Applying Geometric skills to Complex Numbers

AH Mathematics HW

Calculators can be used but <u>working</u> must be shown

<u>Essential knowledge</u>:

1. For the complex numbers $z_1 = 2 + 2i$ and $z_2 = \sqrt{3} - i$
Find the exact values of:

(a) $z_1 - \overline{z_2}$ (b) $z_1 z_2$ (c) $\frac{z_1}{z_2}$ (d) ${z_1}^3$ (e) $|z_2|$ (f) $Arg\, z_1$

2. Find equivalent Cartesian complex numbers for:

(a) $z = 3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ (b) $|z| = 4$ and $Arg\, z = \frac{\pi}{3}$

3. Describe the locus of the points on the complex plane with restrictions:

(a) $|z| = 3$ (b) $Arg\, z = 1$

4. For the complex number $z = 9\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$, Find the values of:

(a) z^2 (b) $\sqrt{z}$ using **DeMoivre's Theorem**

5. Show that $z = -i$ is a solution of $z^4 + 4z^3 + 3z^2 - 4z + 2 = 0$

<u>Unit level</u>:

6. Given $z = -\sqrt{3} + i$, write z in polar form.

7. Convert $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ into Cartesian form and plot z on an Argand diagram.

8. A complex number z has modulus 1 and argument $-\frac{\pi}{6}$

(a) Determine z in Cartesian form using exact values.

(b) Plot z on an Argand Diagram.

<u>Assessment level</u>:

9. Identify the locus in the complex plane given by $|z - i| = 2$ and show in a diagram the region given by $|z - i| \leq 2$.

10. Given that $w = \cos\theta + i\sin\theta$, find $\frac{1}{w}$ using DeMoivre's Theorem.

11. Given the equation $z + 2i\overline{z} = 8 + 7i$, express z in the form $z = x + iy$.

12. Show that $z = 3 + 3i$ is a root of the equation $z^3 - 18z + 108 = 0$ and obtain the remaining roots of the equation.

13. Express -1 as a complex number in polar form and hence determine the solutions to the equation $z^4 + 1 = 0$.

14. Let $z = \cos\theta + i\sin\theta$

(a) Use the binomial expansion to find z^4

(b) Use DeMoivre's theorem to find another expression for z^4

(c) Hence show that: $\frac{\cos 4\theta}{\cos^2\theta} = p\cos^2\theta + q\sec^2\theta + r \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and find the values of p, q and r.

Challenge Questions (optional)

1. Which of the following has the greatest value?

A $cos\ 50°$ **B** $sin50°$ **C** $\tan 50°$ **D** $\frac{1}{\sin 50°}$ **E** $\frac{1}{\cos 50°}$

2. A square has vertices at (0, 0), (1, 0), (1, 1) and (0, 1). Graphs of the following equations are drawn on the same set of axes as the square:

$x^2 + y^2 = 1$ $y = x + 1$ $y = -x^2 + 1$ $y = x$ $y = \frac{1}{x}$

How many of the graphs pass through exactly two vertices of the square?

A 1 **B** 2 **C** 3 **D** 4 **E** 5