Mini-Prelim Examination 2008 / 2009
(Assessing Unit 3 + Units 1 & 2 Revision)

MATHEMATICS

Advanced Higher Grade

Time allowed - 1 hour 20 minutes

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions
3. **Full credit will only be given where the solution contains appropriate working**

All questions should be attempted

1. $P = \begin{pmatrix} 6 & -3 \\ 2 & -5 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}$ and $R = P - 2Q$.

Find R^{-1}, the inverse of R. **3**

2. Obtain algebraically the fixed point of the iterative scheme given by

$$x_{n+1} = \frac{1}{5}\left\{4x_n - \frac{27}{x_n^2}\right\}, \qquad n = 0, 1, 2, \ldots .$$ **3**

3. (*a*) The line *l* has equation $\frac{x-1}{3} = \frac{y+1}{4} = \frac{z-1}{-2}$. This line meets the plane π with equation $2x - y - 4z = 9$ at the point *T*.

Find the coordinates of *T*. **4**

(*b*) Find the size of the angle between the line *l* and the plane π. **4**

(*c*) A second plane α is parallel to the plane π and the line *l* meets the plane α at the point *R* (-5, -9, 5).

Find the equation of the plane α. **3**

4. (*a*) Show that $e^{\int \frac{\sin x}{\cos x} dx} = \sec x$. **2**

(*b*) (i) Find the general solution of the first order linear differential equation

$$\cos x \frac{dy}{dx} + (\sin x)y = 2\cos^3 x \sin x - 1, \quad 0 \le x < \frac{\pi}{2}.$$ **6**

(ii) Find the particular solution corresponding to the condition $y\left(\frac{\pi}{4}\right) = 3\sqrt{2}$. **2**

5. Prove by induction that $\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{1}{2} - \frac{1}{3n+2}$ for all positive integers *n*. **5**

State the value of the limit as $n \to \infty$ of $\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)}$. **1**

6. Express the integer 271 in base 6. **3**

7. Find the Maclaurin expansion of $\ln(1+x)$ as far as the term in x^4. **3**

Given that the Maclaurin expansion of $\ln(\cos x)$ as far as the term in x^4 is $-\frac{x^2}{2}-\frac{x^4}{12}$, find the Maclaurin expansion as far as the term in x^4 of $\ln(\cos x + x\cos x)$. **3**

8. (*a*) Given $A=\begin{pmatrix}1 & 1 & -1\\ -1 & 0 & 2\\ 1 & 2 & -1\end{pmatrix}$ and $B=\begin{pmatrix}4 & 1 & -2\\ -1 & 0 & 1\\ 2 & 1 & -1\end{pmatrix}$, find *AB*. **1**

(*b*) **Hence** solve the system of equations

$$\begin{aligned} 4x + y - 2z &= 1\\ -x \quad + \ z &= -2\\ 2x + y - \ z &= 5. \end{aligned}$$ **3**

9. Find the general solution of the differential equation

$$4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=3x+4.$$

Find the particular solution corresponding to the initial conditions $\frac{dy}{dx}=-3$ and $\frac{d^2y}{dx^2}=4$ when $x = 0$. **10**

[END OF QUESTION PAPER]