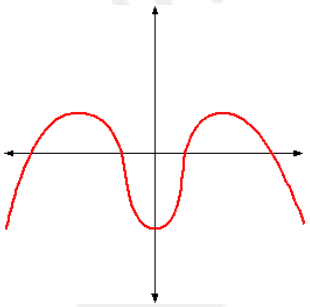
Applying Algebraic & Calculus skills to Properties of Functions

AH Mathematics HW

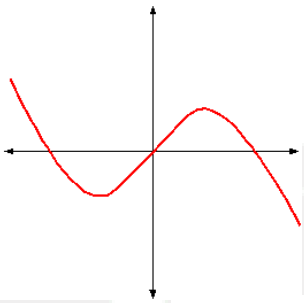
Essential knowledge:

1. State whether each function is odd, even or neither:

a.



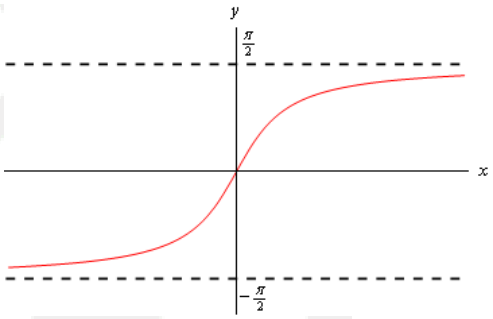
b.



c. $f(x) = e^{x^2} + 3$

d. $f(x) = x^2 + x$

2. For the graph of $f(x) = \tan^{-1} x$ opposite, sketch the graph of:



a. $y = |f(x)|$

b. $y = |f(x)| + \frac{\pi}{2}$

c. $y = |f(x-2)|$

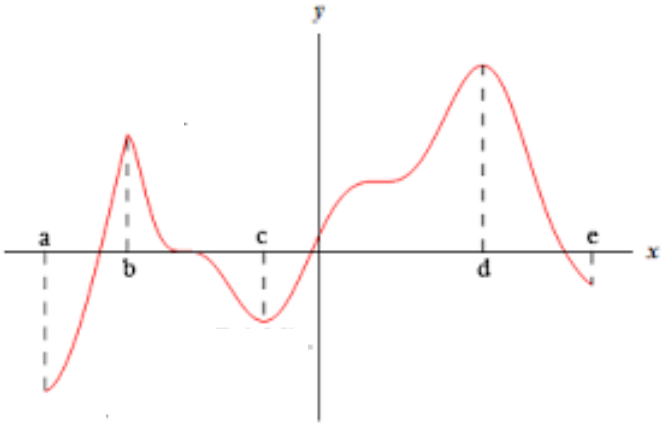
d. $y = 3|f(x)|$

3. State the equations of the vertical asymptotes of:

a. $y = \frac{2x+1}{x-3}$ **b.** $y = x - 1 + \frac{3-x}{x^2-1}$

4. State the equations of the non-vertical asymptotes for **Q3**

5. The graph of $y = f(x)$ over the interval $[a, e)$ is shown.



a. Give a label to each of the extrema points **a** to **e**.

b. How many points of inflexion does the graph show?

Unit level:

6. For the graph of $f(x) = \frac{2x^2+x+1}{x-2}$, $x \in \mathbb{R}, x \neq 2$

a. give the equation of the vertical asymptote

b. determine the equation of the non-vertical asymptote

PTO

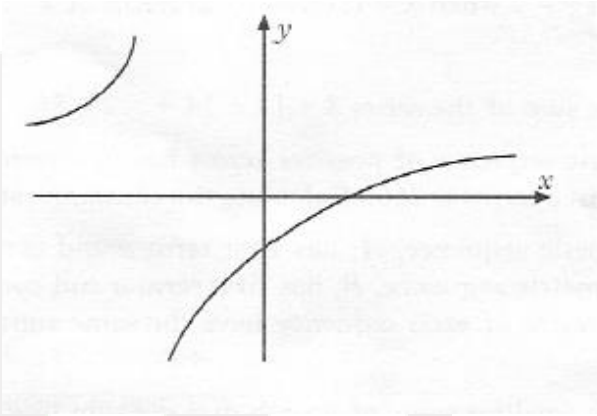
7. Given that $f(x) = \cos 2x$, sketch the graph of $y = |3f(x) - 1|$ where $0 \leq x \leq \pi$

8. Find the coordinates of the point of inflection on the graph of $f(x) = x^3 - 9x^2 + 2$

Assessment level:

9. Show that $f(x) = g(x) - g(-x)$ is an odd function

10. The function f is defined by $f(x) = \frac{x-3}{x+2}$, $x \neq 2$, and the diagram shows part of its graph.



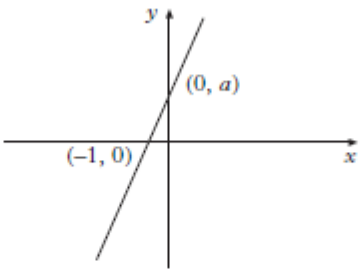
a. Obtain algebraically the asymptotes of the graph.

b. Prove that f has no stationary values

c. Does the graph of f have any points of inflexion? Justify your answer.

d. Sketch the graph of the inverse function f^{-1}, stating the asymptotes and domain of f^{-1}.

11. Given the graph of $y = f(x)$ opposite, sketch the graph of $y = |f^{-1}(x)|$.



Challenge Questions (optional)

1. The lines $y = x$ and $y = mx - 4$ intersect at the point P. What is the sum of the positive integer values of m for which the coordinates of P are also positive integers?

A 3 **B** 5 **C** 7 **D** 8 **E** 10

2. A function, defined on the set of positive integers, is such that $f(xy) = f(x) + f(y)$ for all x and y. It is known that $f(10) = 14$ and $f(40) = 20$. What is the value of $f(500)$?

A 29 **B** 30 **C** 39 **D** 48 **E** 50