

ABRONHILL HIGH SCHOOL

Prelim Examination 2012 / 2013
(Assessing Units 1 & 2)

MATHEMATICS

Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions
3. **Full credit will only be given where the solution contains appropriate working**

All questions should be attempted

1. (a) Given $f(x) = e^{-2x} \tan 4x$, $0 < x < \frac{\pi}{8}$ obtain $f'(x)$ 3
- (b) For $y = \frac{\ln 5x}{x-1}$, where $x > 1$, determine $\frac{dy}{dx}$ in its simplest form 3
2. For what value of t does the system of equations:
- $$\begin{aligned}x + 2y - 3z &= -7 \\4x - y + 2z &= 9 \\3x - 2y + tz &= 13\end{aligned}$$
- have no solution? 5
3. Use the binomial theorem to expand and simplify $\left(a^3 - \frac{3}{a}\right)^4$.
Hence write down the term independent of a . 4
4. Given $y = 2\sec\theta + 3\tan\theta$, find $\frac{d^2y}{dx^2}$ 5
5. Use the substitution $x = (u-1)^2$ to find $\int \frac{1}{(1+\sqrt{x})^3} dx$ 5
6. Find the equation of the locus of $|z-4|=5$ where $z = x+iy$, x and y are real 4
7. For all natural numbers n , prove whether the following statement is true or false:
“ $n^3 + n + 5$ is always prime” 2
8. A curve is defined by the parametric equations
- $$x = 10t, \quad y = 1 + 12t - t^3 \quad \text{for all } t.$$
- (a) Find the coordinates of the stationary points of this curve. 4
- (b) Obtain an expression for $\frac{d^2y}{dx^2}$ and use this to determine the nature of the stationary points found in (a). 3

9. (a) express the function $f(x) = \frac{6x^4 + x^3 - 5x - 4}{x^3 - x}$ in the form:
 $Ax + B + \frac{C}{x} + \frac{D}{x+1} + \frac{E}{x-1}$ where A, B, C, D and E are integers 4
- (b) Hence show that $\int_2^3 f(x)dx = 16 + \ln 6$. 4
10. Given that $w = \cos \theta + i \sin \theta$, show that $\frac{1}{w} = \cos \theta - i \sin \theta$ 1
 Use DeMoivre's theorem to prove that $w^k + w^{-k} = 2 \cos k\theta$ where k is a natural number. 4
11. Use integration by parts to evaluate $\int_0^1 x^2 e^{-x} dx$ 5
12. Let $u_1, u_2, \dots, u_n, \dots$ be an arithmetic sequence and $v_1, v_2, \dots, v_n, \dots$ be a geometric sequence.
 The first terms u_1 and v_1 are both equal to 45 and the third terms u_3 and v_3 are both equal to 5.
- (a) Find u_{11} 3
- (b) Given that $v_1, v_2, \dots, v_n, \dots$ is a sequence of positive numbers, calculate $\sum_{n=1}^{\infty} v_n$ 3
13. Given that $x^2 e^y \frac{dy}{dx} = 1$ and $y = 0$ when $x = 1$, find y in terms of x 4
14. The function f is defined by $f(x) = \frac{x^2 - 25}{x^2 - 4}$
- (a) Decide, giving reasons, whether f is odd, even or neither. 2
- (b) Write down the equation of any vertical asymptote. 2
- (c) Find algebraically the equation of any non vertical asymptote. 3
- (d) Find the coordinate of the only stationary point of the function f . 3

[END OF QUESTION PAPER]