Applying Algebraic and Geometric skills to Methods of Proof

AH Mathematics HW

Essential knowledge:

1. Find an example/counter-example to prove/disprove the following statements:

a. $\exists n$ where n is prime s.t. $2^n + 5$ is also prime.

b. If $p < q$ and $r < s \Rightarrow pr < qs \; \forall \, p, q, r, s \in \mathbb{R}$

2. Write inverse, converse and contrapositive statements for:

a. If ΔABC is right-angled then $a^2 + b^2 = c^2$

b. If n^2 is odd then n is odd.

3. Using proof by contradiction, prove that:

a. For $a \in \mathbb{Z}$, If a^2 is even then a is even

b. If x is irrational $(x > 0)$, then $\sqrt{x}$ is irrational.

4. Using direct proof, prove that:

a. If n is odd then $n^2 + 1$ is even

b. The sum of 2 consecutive odd numbers is always divisible by 4

Unit level:

5. Find a counter-example to disprove these conjectures:

a. If $a^4 < b^4$ then $a < b$ for any real number a and b.

b. If $p < q - 3 \Rightarrow (p + 3)^2 < q^2 \; \forall \, p, q, \in \mathbb{R}$

6. Prove, by contradiction, that:

a. If $n^3 - 3$ is even then n is odd.

b. If $3x$ is irrational $(x > 0)$, then x is irrational

7. Prove directly that the product of two odd numbers is always odd.

Assessment level:

8. For all natural numbers n, prove that $n^3 - n$ is always divisible by 6.

9. For each of the following statements, decide whether it is true or false and prove your conclusion.

A. For all natural numbers m, if m^2 is divisible by 4 then m is divisible by 4.

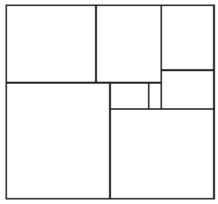
B. The cube of any odd integer p plus the square of any even integer q is always odd.

Challenge Questions (optional)

1. Consider all three-digit numbers formed by using ***different*** digits from 0, 1, 2, 3 and 5. How many of these digits are divisible by 6?

A 4 **B** 7 **C** 10 **D** 15 **E** 20

2. The diagram shows an $n \times (n+1)$ rectangle tiled with $k \times (k+1)$ rectangles, where n and n are integers and k takes each value from 1 to 8 inclusive. What is the value of n?



A 16 **B** 15 **C** 14 **D** 13 **E** 12