Applying Algebraic skills to Sequences and Series

Essential knowledge:

- **1.** Write down the general term for the sequences shown and find u_{20} :
 - **a.** 8, 12, 16,
 - **b.** 6, 12, 24, 48, ...
- 2. Calculate the sum to 30 terms of the series shown:
 - **a.** $1 + 5 + 9 + 13 + \cdots$
 - **b.** $1 + 2 + 4 + 8 + 16 + \cdots$
- **3.** For each geometric series, state whether a sum to infinity exists and if so, find it:

a. $8 + 12 + 18 + 27 + \cdots$ **b.** $64 - 16 + 4 - 1 + \cdots$

$$\sum_{k=1}^{6} (2k+3)$$

Unit level:

- 5. An arithmetic sequence has first term -3 and a common difference of 5.a. Find the 100th term of the arithmetic sequence.
 - **b.** Find the sum of the first 60 terms of this sequence.
- **6.** Two consecutive terms of a geometric sequence are 135 and 405. The 7th term of this sequence is 3645.
 - **a.** Find an expression for the nth term of this sequence.
 - **b.** Find the sum of the first 10 terms of this geometric sequence.
- **7.** Find the first three non-zero terms of the Maclaurin series for $f(x) = \sin(3x)$

Assessment level:

- **8.** Obtain the sum of the series $8 + 11 + 14 + \dots + 56$
- **9.** A geometric sequence of **positive** terms has first term 2, and the sum of the first three terms is 266. Calculate the common ratio.

10. Obtain an expression for $\sum_{k=1}^{n} (2k^3 + k^2 - k)$ in terms on *n*.

Express your answer in a fully factorized form

11. Write down the Maclaurin expansion of e^x as far as the term in x^4 . Deduce the Maclaurin expansion of e^{x^2} as far as the term in x^4 . Hence, or otherwise, find the Maclaurin expansion of e^{x+x^2} as far as the term in x^4 .

Challenge Questions (optional)

1. What is the largest integer k whose square k^2 is a factor of 10!?

A 6 **B**256 **C** 360 **D** 720 **E** 5040

2. A cube is placed with one face on square 1 in the maze shown. The upper face of the cube is covered in wet paint. The cube is then 'rolled' around the maze, rotating about an edge each time until it reaches square 25. It leaves paint on all of the squares on which the painted face lands on, but no others. What is the sum of the numbers on the squares which are now marked with paint?

| 5 | 6 | 7 | 8 | 9 |
|---|----|----|----|----|
| 4 | 19 | 20 | 21 | 10 |
| 3 | 18 | 25 | 22 | 11 |
| 2 | 17 | 24 | 23 | 12 |
| 1 | 16 | 15 | 14 | 13 |

E 625