X100/12/03

NATIONAL QUALIFICATIONS 2014

TUESDAY, 6 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SQA ©

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

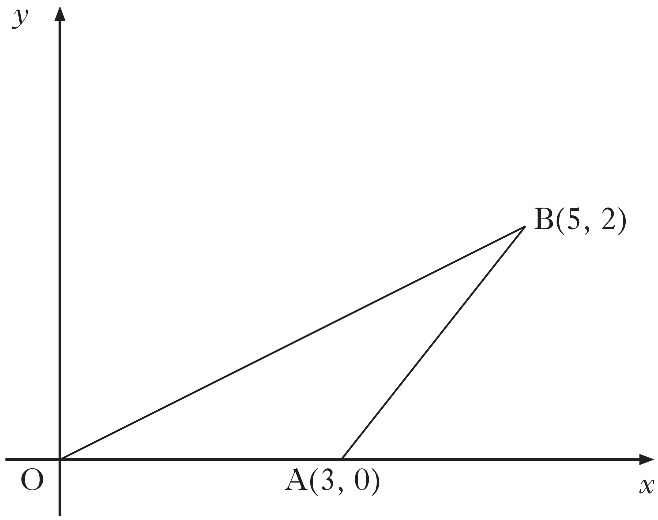
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

ALL questions should be attempted. *Marks*

1. A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



(*a*) Obtain the equation of the perpendicular bisector of AB. **4**

(*b*) The median from A has equation $y + 2x = 6$.

Find T, the point of intersection of this median and the perpendicular bisector of AB. **2**

(*c*) Calculate the angle that AT makes with the positive direction of the x-axis. **2**

2. A curve has equation $y = x^4 - 2x^3 + 5$.

Find the equation of the tangent to this curve at the point where $x = 2$. **4**

3. Functions f and g are defined on suitable domains by

$$f(x) = x(x-1) + q \quad \text{and} \quad g(x) = x + 3.$$

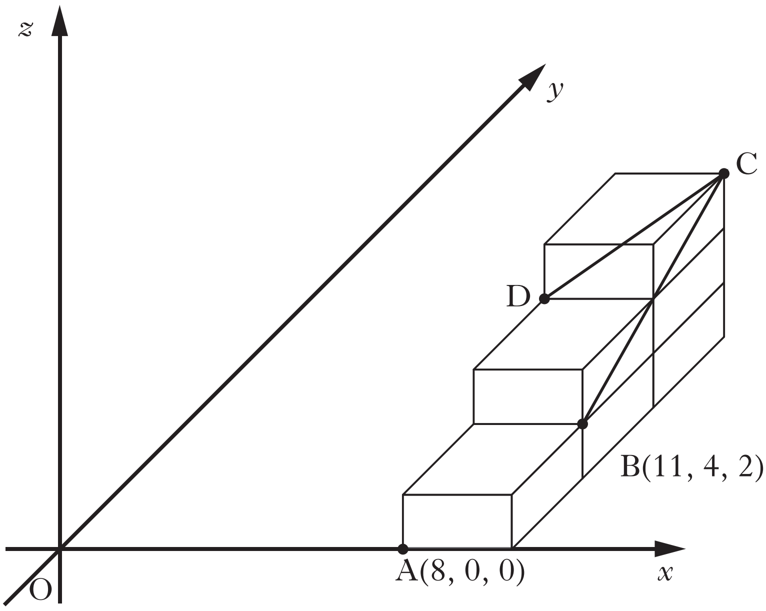
(*a*) Find an expression for $f(g(x))$. **2**

(*b*) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots. **4**

[Turn over

Marks

4. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D. **2**

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. **2**

(*c*) Find the size of the angle BCD. **5**

5. Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$, find the value of t. **5**

6. Solve the equation

$$\sin x - 2\cos 2x = 1 \qquad \text{for } 0 \le x < 2\pi.$$ **5**

Marks

7. Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,\text{m}^2$ of land.

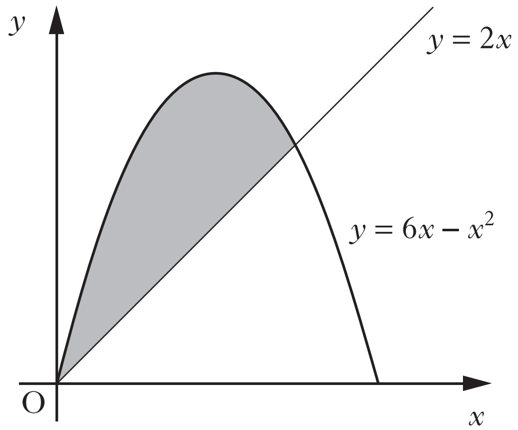


Diagram 1

(*a*) Calculate the area of land being developed. **5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

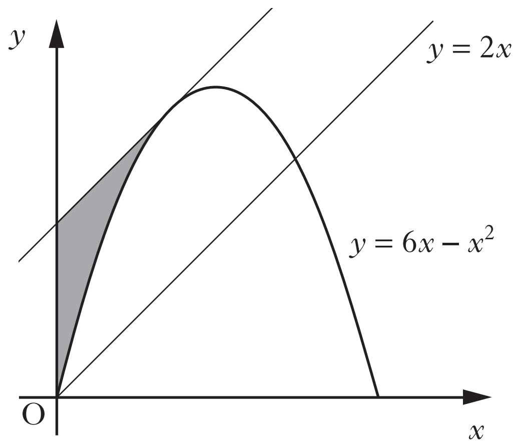


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park. **5**

[Turn over

Marks

8. Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p. **5**

9. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(*c*) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

[*END OF QUESTION PAPER*]

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