

Marking Scheme – Advanced Higher Grade 2008 / 2009
Mini-Prelim (Assessing Unit 3 + Units 1 & 2 Revision)

	Give one mark for each •	Illustrations for awarding each mark
1.	ans: $R^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$ 3 marks <ul style="list-style-type: none"> • finds $P - 2Q$ correctly • knows how to find inverse • knows how to find inverse 	<ul style="list-style-type: none"> • $R = \begin{pmatrix} 4 & -1 \\ -2 & 1 \end{pmatrix}$ • $R^{-1} = \frac{1}{4-2} \begin{pmatrix} & \\ & \end{pmatrix}$ • $R^{-1} = \dots \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$
2.	ans: -3 3 marks <ul style="list-style-type: none"> • correct equation • rearranges correctly • correct fixed point 	<ul style="list-style-type: none"> • $x = \frac{1}{5} \left(4x - \frac{27}{x^2} \right)$ • $5x^3 = 4x^3 - 27$ • $x = -3$
3(a)	ans: $T(4, 3, -1)$ 4 marks <ul style="list-style-type: none"> • correct equations • substitutes correctly • solves correctly • correct point 	<ul style="list-style-type: none"> • $x = 3t + 1, y = 4t - 1 \& z = -2t + 1$ • $2(3t + 1) - (4t - 1) - 4(-2t + 1) = 9$ • $t = 1$ • $T(4, 3, -1)$
3(b)	ans: $23 \cdot 9^0$ 4 marks <ul style="list-style-type: none"> • correct vectors • knows how to find angle • correct angle • correct answer 	<ul style="list-style-type: none"> • $\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} \& \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ • $\cos \theta^0 = \frac{6 - 4 + 8}{\sqrt{21}\sqrt{29}}$ • $66 \cdot 1^0$ • $23 \cdot 9^0$

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3(c)	ans: $2x - y - 4z = -21$ 3 marks <ul style="list-style-type: none"> • uses correct normal vector • substitutes point correctly • correct equation 	<ul style="list-style-type: none"> • $2x - y - 4z = k$ • $2(-5) - (-9) - 4(5) = k$ • $k = -21$
4(a)	ans: Proof 2 marks <ul style="list-style-type: none"> • integrates correctly • completes proof correctly 	<ul style="list-style-type: none"> • $e^{-\ln \cos x }$ • $e^{\ln \cos x ^{-1}} = \dots = \sec x$
4(b) (i)	ans: $y = -\frac{1}{2} \cos x \cos 2x - \sin x + C \cos x$ 6 marks <ul style="list-style-type: none"> • rearranges correctly • correct integrating factor • correct method • simplifies correctly • integrates correctly • correct general solution 	<ul style="list-style-type: none"> • $\frac{dy}{dx} + \frac{\sin x}{\cos x} y = 2 \cos^2 x \sin x - \frac{1}{\cos x}$ • $IF = e^{\int \frac{\sin x}{\cos x} dx} = \sec x$ • $(\sec x)y = \int \sec x \left(2 \cos^2 x \sin x - \frac{1}{\cos x} \right) dx$ • $(\sec x)y = \int (\sin 2x - \sec^2 x) dx$ • $(\sec x)y = -\frac{1}{2} \cos 2x - \tan x + C$ • $y = -\frac{1}{2} \cos x \cos 2x - \sin x + C \cos x$
4(b) (ii)	ans: $y = -\frac{1}{2} \cos x \cos 2x - \sin x + 7 \cos x$ 2 marks <ul style="list-style-type: none"> • substitutes correctly • solves correctly 	<ul style="list-style-type: none"> • $3\sqrt{2} = -\frac{1}{2} \cos \frac{\pi}{4} \cos \frac{\pi}{2} - \sin \frac{\pi}{4} + C \cos \frac{\pi}{4}$ • $C = 7$
5.	ans: Proof 6 marks $\frac{1}{2}$ <ul style="list-style-type: none"> • verifies result for $n = 1$ (e.g.) • states correct assumption for $n = k$ • states correct result for $n = k + 1$ • continues proof correctly • completes proof correctly • correct value 	<ul style="list-style-type: none"> • $LHS = \frac{3}{10} = RHS \Rightarrow T$ for $n = 1$ • $\sum_{r=1}^k \frac{3}{(3r-1)(3r+2)} = \frac{1}{2} - \frac{1}{3k+2}$ • $\sum_{r=1}^{k+1} \frac{3}{(3r-1)(3r+2)} = \frac{1}{2} - \frac{1}{3k+5}$ • $\dots = \frac{1}{2} - \frac{3k+2}{(3k+2)(3k+5)}$ • \therefore Since result is T for $n = 1$ and (T for $n = k \Rightarrow T$ for $n = k + 1$), the result is $T \forall n \in \mathbb{N}$ • $\frac{1}{2}$

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6.	<p>ans: 1131_6 3 marks</p> <ul style="list-style-type: none"> • knows to apply repeated division by 6 • applies repeated division correctly • correct answer 	<ul style="list-style-type: none"> • $271 \div 6 = 45r1$ • $45 \div 6 = 7r3$ • $7 \div 6 = 1r1$ • $1 \div 6 = 0r1$ • 1131_6
7.	<p>ans: $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$</p> <p>$\ln(\cos x + x \cos x) = x - x^2 + \frac{1}{3}x^3 - \frac{1}{3}x^4$ 6 marks</p> <ul style="list-style-type: none"> • starts correctly • continues correctly • correct expansion • correct expression • correct expansion • correctly simplified expansion 	<ul style="list-style-type: none"> • $f(0) = 0, f'(0) = 1 \& f''(0) = -1$ • $f'''(0) = 2 \& f''''(0) = -6$ • $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$ • $\ln(\cos x(1+x))$ • $-\frac{x^2}{2} - \frac{x^4}{12} + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$ • $x - x^2 + \frac{1}{3}x^3 - \frac{1}{3}x^4$
8(a)	<p>ans: $AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 1 mark</p> <ul style="list-style-type: none"> • correct matrix 	<ul style="list-style-type: none"> • $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
8(b)	<p>ans: $x = -6, y = 9, z = -8$ 3 marks</p> <ul style="list-style-type: none"> • correct matrix equation • pre-multiplies both sides by A • correct solution 	<ul style="list-style-type: none"> • $\begin{pmatrix} 4 & 1 & -2 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ • $\begin{pmatrix} 1 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 4 & 1 & -2 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ • $\begin{pmatrix} 1 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ • $x = -6, y = 9, z = -8$

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9.	<p>ans: $y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x} + 3x - 8$</p> <p>$y = 8e^{-\frac{1}{2}x} - 2xe^{-\frac{1}{2}x} + 3x - 8$</p> <p style="text-align: right;">10 marks</p> <ul style="list-style-type: none"> • correct auxiliary equation • solves auxiliary equation correctly • correct complementary function • correct form of particular integral • correct particular integral • correct general solution • correct first derivative • correct second derivative • correct value for A • correct value for B 	<ul style="list-style-type: none"> • $4m^2 + 4m + 1 = 0$ • $m = -\frac{1}{2}$ (twice) • $y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x}$ • $y = Cx + D$ • $y = 3x - 8$ • $y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x} + 3x - 8$ • $\frac{dy}{dx} = -\frac{1}{2}Ae^{-\frac{1}{2}x} + Be^{-\frac{1}{2}x} - \frac{1}{2}Bxe^{-\frac{1}{2}x} + 3$ • • $\frac{d^2y}{dx^2} = \frac{1}{4}Ae^{-\frac{1}{2}x} - \frac{1}{2}Be^{-\frac{1}{2}x} - \frac{1}{2}Be^{-\frac{1}{2}x} + \frac{1}{4}Bxe^{-\frac{1}{2}x}$ • $A = 8$ • $B = -2$

56 Marks