

**Marking Scheme – Advanced Higher Grade 2008 / 2009
Prelim (Assessing Units 1 & 2)**

	Give one mark for each •	Illustrations for awarding each mark
1.	<p>ans: $\frac{dy}{dx} = \frac{\sqrt{1-t^2}}{t}$ 3 marks</p> <ul style="list-style-type: none"> • knows how to use parametric differentiation • differentiates correctly • simplifies correctly 	<ul style="list-style-type: none"> • $\frac{y'(t)}{x'(t)}$ • $\frac{1}{\frac{t}{1}}$ • $\frac{\sqrt{1-t^2}}{\frac{t}{1}}$ • $\frac{\sqrt{1-t^2}}{t}$
2.	<p>ans: 1120 5 marks</p> <ul style="list-style-type: none"> • finds correct general term • simplifies to find correct expression for power of x • solves for r correctly • substitutes correctly • finds correct coefficient of x^{-4} 	<ul style="list-style-type: none"> • $\binom{8}{r} \left(\frac{x^2}{2}\right)^{8-r} \left(-\frac{4}{x^3}\right)^r$ • x^{16-5r} • $r = 4$ • $\binom{8}{4} \frac{(-4)^4}{2^4} x^{-4}$ • 1120
3.	<p>ans: $\frac{-1}{x} + \frac{3}{x^2} + \frac{x-2}{x^2+2}$ 5 marks</p> <ul style="list-style-type: none"> • starts correctly • finds B correctly • finds D correctly • finds A correctly • finds C correctly 	<ul style="list-style-type: none"> • $x^2 - 2x + 6 \equiv$ • $Ax(x^2 + 2) + B(x^2 + 2) + (Cx + D)x^2$ • $B = 3$ • $D = -2$ • $A = -1$ • $C = 1$

4.	<p>ans: No solutions</p> <p>5 marks</p> <ul style="list-style-type: none"> • correct augmented matrix • first modified system correct • second modified system correct • third modified system correct • correct conclusion 	<ul style="list-style-type: none"> • $\begin{bmatrix} 2 & -7 & 10 & -1 \\ 1 & -3 & 4 & 2 \\ 5 & -18 & 26 & -6 \end{bmatrix}$ • $\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & -1 & 2 & -5 \\ 5 & -18 & 26 & -6 \end{bmatrix}$ • $\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & -1 & 2 & -5 \\ 0 & -3 & 6 & -16 \end{bmatrix}$ • $\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
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5.	<p>$z = 2 + i$ ans: $z = 2 - i, z = i \text{ \& } z = -i$</p> <p style="text-align: right;">6 marks</p> <ul style="list-style-type: none"> • starts correctly • simplifies correctly • verifies correctly • correct conjugate root • correct equation • correct roots 	<ul style="list-style-type: none"> • $\frac{5i}{1+2i} \frac{1-2i}{1-2i}$ • $2+i$ • '$R = 0$' • $2 - i$ • $z^2 + 1 = 0$ • $\pm i$
6.	<p>ans: Proof</p> <ul style="list-style-type: none"> • verifies result for $n = 1$ (e.g.) • states correct assumption for $n = k$ • states correct result for $n = k + 1$ • continues proof correctly • concludes proof correctly <p style="text-align: right;">5 marks</p>	<ul style="list-style-type: none"> • $8^1 - 7(1) + 6 = 7$ • $8^k - 7k + 6 = 7m$ • $8^{k+1} - 7(k+1) + 6 = 7l$ • $8^{k+1} - 7(k+1) + 6 = \dots = 7(8m + 7k - 7)$ • \therefore Since T for $n = 1$ & (T for $n = k \Rightarrow$ T for $n = k+1$), the result is T $\forall n \in N$
7.	<p>$\frac{dy}{dx} = \frac{2y - 3x^2 y^2}{2x^3 y - 2x}$ ans: $3y = -8x - 14$</p> <p style="text-align: right;">6 marks</p> <ul style="list-style-type: none"> • uses implicit differentiation correctly • rearranges correctly • correct answer • correct y value • correct gradient • correct equation 	<ul style="list-style-type: none"> • $3x^2 y^2 + 2x^3 y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 0$ • $(2x^3 y - 2x) \frac{dy}{dx} = 2y - 3x^2 y^2$ • $\frac{dy}{dx} = \frac{2y - 3x^2 y^2}{2x^3 y - 2x}$ • $y = -2$ • $\frac{-8}{3}$ • $y + 2 = \frac{-8}{3}(x + 1)$
8(a)	<p>ans: 37</p> <ul style="list-style-type: none"> • correct expansion • correct answer <p style="text-align: right;">2 marks</p>	<ul style="list-style-type: none"> • $16 \times \left(\frac{3}{4}\right)^0 + 16 \times \left(\frac{3}{4}\right)^1 + 16 \times \left(\frac{3}{4}\right)^2$ • 37
8(b)	<p>ans: $-1 < \frac{3}{4} < 1$ 64</p> <ul style="list-style-type: none"> • correct explanation • uses correct formula • correct answer <p style="text-align: right;">3 marks</p>	<ul style="list-style-type: none"> • $-1 < \frac{3}{4} < 1$ • $S_n = \frac{16}{1 - \frac{3}{4}}$ • 64

	Give one mark for each •	Illustrations for awarding each mark
9(a)	ans: $I = \int_1^3 \frac{1}{2} ue^u du$ 3 marks <ul style="list-style-type: none"> • starts substitution correctly • continues substitution correctly • changes limits correctly 	<ul style="list-style-type: none"> • $\frac{du}{dx} = 2(4x+1)^{-\frac{1}{2}}$ • $\frac{u}{2} du = dx$ • $0 \rightarrow 1 \& 2 \rightarrow 3$
9(b)	ans: e^3 5 marks <ul style="list-style-type: none"> • uses integration by parts correctly • evaluates correctly • integrates correctly • evaluates correctly • correct answer 	<ul style="list-style-type: none"> • $\left[\frac{u}{2} e^u \right] - \int \frac{1}{2} e^u du$ • $\frac{3}{2} e^3 - \frac{1}{2} e$ • ... $-\left[\frac{1}{2} e^u \right]$ • ... $-\frac{1}{2} e^3 + \frac{1}{2} e$ • e^3
10(a)	ans: Neither 2 marks <ul style="list-style-type: none"> • correct formula for $g(-x)$ • correct conclusion 	<ul style="list-style-type: none"> • $g(-x) = e^{-2x} \sin(-2x)$ • Neither
10(b)	ans: $\left(\frac{3\pi}{8}, \frac{e^{\frac{3\pi}{4}}}{\sqrt{2}} \right)$ 5 marks <ul style="list-style-type: none"> • differentiates correctly • sets derivative equal to zero • solves correctly • solves correctly • correct y-value 	<ul style="list-style-type: none"> • $g'(x) = 2e^{2x} \sin 2x + 2e^{2x} \cos 2x$ • $2e^{2x} (\sin 2x + \cos 2x) = 0$ • $e^{2x} \neq 0$ • $x = \frac{3\pi}{8}$ • $y = \frac{e^{\frac{3\pi}{4}}}{\sqrt{2}}$
10(c)	ans: $g''(x) = 8e^{2x} \cos 2x$ 1 mark <ul style="list-style-type: none"> • correct answer 	<ul style="list-style-type: none"> • $g''(x) = 8e^{2x} \cos 2x$
10(d)	ans: Maximum turning point 2 marks <ul style="list-style-type: none"> • correct substitution • correct conclusion 	<ul style="list-style-type: none"> • $g''\left(\frac{3\pi}{8}\right) = \frac{8e^{\frac{3\pi}{4}}}{-\sqrt{2}}$ • Maximum turning point

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11(a)	ans: $z^3 = \cos 3\theta + i \sin 3\theta$ 1 mark <ul style="list-style-type: none"> • correct answer 	<ul style="list-style-type: none"> • $z^3 = \cos 3\theta + i \sin 3\theta$
11(b)	ans: $z^3 = \cos^3 \theta + 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta - i \sin^3 \theta$ 2 marks <ul style="list-style-type: none"> • expands correctly • simplifies correctly 	<ul style="list-style-type: none"> • $(\cos \theta)^3 + 3(\cos \theta)^2(i \sin \theta) + 3(\cos \theta)(i \sin \theta)^2 + (i \sin \theta)^3$ • $\cos^3 \theta + 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta - i \sin^3 \theta$
11(c) (i)	ans: $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ 2 marks <ul style="list-style-type: none"> • equates real parts correctly • correct expression 	<ul style="list-style-type: none"> • $\cos 3\theta = \cos^3 \theta - 3\cos \theta \sin^2 \theta$ • $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$
11(c) (ii)	ans: $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$ 2 marks <ul style="list-style-type: none"> • equates imaginary parts correctly • correct expression 	<ul style="list-style-type: none"> • $\sin 3\theta = 3\cos^2 \theta \sin \theta - \sin^3 \theta$ • $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$
11(d)	ans: Proof 4 marks <ul style="list-style-type: none"> • starts correctly • simplifies correctly • substitutes correctly • completes proof 	<ul style="list-style-type: none"> • $\frac{\cos 3\theta}{\sin 3\theta}$ • $\frac{4 - 3\sec^2 \theta}{3 \tan \theta \sec^2 \theta - 4 \tan^3 \theta}$ • $\sec^2 \theta = 1 + \tan^2 \theta$ • $\cot 3\theta = \frac{1 - 3 \tan^2 \theta}{3 \tan \theta - \tan^3 \theta}$

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12(a)	ans: $1 + \frac{9}{x-1}$ 2 marks <ul style="list-style-type: none"> • knows to use algebraic long division • divides correctly 	<ul style="list-style-type: none"> • $1 + \frac{9}{x-1}$
12(b)	ans: Proof 6 marks <ul style="list-style-type: none"> • knows to integrate • correct integral • correct limits • integrates correctly • substitutes correctly • completes proof 	<ul style="list-style-type: none"> • $A = \int \dots$ • $\int \left(12 - x - 1 - \frac{9}{x-1} \right) dx$ • $\int_2^{10} \dots$ • $\left[11x - \frac{x^2}{2} - 9 \ln x-1 \right]$ • $(110 - 50 - 9 \ln 9) - (22 - 2 - 9 \ln 1)$ • $40 - \ln 9^9$.

TOTAL MARKS = 77