Applying Algebraic and Geometric skills to Vectors

AH Mathematics HW

Essential knowledge:

1. Find the direction vector, ratio and cosines for $\boldsymbol{a} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$

2. Find the Cartesian/symmetric equation of the line through $(2, 3, -1)$ and parallel to the vector $\boldsymbol{i} - \boldsymbol{2}j + 5\boldsymbol{k}$

3. Find the equation of the plane with normal $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ through $(2, 3, -1)$:

4. Calculate the volume of the parallelepiped that has $\boldsymbol{u} = -2\boldsymbol{i} + 5\boldsymbol{k}$, $\boldsymbol{v} = 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{w} = -\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$

5. For the lines: $L_1: x - 1 = y = z - 1$ and $L_2: x = 1 + t$, $y = 5t$ and $z = -t$ and the planes: $\pi_1: x + 2y + z = 0$ and $\pi_2: x + y = 0$ find the angle between:- (a) L_1 and L_2 (b) π_1 and π_2 (c) L_1 and π_2

Unit level:

6. Given the vectors $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, calculate $\boldsymbol{a} \times \boldsymbol{b}$.

7. Find the equation of the line joining $(1, 0, 2)$ and $(2, 1, 0)$.

8. Find, in symmetric form, an equation of the line through the point $(0, 5, -2)$ which is parallel to the line $\boldsymbol{r} = (\boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{k}) + \lambda(\boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{k})$

9. State, in parametric form, the equation of the plane which is parallel to the vectors $8\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$ and $-4\boldsymbol{i} + 5\boldsymbol{j} + 7\boldsymbol{k}$ and passes through the point $(-1, 2, 3)$.

Assessment level:

10. Find the point of intersection between the lines $L_1: \frac{x-1}{3} = \frac{y-4}{-1} = \frac{z+7}{2}$ and $L_2: \frac{x+4}{4} = \frac{y-3}{-1} = \frac{z-3}{1}$

11. Let A, B, C be the points $(2, 1, 0)$, $(3, 3, -1)$ and $(5, 0, 2)$ respectively.

Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and obtain the equation of the plane containing A, B and C

12. Three planes have equations: $\pi_1: x - 4y - z = 3$, $\pi_2: 2x - 2y + z = 6$ and $\pi_3: 3x - 11y - 2z = 10$

a. Find the **acute** angle between π_1 and π_2

b. By using Gaussian elimination, show that the three planes intersect at a point Q, and obtain the coordinates of Q.

c. Find, in Cartesian form, the equation for the line L in which π_1 and π_2 intersect, and the point R in which L intersects the xy −plane.

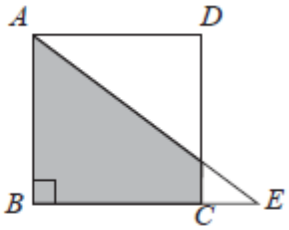
d. Find the shortest distance from R to π_3

Challenge Questions (optional)

1. Four different straight lines are drawn on a flat piece of paper. The number of points where two or more lines intersect is counted. Which of the following could **not** be the number of such points?

A 1 **B** 2 **C** 3 **D** 4 **E** 5

2. The diagram shows a square $ABCD$ and a right-angled triangle ABE. The length of BC is 3. The length of BE is 4. What is the area of the shaded region?



A $5\frac{1}{4}$ **B** $5\frac{3}{8}$ **C** $5\frac{1}{2}$ **D** $5\frac{5}{8}$ **E** $5\frac{3}{4}$