ELGIN ACADEMY

Prelim Examination 2007 / 2008

MATHEMATICS
Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (*a*) Given $f(x) = 6\tan^{-1}\sqrt{x}$, where $x > 0$, obtain $f'(x)$ and simplify your answer. **3**

(*b*) Given $y = x^{x-2}$, where $x > 2$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x. **3**

2. $z_1 = 2i$ and $z_2 = 1 - i$.

(*a*) Express $\frac{z_1}{z_2}$ in the form $a + bi$ (where a and b are real numbers). **2**

(*b*) Find $\arg\left(\frac{z_1}{z_2}\right)$. **1**

3. Find the term independent of p in the expansion of $\left(3p^3 - \frac{2}{p}\right)^4$. **4**

4. Prove by induction that for all natural numbers n, $2^{3n} - 1$ is divisible by 7. **5**

5. (*a*) Show that the matrix $A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ is non-singular. **3**

(*b*) Use elementary row operations to find A^{-1}. **5**

6. Use integration by parts to evaluate

$$\int_0^1 2\tan^{-1} x \; dx.$$ **4**

7. A curve is defined by the parametric equations

$$x = t^2 - 2t\,, \quad y = 1 - t^4\,.$$

Find the equation of the tangent to the curve at the point where $t = -1$. **4**

8. Express the improper rational function $f(x) = \dfrac{x^3 + 3x^2 - 8x + 2}{x^2 - 2x + 1}$ in the form

$$f(x) = g(x) + h(x)\,,$$

where $g(x)$ is a polynomial function and $h(x)$ is a proper rational function expressed in partial fractions. **6**

9. By using the substitution $t = 1 + \tan x$, show that:

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1 + \tan x}\, dx$$ **5**

10. (*a*) Calculate the sum of all the two digit natural numbers which are divisible by 3. **4**

(*b*) Find the value of θ, $0 < \theta < \dfrac{\pi}{2}$, such that:

$$1 + \sin^2\theta + \sin^4\theta + \sin^6\theta + \ldots = 2\,.$$ **5**

11. A scientist constructs the differential equation

$$\frac{dy}{dx} = e^{x+y}$$

to describe the relationship between two quantities x and y.

(*a*) Find the general solution of the differential equation. **4**

(*b*) Given that $y = 0$ when $x = 1$, find the particular solution, expressing y in terms of x. **2**

12. The function f is defined by $f(x) = \dfrac{x^2 + 3}{x + 1}, x \neq -1, x \in R$.

(a) (i) Write down the equation of the vertical asymptote of f. **1**

(ii) Show that f has a non-vertical asymptote and obtain its equation. **2**

(iii) Find the point(s) of intersection with the x- and y- axes. **2**

(b) Find the coordinates and nature of the stationary points of f. **5**

(c) Sketch the graph of $y = f(x)$, indicating the features found in (a) and (b). **3**

[*END OF QUESTION PAPER*]