

## Applying Calculus skills through techniques of Integration part 2

AH Mathematics HW

### **Essential knowledge:**

1. Use integration by parts to obtain an expression for:

(a)  $\int (4x - 3)e^{2x} dx$       (b)  $\int_0^{\frac{\pi}{4}} x \cos 2x dx$

2. Find the general solution of following differential equations:

(a)  $\frac{dy}{dx} = \frac{y+3}{x^2}$       (b)  $\frac{dy}{dx} + \frac{y}{x} = 1$       (c)  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

3. Find the particular solution of following differential equations:

(a)  $\frac{dy}{dx} = x(y - 2)$   $x = 0$  when  $y = 5$       (b)  $\frac{dy}{dx} + \frac{2y}{x} = x^2$   $x = 1$  when  $y = 1$

### **Unit level:**

4. Using integration by parts, evaluate:  $\int_1^e x \ln x dx$

5. Find the general solution of the differential equation:  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

6. Find the general solution, in the form  $y = f(x)$ , of the first order linear differential equation:  $\frac{dy}{dx} + \frac{x+1}{x}y = e^{-x}$

7. Find the particular solution of the second order differential equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0 \text{ when } x = 0, y = 5 \text{ and } \frac{dy}{dx} = 11.$$

### **Assessment level:**

8. Use integration by parts to obtain  $\int 8x^2 \sin 4x dx$

9. Given that  $y > -1$  and  $x > -1$ , obtain the general solution of the differential equation

$$\frac{dy}{dx} = 3(1+y)\sqrt{1+x}$$

Expressing your answer in the form  $y = f(x)$

PTO

**10.** A mathematical biologist believes that the differential equation  $x \frac{dy}{dx} - 3y = x^4$  models a process. Given that  $y = 2$  when  $x = 1$ , find the particular solution, expressing  $y$  in terms of  $x$

**11.** Obtain the general solution of the equation  $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0$

**12.** Determine the solution of the differential equation

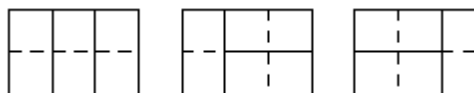
$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 5y = 4 \cos x \text{ which satisfies } y(0) = 0 \text{ and } y'(0) = 1$$

**Challenge Questions (optional)**

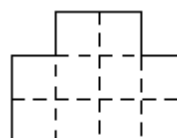
**1.** What is the smallest prime number that is equal to the sum of two prime numbers and is also equal to the sum of three **different** prime numbers?

- A** 7      **B** 11      **C** 13      **D** 17      **E** 19

**2.** A  $2 \times 3$  grid of squares can be divided into  $1 \times 2$  rectangles in three different ways.



How many ways are there of dividing this shape into  $1 \times 2$  rectangles?



- A** 1      **B** 4      **C** 6      **D** 7      **E** 8

**3.** The *primorial* of a number is the product of all of the prime numbers less than or equal to that number. For example, the primorial of 6 is  $2 \times 3 \times 5 = 30$ . How many different whole numbers have a primorial of 210?

- A** 1      **B** 2      **C** 3      **D** 4      **E** 5