Binomial theorem

$$\left(a+b\right)^{n} = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^{r} \text{ where } \binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

```
                        1
                      1   1
                    1   2   1
                  1   3   3   1
                1   4   6   4   1
              1   5  10  10   5   1
            1   6  15  20  15   6   1
          1   7  21  35  35  21   7   1
        1   8  28  56  70  56  28   8   1
      1   9  36  84 126 126  84  36   9   1
    1  10  45 120 210 252 210 120  45  10   1
```

Essential knowledge:

1. Expand: (a) $(a + b)^7$
(b) $(2 - x)^4$ using Pascal's Triangle

2. Use your expansion of $(2 - x)^4$ to find the value of $(1.9)^4$

3. Expand $(u + 3v)^4$ using the binomial theorem and simplify as far as possible.

4. Solve for n: $\binom{n+1}{2} - \binom{n}{1} = 36$

Unit level:

5. Expand $(2a + 1)^5$ using the binomial theorem and simplify as far as possible.

6. Write down the binomial expansion of $(3y - 2)^4$ and simplify your answer as far as possible.

Assessment level:

7. Write down the general term for the binomial expansion of $\left(x^2 - \frac{2}{x}\right)^4$ $x \neq 0$ and simplify your answer as far as possible.

8. Write down and simplify the general term for the binomial expansion of $\left(3x - \frac{2}{x^2}\right)^4$ and simplify your answer. Hence, or otherwise obtain the term in x

9. Show that $\binom{n+2}{3} - \binom{n}{3} = n^2$ for all integers n, where $n \geq 3$

10. Show that $\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$ where the integer n is greater than or equal to 3

Challenge Questions (optional)

1. There are 120 different arrangements of the five letters in the word ANGLE. If all 120 are listed in alphabetical order starting with AEGLN and finishing with NLGEA, which position in the list does ANGLE occupy?

     A 18th **B** 20th **C** 22nd **D** 24th **E** 26th

2. The *primorial* of a number is the product of all of the prime numbers less than or equal to that number. For example, the *primorial* of 6 is $2 \times 3 \times 5 = 30$. How many different whole numbers have a *primorial* of 210?

     A 1   **B** 2   **C** 3   **D** 4   **E** 5

3. What is the largest integer k whose square k^2 is a factor of 10!?
$[10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1.]$

     A 6   **B** 256   **C** 360   **D** 720   **E** 5040