X747/76/11
Mathematics
Paper 1
(Non-Calculator)
WEDNESDAY, 20 MAY
9:00AM-10:10AM

## Total marks - 60

Attempt ALL questions.
You may NOT use a calculator.
Full credit will be given only to solutions which contain appropriate working.
State the units for your answer where appropriate.
Answers obtained by readings from scale drawings will not receive any credit.
Write your answers clearly in the spaces in the answer booklet provided. Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.
Use blue or black ink.
Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

## FORMULAE LIST

Circle:
The equation $x^{2}+y^{2}+2 g x+2 f y+c=0$ represents a circle centre $(-g,-f)$ and radius $\sqrt{g^{2}+f^{2}-c}$. The equation $(x-a)^{2}+(y-b)^{2}=r^{2}$ represents a circle centre $(a, b)$ and radius $r$.

## Scalar Product:

$\mathbf{a} \cdot \mathbf{b}=|\mathbf{a}||\mathbf{b}| \cos \theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$
or

$$
\text { a.b }=a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3} \text { where } \mathbf{a}=\left(\begin{array}{l}
a_{1} \\
a_{2} \\
a_{3}
\end{array}\right) \text { and } \mathbf{b}=\left(\begin{array}{l}
b_{1} \\
b_{2} \\
b_{3}
\end{array}\right) .
$$

Trigonometric formulae:

$$
\begin{aligned}
\sin (\mathrm{A} \pm \mathrm{B}) & =\sin \mathrm{A} \cos \mathrm{~B} \pm \cos \mathrm{A} \sin \mathrm{~B} \\
\cos (\mathrm{~A} \pm \mathrm{B}) & =\cos \mathrm{A} \cos \mathrm{~B} \mp \sin \mathrm{~A} \sin \mathrm{~B} \\
\sin 2 \mathrm{~A} & =2 \sin \mathrm{~A} \cos \mathrm{~A} \\
\cos 2 \mathrm{~A} & =\cos ^{2} \mathrm{~A}-\sin ^{2} \mathrm{~A} \\
& =2 \cos ^{2} \mathrm{~A}-1 \\
& =1-2 \sin ^{2} \mathrm{~A}
\end{aligned}
$$

Table of standard derivatives:

| $f(x)$ | $f^{\prime}(x)$ |
| :--- | :---: |
| $\sin a x$ <br> $\cos a x$ | $a \cos a x$ <br> $-a \sin a x$ |

Table of standard integrals:

| $f(x)$ | $\int f(x) d x$ |
| :---: | :---: |
| $\sin a x$ | $-\frac{1}{a} \cos a x+c$ |
| $\cos a x$ | $\frac{1}{a} \sin a x+c$ |

## Attempt ALL questions

Total marks - 60

1. Vectors $\mathbf{u}=8 \mathbf{i}+2 \mathbf{j}-\mathbf{k}$ and $\mathbf{v}=-3 \mathbf{i}+t \mathbf{j}-6 \mathbf{k}$ are perpendicular.

Determine the value of $t$.
2. Find the equation of the tangent to the curve $y=2 x^{3}+3$ at the point where $x=-2$.
3. Show that $(x+3)$ is a factor of $x^{3}-3 x^{2}-10 x+24$ and hence factorise $x^{3}-3 x^{2}-10 x+24$ fully.
4. The diagram shows part of the graph of the function $y=p \cos q x+r$.


Write down the values of $p, q$ and $r$.
5. A function $g$ is defined on $\mathbb{R}$, the set of real numbers, by $g(x)=6-2 x$.
(a) Determine an expression for $g^{-1}(x)$.
(b) Write down an expression for $g\left(g^{-1}(x)\right)$.
6. Evaluate $\log _{6} 12+\frac{1}{3} \log _{6} 27$.
7. A function $f$ is defined on a suitable domain by $f(x)=\sqrt{x}\left(3 x-\frac{2}{x \sqrt{x}}\right)$. Find $f^{\prime}(4)$.
8. ABCD is a rectangle with sides of lengths $x$ centimetres and $(x-2)$ centimetres, as shown.


If the area of $A B C D$ is less than $15 \mathrm{~cm}^{2}$, determine the range of possible values of $x$.
9. $\mathrm{A}, \mathrm{B}$ and C are points such that AB is parallel to the line with equation $y+\sqrt{3} x=0$ and BC makes an angle of $150^{\circ}$ with the positive direction of the $x$-axis.
Are the points $\mathrm{A}, \mathrm{B}$ and C collinear?
10. Given that $\tan 2 x=\frac{3}{4}, 0<x<\frac{\pi}{4}$, find the exact value of
(a) $\cos 2 x$
(b) $\cos x$.
11. $\mathrm{T}(-2,-5)$ lies on the circumference of the circle with equation

$$
(x+8)^{2}+(y+2)^{2}=45 .
$$

(a) Find the equation of the tangent to the circle passing through T .
(b) This tangent is also a tangent to a parabola with equation $y=-2 x^{2}+p x+1-p$, where $p>3$.

Determine the value of $p$.
12. The diagram shows part of the graph of $y=a \cos b x$.

The shaded area is $\frac{1}{2}$ unit $^{2}$.


What is the value of $\int_{0}^{\frac{3 \pi}{4}}(a \cos b x) d x$ ?
13. The function $f(x)=2^{x}+3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y=f(x)$ passes through the point $\mathrm{P}(1, b)$ and cuts the $y$-axis at Q as shown in the diagram.

(a) What is the value of $b$ ?
(b) (i) Copy the above diagram.

On the same diagram, sketch the graph with equation $y=f^{-1}(x)$.
(ii) Write down the coordinates of the images of P and Q .
(c) $\mathrm{R}(3,11)$ also lies on the graph with equation $y=f(x)$.

Find the coordinates of the image of R on the graph with equation $y=4-f(x+1)$.
14. The circle with equation $x^{2}+y^{2}-12 x-10 y+k=0$ meets the coordinate axes at exactly three points.
What is the value of $k$ ?
15. The rate of change of the temperature, $T^{\circ} \mathrm{C}$ of a mug of coffee is given by

$$
\frac{d T}{d t}=\frac{1}{25} t-k, 0 \leq \mathrm{t} \leq 50
$$

- $t$ is the elapsed time, in minutes, after the coffee is poured into the mug
- $k$ is a constant
- initially, the temperature of the coffee is $100^{\circ} \mathrm{C}$
- 10 minutes later the temperature has fallen to $82^{\circ} \mathrm{C}$.

Express $T$ in terms of $t$.
[BLANK PAGE]
DO NOT WRITE ON THIS PAGE
[BLANK PAGE]
DO NOT WRITE ON THIS PAGE

