ABRONHILL HIGH SCHOOL

Prelim Examination 2010 / 2011
(Assessing Units 1 & 2)

MATHEMATICS

Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions
3. **Full credit will only be given where the solution contains appropriate working**

All questions should be attempted

1. (a) Given $f(x) = e^{-2x}\tan 4x$, $0 < x < \frac{\pi}{8}$ obtain $f'(x)$ **3**

(b) For $y = \frac{\ln 5x}{x-1}$, where $x > 1$, determine $\frac{dy}{dx}$ in its simplest form **3**

2. For what value of *t* does the system of equations:

$$\begin{aligned} x + 2y - 3z &= -7 \\ 4x - y + 2z &= 9 \\ 3x - 2y + tz &= 13 \end{aligned}$$

have no solution? **5**

3. Use the binomial theorem to expand and simplify $\left(a^3 - \frac{3}{a}\right)^4$.

Hence write down the term independent of a. **4**

4. Given $y = 2\sec\theta + 3\tan\theta$, find $\frac{d^2y}{dx^2}$ **5**

5. Use the substitution $x = (u-1)^2$ to find $\int \frac{1}{\left(1+\sqrt{x}\right)^3}dx$ **5**

6. Find the equation of the locus of $|z-4| = 5$ where $z = x + iy$, x and y are real **4**

7. For all natural numbers *n*, prove whether the following statement is true or false:

"$n^3 + n + 5$ is always prime" **2**

8. A curve is defined by the parametric equations

$x = 10t$, $y = 1 + 12t - t^3$ for all *t*.

(a) Find the coordinates of the stationary points of this curve. **4**

(b) Obtain an expression for $\frac{d^2y}{dx^2}$ and use this to determine the nature of the stationary points found in **(a).** **3**

9. (*a*) express the function $f(x) = \dfrac{6x^4 + x^3 - 5x - 4}{x^3 - x}$ in the form:

$$Ax + B + \frac{C}{x} + \frac{D}{x+1} + \frac{E}{x-1}$$ where A, B, C, D and E are integers **4**

(*b*) Hence show that $\int_2^3 f(x)dx = 16 + \ln 6$. **4**

10. Given that $w = \cos\theta + i\sin\theta$, show that $\dfrac{1}{w} = \cos\theta - i\sin\theta$ **1**

Use DeMoivre's theorem to prove that $w^k + w^{-k} = 2\cos k\theta$ where *k* is a natural number. **4**

11. Use integration by parts to evaluate $\int_0^1 x^2 e^{-x} dx$ **5**

12. Let $u_1, u_2 ....., u_n, ...$ be an arithmetic sequence and $v_1, v_2 ....., v_n, ...$ be a geometric sequence.
The first terms u_1 and v_1 are both equal to 45 and the third terms u_3 and v_3 are both equal to 5.

(*a*) Find u_{11} **3**

(*b*) Given that $v_1, v_2 ....., v_n, ...$ is a sequence of positive numbers, calculate $\sum_{n=1}^{\infty} v_n$ **3**

13. Given that $x^2 e^y \dfrac{dy}{dx} = 1$ and $y = 0$ when $x = 1$, find *y* in terms of *x* **4**

14. The function *f* is defined by $f(x) = \dfrac{x^2 - 25}{x^2 - 4}$

(*a*) Decide, giving reasons, whether *f* is odd, even or neither. **2**

(b) Write down the equation of any vertical asymptote. **2**

(c) Find algebraically the equation of any non vertical asymptote. **3**

(d) Find the coordinate of the only stationary point of the function *f*. **3**

[END OF QUESTION PAPER]