

Applying Algebraic skills to Matrices and Systems of Equations

AH Mathematics HW

Essential knowledge:

1. For matrices: $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

Find:

- (a) $2A - 3B$ (b) CA (c) $A'C$ (d) C^{-1} (e) D^3

2. What value of x makes this matrix singular? $\begin{pmatrix} 2 & x \\ -1 & 3 \end{pmatrix}$

3. Use Gaussian elimination to solve:

$$\begin{aligned} 2x - y + 2z &= 1 \\ x + y - 2z &= 2 \\ x + 2y + 4z &= -1 \end{aligned}$$

4. A is the matrix $\begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}$. For $A^2 = pA + qI$, find the values of p and q .

5. Write down the 2×2 matrix that represents an anti-clockwise rotation of 45° about the origin.

Unit level:

6. For matrices: $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & p \\ 0 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & q & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 0 \\ -3 & 5 \\ 8 & r \end{pmatrix}$

Find: (a) $4A - 3B$ (b) $CD + D$

7. Use Gaussian elimination to solve:

$$\begin{aligned} x + y + z &= 10 \\ 2x - y + 3z &= 4 \\ x + 2z &= 20 \end{aligned}$$

8. For matrices: $E = \begin{pmatrix} 4 & -1 \\ -5 & 3 \end{pmatrix}$ and $F = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ -1 & k-1 & 6 \end{pmatrix}$

(a) Find E^{-1}

(b) Determine the value(s) of k for which F is singular.

Assessment level:

9. The matrix A is such that $A^2 = 4A - 3I$, find integers p and q such that $A^4 = pA + qI$

10. Obtain the 2×2 matrix M associated with an enlargement, scale factor 2, followed by a **clockwise** rotation of 60° about O.

11. (a) Use elementary row operations to reduce the following system of equations to upper triangular form

$$\begin{aligned}x + y + 3z &= 1 \\3x + ay + z &= 1 \\x + y + z &= -1\end{aligned}$$

- (b) Express x and y in terms of the parameter a
 (c) Explain what happens when $a = 3$

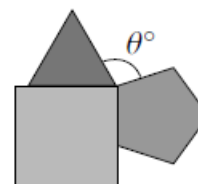
12. Matrices A and B are defined by

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} x+2 & x-2 & x+3 \\ -4 & 4 & 2 \\ 2 & -2 & 3 \end{pmatrix}$$

- (a) Find the product AB
 (b) Obtain the determinants of A and AB
 (c) Hence obtain an expression for $\det(B)$

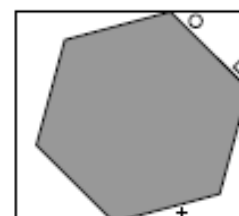
Challenge Questions (optional)

1. The diagram shows an equilateral triangle, a square and a regular pentagon which all share a common vertex. What is the value of θ ?



- A** 98 **B** 102 **C** 106 **D** 110 **E** 112

2. The diagram shows a regular hexagon inside a rectangle. What is the sum of the four marked angles?



- A** 90° **B** 120° **C** 150° **D** 180° **E** 210°