Abronhill High School

Prelim Examination 2010 / 2011
(Assessing Unit 3)

MATHEMATICS

Advanced Higher Grade

Time allowed - 1 hour

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used in this paper.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. A is the 3 x 3 matrix $A = \begin{pmatrix} x-2 & -1 & 2 \\ 1 & x-1 & 3 \\ 2 & 1 & x \end{pmatrix}$

   (*a*) Show that the determinant, $\det(A) = x^3 - 3x^2 - 4x + 6$ (3)

   (*b*) Find the integer value of *x* such that matrix A has no inverse (3)

2. (*a*) Use the Euclidean algorithm to determine the greatest common divisor of 407 and 592 (3)

   (*b*) Hence find the integers *x* and *y* such that $d = 407x + 592y$ (2)

3. Prove by induction that $\sum_{r=1}^{n}(19r - 18) = \frac{n}{2}(19n - 17)$ where *n* is a positive integer. (5)

4. Find the first five non-zero terms of the Maclaurin series for $\ln(1+x)$. (5)

5. (*a*) Use the vector product to calculate the area of a triangle with vertices P(2, -1, 0), Q(1, 1, -1) and R(3, 4, 2). (4)

   (*b*) Find the equation of the plane passing through triangle PQR. (3)

   (*c*) Determine the point of intersection of the line

   $$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{3}$$

   with this plane. (4)

6. A recurrence relation is defined by the formula:

   $$x_{n+1} = \frac{14 - 5x_n}{x_n}$$

   Find algebraically the fixed points of the recurrence relation (3)

7. Find the general solution of the differential equation

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 3x + 4.$$

Find the particular solution corresponding to the initial conditions $\frac{dy}{dx} = -3$ and $\frac{d^2y}{dx^2} = 4$ when $x = 0$. (10)