3$  (b)  $3x^2$  (c)  $\frac{2p^5}{3}$  (d)  $a^2b^4$  (e)  $x^3y^5$

(f)  $p^4q^4$  (g)  $\frac{1}{b^4}$  (h)  $\frac{2}{x^5}$  (i)  $\frac{3}{5p}$  (j)  $\frac{2a^2}{b^2}$

(k)  $\frac{3x^3}{5y^2}$  (l)  $\frac{q^2}{2p^2}$

Q2 (a)  $\frac{3}{x-5}$  (b)  $\frac{5}{x-2}$  (c)  $x + 3$  (d)  $\frac{2x+5}{2x-1}$  (e)  $\frac{2}{x+4}$

(f)  $\frac{3(x+2)}{(3x-1)}$  (g)  $\frac{x-1}{3}$  (h)  $\frac{x+2}{x-1}$  (i)  $\frac{5}{2x-1}$  (j)  $\frac{7}{2x+3}$

(k)  $\frac{3x+1}{x-2}$  (l)  $\frac{x+1}{5x+1}$

### N2



# Algebraic Fractions

Q1 (a)  $\frac{a^2-b^2}{ab}$  (b)  $\frac{x^2+y^2}{xy}$  (c)  $\frac{a^2+2}{a}$  (d)  $\frac{4a^2-3b^2}{6ab}$  (e)  $\frac{2x^2+15y^2}{10xy}$   
(f)  $\frac{a^2+4}{4a}$  (g)  $\frac{a-2}{ab}$  (h)  $\frac{3x+y}{x^2y}$  (i)  $\frac{3+a}{a^2}$

Q2 (a)  $\frac{3p+5}{p(p+5)}$  (b)  $\frac{3-x}{x(x+1)}$  (c)  $\frac{8-a}{a(a+4)}$  (d)  $\frac{2p+14}{(p+1)(p+5)}$  (e)  $\frac{6x}{(x-1)(x+2)}$   
(f)  $\frac{5a-8}{(a-3)(a+4)}$  (g)  $\frac{p^2+p-1}{p(p-1)}$  (h)  $\frac{-4x^2-6x+2}{(x+2)(x+1)}$  (i)  $\frac{2a^2+3a-10}{(a+2)(a+4)}$  (j)  $\frac{5p+15}{6}$   
(k)  $\frac{x}{3}$  (l)  $\frac{a+17}{10}$

N3

Q1 (a)  $\frac{5p}{4}$  (b)  $\frac{3s}{2}$  (c)  $\frac{4x^4y^2}{3}$  (d)  $\frac{4pr^5}{5}$  (e)  $\frac{7}{3s^3t^3}$   
(f)  $\frac{p^3}{14}$

Section D

Q1  $S = \frac{y^2}{4x}$  km/h

Q2  $D = \frac{10xt^3}{3}$  metres

Q3  $A = \frac{x+6}{x(x+2)}$  as required.

Q4  $\frac{1}{x(2x+1)}$  as required.