

## Applying Calculus skills through techniques of Integration part 1

AH Mathematics HW

### **Essential knowledge:**

1. Integrate the following functions:

(a)  $f(x) = e^{2x} + e^{-2x}$       (b)  $f(x) = \frac{1}{2x+3}$       (c)  $\int \frac{1}{25+x^2} dx$

2. By choosing a suitable substitution, find the following integrals:

(a)  $\int \sin^2 x \cos x dx$       (b)  $\int \frac{x^4}{(x^5+1)^3} dx$       (c)  $\int \frac{e^x}{3e^x-1} dx$

3. Use partial fractions to integrate:

(a)  $\int \frac{dx}{(x-3)(x+2)}$       (b)  $\int \frac{x+4}{(x+1)(x+2)^2} dx$

### **Unit level:**

4. Find:

(a)  $\int \frac{5}{\sqrt{1-(3x)^2}} dx$       (b)  $\int \frac{3}{\sqrt{1-25x^2}} dx$       (c)  $\int \frac{3}{2x-5} dx$

(d)  $\int 2e^{2x+1} dx$       (d)  $\int_0^{\frac{\pi}{18}} \sec^2 6x dx$

5. Using the substitution  $u = \ln x$ , find  $\int \frac{2(\ln x)^3}{x} dx$

### **Assessment level:**

6. Use the substitution  $x = 1 + \sin \theta$  to evaluate  $\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{(1+\sin \theta)^3} d\theta$

7. Find  $\int \frac{x^3}{x^2-1} dx$ ,  $x > 1$

8. Use the substitution  $u = 1 + x$  to evaluate  $\int_0^3 \frac{x}{\sqrt{1+x}} dx$

9. Given that  $\int_4^6 \frac{2x^2-9x-6}{x(x^2-x-6)} dx = \ln\left(\frac{m}{n}\right)$  determine the values for the integers  $m$  and  $n$ .

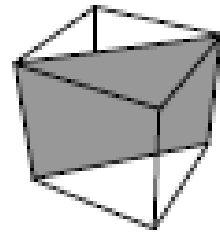
### Challenge Questions (optional)

1. A rectangle has area  $120\text{cm}^2$  and perimeter  $46\text{cm}$ . Which of the following is the length of the diagonals?

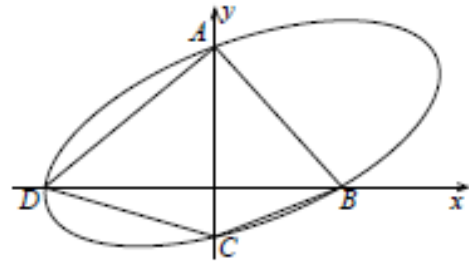
- A**  $15\text{cm}$       **B**  $16\text{cm}$       **C**  $17\text{cm}$       **D**  $18\text{cm}$       **E**  $19\text{cm}$

2. A solid cube of side  $2\text{cm}$  is cut into two triangular prisms by a plane passing through four vertices, as shown. What is the total surface area of these two prisms?

- A**  $8(3 + \sqrt{2})$       **B**  $2(8 + \sqrt{2})$       **C**  $8(3 + 2\sqrt{2})$   
**D**  $16(3 + \sqrt{2})$       **E**  $8\sqrt{2}$



3. The diagram shows the ellipse whose equation is  $x^2 + y^2 - xy + x - 4y = 12$ . The curve cuts the y-axis at points A and C and cuts the x-axis at points B and D. What is the area of the inscribed quadrilateral ABCD?



- A** 28      **B** 36      **C** 42      **D** 48      **E** 56