Prelim Examination 2008 / 2009
(Assessing Units 1 & 2)

MATHEMATICS

Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions
3. **Full credit will only be given where the solution contains appropriate working**

All questions should be attempted

1. Given $x = \sin^{-1} t,\ \ y = \ln t$ where $0 < t < 1$, use parametric differentiation to obtain $\frac{dy}{dx}$ in terms of t. Simplify your answer. **3**

2. Find the coefficient of x^{-4} in the expansion of $\left(\frac{x^2}{2} - \frac{4}{x^3}\right)^8$. **5**

3. Express $\frac{x^2 - 2x + 6}{x^2(x^2+2)}$ in partial fractions. **5**

4. Use Gaussian elimination to solve the system of equations

$$\begin{aligned} 2x - 7y + 10z &= -1 \\ x - 3y + 4z &= 2 \\ 5x - 18y + 26z &= -6. \end{aligned}$$ **5**

5. Express $z = \frac{5i}{1+2i}$ in the form $a + ib$ where a and b are real numbers. **2**

Verify that z is a solution of the equation $z^4 - 4z^3 + 6z^2 - 4z + 5 = 0$ and find the other three roots. **4**

6. Prove by induction that $8^n - 7n + 6$ is divisible by 7 for all natural numbers n. **5**

7. A curve is defined by the equation $x^3y^2 - 2xy + 8 = 0$, $x < 0$ and $y < 0$.

Use implicit differentiation to find $\frac{dy}{dx}$. **3**

Hence find the equation of the tangent to the curve at the point where $x = -1$. **3**

8. (*a*) Evaluate $\sum_{r=1}^{3} 16 \times \left(\frac{3}{4}\right)^{r-1}$. **2**

(*b*) Explain why the sum to infinity of the geometric series 16 + 12 + 9 + … exists and find this sum. **3**

9. $I = \int_0^2 e^{\sqrt{4x+1}}\,dx.$

(*a*) Use the substitution $u = \sqrt{4x+1}$ to express *I* in the form $\int_a^b \frac{1}{k} u e^u \, du$, where *a*, *b* and *k* are integers. **3**

(*b*) Use integration by parts to evaluate the integral found in (*a*). **5**

10. The function *g* is given by $g(x) = e^{2x} \sin 2x$.

(*a*) Determine whether *g* is odd, even or neither. **2**

(*b*) Find the coordinates of the stationary point of *g* in the interval $0 < x < \frac{\pi}{2}$. **5**

(*c*) Obtain a formula for $y = g''(x)$. **1**

(*d*) Use your answer to (*c*) to determine the nature of the stationary point found in (*b*). **2**

11. Let $z = \cos\theta + i\sin\theta$.

(*a*) Use de Moivre's theorem to express z^3 in terms of 3θ. **1**

(*b*) Use the binomial theorem to express z^3 in terms of $\sin\theta$ and $\cos\theta$. **2**

(*c*) Hence express

(i) $\cos 3\theta$ in terms of $\cos\theta$

(ii) $\sin 3\theta$ in terms of $\sin\theta$. **2,2**

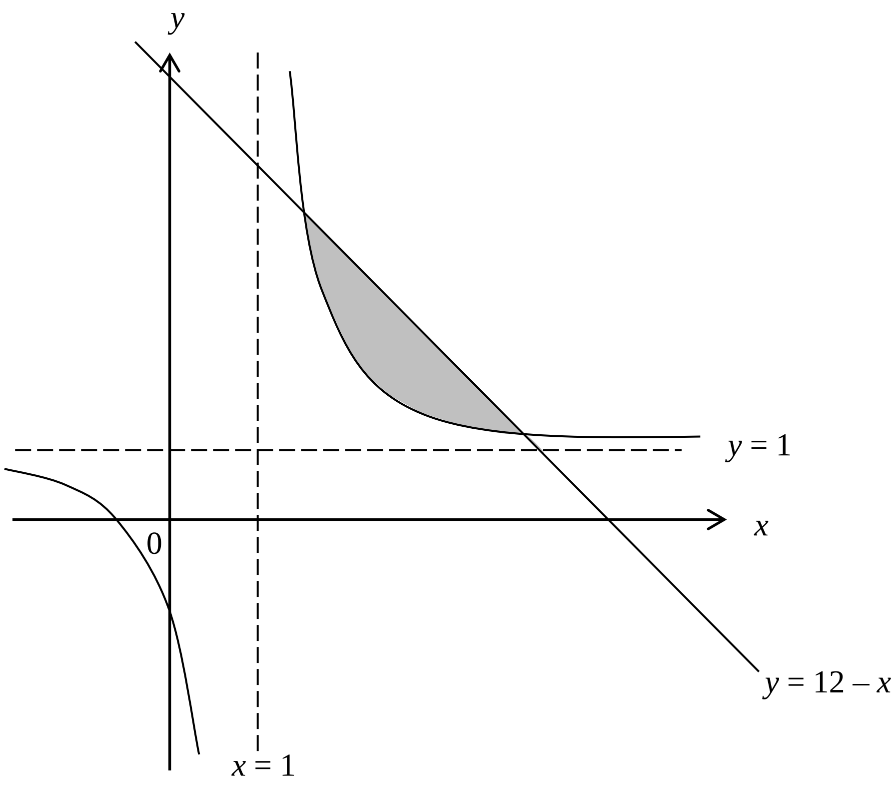
(*d*) Use your answers to (*c*)(i) and (*c*)(ii) to show that

$$\cot 3\theta = \frac{1 - 3\tan^2\theta}{3\tan\theta - \tan^3\theta}.$$ **4**

12. (*a*) Express $\dfrac{x+8}{x-1}$ in the form $A+\dfrac{B}{x-1}$. **2**

(*b*) The diagram below shows the curve with equation $y=\dfrac{x+8}{x-1}$ and the line with equation $y=12-x$.

Show that the shaded area can be written as $40-\ln 9^9$. **6**



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