Applying Calculus skills through techniques of Differentiation part 2

AH Mathematics HW

Essential knowledge:

1. Using Implicit Differentiation:

(a) $xy + y^2 = 2$ find $\frac{dy}{dx}$ (b) $xy - x = 4$ find $\frac{d^2y}{dx^2}$

2. Using Logarithmic Differentiation:

(a) $y = 3^x$ find $\frac{dy}{dx}$ (b) $y = x^{2x}$ find $\frac{dy}{dx}$

3. Using parametric differentiation:

(a) $x = 2\sec\theta$ and $y = 3\sin\theta$ find $\frac{dy}{dx}$

(b) $x = t^2 + t + 1$ and $y = 2t^2 - t + 1$ find $\frac{d^2y}{dx^2}$

Unit level:

4. If $y^3 + 3xy - 3x^2 = 5$, use implicit differentiation to find $\frac{dy}{dx}$

5. A curve is given by the parametric equations $x = 5\cos\theta$ and $y = 5\sin\theta$. Find $\frac{dy}{dx}$ in terms of θ.

6. Find the speed of a particle defined by the following pair of parametric equations when $t = \frac{\pi}{8}$:

$$x = \cos 2t \qquad y = \sin 2t$$

Assessment level:

7. Given the equation $2y^2 - 2xy - 4y + x^2 = 0$ of a curve, obtain the x-coordinate of each point at which the curve has a horizontal tangent.

8. The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$.

9. Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point (3, -1)

10. Given $y = (x+1)^2(x+2)^{-4}$ and $x > 0$, use logarithmic differentiation to show that $\frac{dy}{dx}$ can be expressed in the form $\left(\frac{a}{x+1} + \frac{b}{x+2}\right)y$, stating the values of the constants a and b.

Challenge Questions (optional)

1. Steve travelled 150 miles on a motorbike and used 10 litres of petrol. Given that 1 gallon ≈ 4.5 litres, roughly how many miles per gallon did Steve achieve on his journey?

A 10 **B** 20 **C** 40 **D** 50 **E** 70

2. A triangle has two edges of length 5. What should be chosen for the third side of the triangle so as to maximize the area within the triangle?

A 5 **B** 6 **C** $5\sqrt{2}$ **D** 8 **E** $5\sqrt{3}$

3. Suppose that $x - \frac{1}{x} = y - \frac{1}{y}$ and $x \neq y$. Which is the value of xy?

A 4 **B** 1 **C** -1 **D** -4 **E** More information is needed