

| | Give 1 mark for each • | Illustration(s) for awarding each mark |
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| 1(a) | ans: Proof <ul style="list-style-type: none"> •¹ knows to find determinant •² correct algebra •³ conclusion | (3 marks) <ul style="list-style-type: none"> •¹ $x-2 \begin{vmatrix} x-1 & 3 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & x \end{vmatrix} + 2 \begin{vmatrix} 1 & x-1 \\ 2 & 1 \end{vmatrix}$ •² $(x-2)(x^2 - x - 3) + (x-6) + 2(3-2x)$ •³ $x^3 - 3x^2 - 4x + 6$ |
| | (b) ans: $x = 1$ <ul style="list-style-type: none"> •¹ knows to let $\det(A) = 0$ •² solves cubic equation •³ answer | (3 marks) <ul style="list-style-type: none"> •¹ nested calculation •² $(x-1)(x^2 - 2x - 6) = 0$ •³ $x = 1, x^2 - 2x - 6$ has no solution |
| 2(a) | ans: $\text{gcd} = 37$ <ul style="list-style-type: none"> •¹ method •² remaining steps •³ answer | (3 marks) <ul style="list-style-type: none"> •¹ $592 = 1 \times 407 + 185$ •² $407 = 2 \times 185 + 37; 185 = 5 \times 37 + 0$ •³ $\text{gcd} = 37$ |
| | (b) ans: $x = 3, y = 2$ <ul style="list-style-type: none"> •¹ method •² complete process | (2 marks) <ul style="list-style-type: none"> •¹ $37 = 407 - 2(185); 37 = 407 - 2(592 - 407)$ •² $37 = 3 \times 407 - 2 \times 592$ |
| 3 | ans: Proof <ul style="list-style-type: none"> •¹ prove true for $n = 1$ •² assume true for $n = k$ •³ considers $n = k + 1$ •⁴ simplifies •⁵ statement | (5 marks) <ul style="list-style-type: none"> •¹ LHS = RHS = 1 •² $\frac{k}{2}(19k-17)$ •³ $\sum_{r=1}^k (19r-18) + (19(k+1)-18)$ •⁴ $\frac{k+1}{2}(19(k+1)-17)$ •⁵ valid for $n=1 \dots\dots\dots$ |
| 4. | ans: $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$ <ul style="list-style-type: none"> •¹ finds $f(0)$ and $f'(0)$ •² finds $f''(0)$ •³ finds values for higher order derivatives •⁴ method •⁵ final statement | (5 marks) <ul style="list-style-type: none"> •¹ $f(0) = 0$ and $f'(0) = 1$ •² $f''(0) = -1$ •³ 2, -6 and 24 •⁴ $f(x) \approx f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \dots$ •⁵ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$ |

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| 5(a) | <p>ans: $\frac{\sqrt{75}}{2} = \frac{5\sqrt{3}}{2}$</p> <ul style="list-style-type: none"> •¹ finds direction vectors •² use vector product correctly •³ knows how to find the area •⁴ finds area | <p style="text-align: right;">(4 marks)</p> <ul style="list-style-type: none"> •¹ $\vec{PQ} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ and $\vec{PR} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ •² $\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ 1 & 5 & 2 \end{vmatrix} = 9i + j - 7k$ •³ $A = \frac{1}{2} 9i + j - 7k$ •⁴ $\frac{\sqrt{75}}{2} = \frac{5\sqrt{3}}{2}$ |
| (b) | <p>ans: $9x + y - 7z = 17$</p> <ul style="list-style-type: none"> •¹ knows to use vector product for normal •² uses coefficients of n for equation of plane •³ finds value of k | <p style="text-align: right;">(3 marks)</p> <ul style="list-style-type: none"> •¹ $n = 9i + j - 7k$ •² $9x + y - 7z = k$ •³ $k = 9(2) + (-1) - 7(0) = 17$ |
| (c) | <p>ans: $(-15, -9, -23)$</p> <ul style="list-style-type: none"> •¹ writes equation in parametric form •² subst. parametric into equation of plane •³ finds t •⁴ finds point of intersection | <p style="text-align: right;">(4 marks)</p> <ul style="list-style-type: none"> •¹ $x = 1 + 2t; y = t - 1; z = 1 + 3t$ •² $9(1 + 2t) + (t - 1) - 7(1 + 3t) = 17$ •³ $t = -8$ •⁴ $x = -15; y = -9; z = -23$ |
| 6 | <p>ans: $x = -7$ or $x = 2$</p> <ul style="list-style-type: none"> •¹ correct strategy •² correct re-arrangement •³ correct fixed points | <p style="text-align: right;">(3 marks)</p> <ul style="list-style-type: none"> •¹ $x = \frac{14 - 5x}{x}$ •² $x^2 + 5x - 14 = 0$ •³ $x = -7$ or $x = 2$ |

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| 7 | <p>ans: $y = 8e^{-\frac{1}{2}x} - 2xe^{-\frac{1}{2}x} + 3x - 8$</p> <ul style="list-style-type: none"> ●¹ correct auxiliary equation ●² solves auxiliary equation ●³ correct CF ●⁴ correct format of PI ●⁵ correct PI ●⁶ correct general solution ●⁷ correct 1st derivative ●⁸ correct 2nd derivative ●⁹ correct A value ●¹⁰ correct B value | <p style="text-align: right;">(10 marks)</p> <ul style="list-style-type: none"> ●¹ $4m^2 + 4m + 1 = 0$ ●² $m = -\frac{1}{2}$ ●³ $y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x}$ ●⁴ $y = Cx + D$ ●⁵ $y = 3x - 8$ ●⁶ $y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x} + 3x - 8$ ●⁷ $\frac{dy}{dx} = -\frac{1}{2}Ae^{-\frac{1}{2}x} + Be^{-\frac{1}{2}x} - \frac{1}{2}Bxe^{-\frac{1}{2}x} + 3$ ●⁸ $\frac{d^2y}{dx^2} = \frac{1}{4}Ae^{-\frac{1}{2}x} - \frac{1}{2}Be^{-\frac{1}{2}x} - \frac{1}{2}Be^{-\frac{1}{2}x} + \frac{1}{4}Bxe^{-\frac{1}{2}x}$ ●⁹ $A = 8$ ●¹⁰ $B = -2$ <div style="text-align: right; border: 1px solid black; padding: 5px; width: fit-content; margin: 20px auto;"> <p>Total: 45 marks</p> </div> |