X100/701

NATIONAL QUALIFICATIONS 2011

WEDNESDAY, 18 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**

Marks

Answer all the questions.

1. Express $\dfrac{13-x}{x^2+4x-5}$ in partial fractions and hence obtain

$$\int \frac{13-x}{x^2+4x-5}dx.$$ **5**

2. Use the binomial theorem to expand $\left(\frac{1}{2}x-3\right)^4$ and simplify your answer. **3**

3. (*a*) Obtain $\dfrac{dy}{dx}$ when y is defined as a function of x by the equation

$$y + e^y = x^2.$$ **3**

(*b*) Given $f(x) = \sin x \cos 2x$, use logarithmic differentiation to obtain $f'(x)$.

Express $f'(x)$ in the form $\dfrac{g(x)f(x)}{\sin x \ \cos x}$. **3**

4. (*a*) For what value of λ is $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ -1 & \lambda & 6 \end{pmatrix}$ singular? **3**

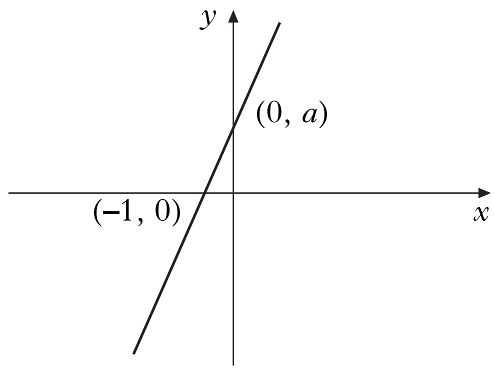
(*b*) For $A = \begin{pmatrix} 2 & 2\alpha-\beta & -1 \\ 3\alpha+2\beta & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$, obtain values of α and β such that

$$A' = \begin{pmatrix} 2 & -5 & -1 \\ -1 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}.$$ **3**

5. Obtain the first four terms in the Maclaurin series of $\sqrt{1+x}$, and hence write down the first four terms in the Maclaurin series of $\sqrt{1+x^2}$. **4**

Hence obtain the first four terms in the Maclaurin series of $\sqrt{(1+x)(1+x^2)}$. **2**

6.



The diagram shows part of the graph of a function $f(x)$. Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes. **4**

7. A curve is defined by the equation $y = \dfrac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$ for $x < 1$.

Calculate the gradient of the curve when $x = 0$. **4**

8. Write down an expression for $\displaystyle\sum_{r=1}^{n} r^3 - \left(\sum_{r=1}^{n} r\right)^2$ **1**

and an expression for

$$\sum_{r=1}^{n} r^3 + \left(\sum_{r=1}^{n} r\right)^2.$$ **3**

9. Given that $y > -1$ and $x > -1$, obtain the general solution of the differential equation

$$\frac{dy}{dx} = 3(1+y)\sqrt{1+x}$$

expressing your answer in the form $y = f(x)$. **5**

[Turn over

Marks

10. Identify the locus in the complex plane given by

$$|z-1| = 3.$$

Show in a diagram the region given by $|z-1| \leq 3$. **5**

11. (*a*) Obtain the exact value of $\int_0^{\pi/4} (\sec x - x)(\sec x + x)\,dx$. **3**

(*b*) Find $\int \frac{x}{\sqrt{1-49x^4}}\,dx$. **4**

12. Prove by induction that $8^n + 3^{n-2}$ is divisible by 5 **for all integers $n \geq 2$**. **5**

13. The first three terms of an arithmetic sequence are $a, \frac{1}{a}, 1$ where $a < 0$.
Obtain the value of a and the common difference. **5**

Obtain the smallest value of n for which the sum of the first n terms is greater than 1000. **4**

14. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12.$$ **7**

Find the particular solution for which $y = -\frac{3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. **3**

Marks

15. The lines L_1 and L_2 are given by the equations

$$\frac{x-1}{k} = \frac{y}{-1} = \frac{z+3}{1} \text{ and } \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2},$$

respectively.

Find:

(*a*) the value of k for which L_1 and L_2 intersect and the point of intersection; **6**

(*b*) the acute angle between L_1 and L_2. **4**

16. Define $I_n = \int_0^1 \frac{1}{(1+x^2)^n}dx$ for $n \geq 1$.

(*a*) Use integration by parts to show that

$$I_n = \frac{1}{2^n} + 2n\int_0^1 \frac{x^2}{(1+x^2)^{n+1}}dx.$$ **3**

(*b*) Find the values of A and B for which

$$\frac{A}{(1+x^2)^n} + \frac{B}{(1+x^2)^{n+1}} = \frac{x^2}{(1+x^2)^{n+1}}$$

and hence show that

$$I_{n+1} = \frac{1}{n \times 2^{n+1}} + \left(\frac{2n-1}{2n}\right)I_n.$$ **5**

(*c*) Hence obtain the exact value of $\int_0^1 \frac{1}{(1+x^2)^3}dx$. **3**

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