

**Marking Scheme - Advanced Higher Grade 2007/2008 Prelim**

	<b>Give one mark for each •</b>	<b>Illustrations for awarding each mark</b>
1(a)	<p><b>ans:</b> <math>f'(x) = \frac{3}{\sqrt{x}(1+x)}</math></p> <p align="right"><b>3 marks</b></p> <ul style="list-style-type: none"> <li>• differentiates <math>\tan^{-1}</math> correctly</li> <li>• applies chain rule correctly</li> <li>• Simplifies correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>6 \cdot \frac{1}{1+(\sqrt{x})^2}</math></li> <li>• <math>\frac{1}{2} x^{-\frac{1}{2}}</math></li> <li>• <math>\frac{3}{\sqrt{x}(1+x)}</math></li> </ul>
1(b)	<p><b>ans:</b> <math>\frac{dy}{dx} = x^{x-2} \left( \ln x + 1 - \frac{2}{x} \right)</math></p> <p align="right"><b>3 marks</b></p> <ul style="list-style-type: none"> <li>• takes logs of both sides &amp; simplifies</li> <li>• differentiates correctly</li> <li>• expresses <math>\frac{dy}{dx}</math> in terms of <math>x</math> correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\ln y = (x-2)\ln x</math></li> <li>• <math>\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + (x-2) \cdot \frac{1}{x}</math></li> <li>• <math>\frac{dy}{dx} = x^{x-2} \left( \ln x + 1 - \frac{2}{x} \right)</math></li> </ul>
2(a)	<p><b>ans:</b> <math>\frac{z_1}{z_2} = -1 + i</math></p> <p align="right"><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• knows how to find fraction</li> <li>• correct simplification</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{2i}{1-i} \cdot \frac{1+i}{1+i}</math></li> <li>• <math>-1 + i</math></li> </ul>
2(b)	<p><b>ans:</b> <math>\frac{3\pi}{4}</math></p> <p align="right"><b>1 mark</b></p> <ul style="list-style-type: none"> <li>• finds correct principal argument</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{3\pi}{4}</math></li> </ul>
3	<p><b>ans:</b> -96</p> <p align="right"><b>4 marks</b></p> <ul style="list-style-type: none"> <li>• applies binomial theorem correctly</li> <li>• simplifies expression correctly</li> <li>• finds correct value of <math>r</math></li> <li>• finds correct term</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\sum_{r=0}^4 \binom{4}{r} (3p^3)^{4-r} \left( \frac{-2}{p} \right)^r</math></li> <li>• <math>\sum_{r=0}^4 \binom{4}{r} 3^{4-r} (-2)^r p^{12-4r}</math></li> <li>• <math>r = 3</math></li> <li>• -96</li> </ul>

	Give one mark for each •	Illustrations for awarding each mark
4	<b>ans: Proof</b> <span style="float: right;"><b>5 marks</b></span> <ul style="list-style-type: none"> <li>Proves result for <math>n = 1</math></li> <li>States correct result for <math>n = k</math></li> <li>States correct result for <math>n = k + 1</math></li> <li>Continues proof correctly</li> <li>Completes proof correctly</li> </ul>	<ul style="list-style-type: none"> <li><math>2^3 - 1 = 7</math></li> <li><math>2^{3k} - 1 = 7m</math></li> <li><math>2^{3(k+1)}</math> is divisible by 7</li> <li><math>2^{3(k+1)} = \dots = 7(8m + 1)</math></li> <li>Since result is T for <math>n = 1</math> &amp; (T for <math>n = k \Rightarrow</math> T for <math>n = k + 1</math>, the result is T <math>\forall n \in N</math>.</li> </ul>
5(a)	<b>ans: <math>\det A = 3 \neq 0</math></b> <span style="float: right;"><b>3 marks</b></span> <ul style="list-style-type: none"> <li>knows how to find the determinant of a <math>3 \times 3</math> matrix</li> <li>finds determinant correctly</li> <li>correct explanation</li> </ul>	<ul style="list-style-type: none"> <li><math>\det A = \begin{vmatrix} 2 &amp; 1 &amp; 4 \\ 1 &amp; 0 &amp; 2 \\ 2 &amp; 3 &amp; 1 \end{vmatrix}</math></li> <li>3</li> <li>Since <math> A  \neq 0</math>, <math>A</math> is non-singular</li> </ul>
5(b)	<b>ans: <math>A^{-1} = \frac{1}{3} \begin{pmatrix} -6 &amp; 11 &amp; 2 \\ 3 &amp; -6 &amp; 0 \\ 3 &amp; -4 &amp; -1 \end{pmatrix}</math></b> <span style="float: right;"><b>5 marks</b></span> <ul style="list-style-type: none"> <li>correct augmented matrix</li> <li>one correct row</li> <li>a second correct row</li> <li>the third row correct</li> <li>identifies <math>A^{-1}</math></li> </ul>	<ul style="list-style-type: none"> <li><math>\begin{pmatrix} 2 &amp; 1 &amp; 41 &amp; 0 &amp; 0 \\ 1 &amp; 0 &amp; 20 &amp; 1 &amp; 0 \\ 2 &amp; 3 &amp; 10 &amp; 0 &amp; 1 \end{pmatrix}</math></li> <li><math>\begin{pmatrix} 0 &amp; 1 &amp; 01 &amp; -2 &amp; 0 \end{pmatrix}</math></li> <li><math>\begin{pmatrix} 0 &amp; 0 &amp; 11 &amp; \frac{-4}{3} &amp; \frac{-1}{3} \end{pmatrix}</math></li> <li><math>\begin{pmatrix} 1 &amp; 0 &amp; 0-2 &amp; \frac{11}{3} &amp; \frac{2}{3} \end{pmatrix}</math></li> <li><math>A^{-1} = \begin{pmatrix} -2 &amp; \frac{11}{3} &amp; \frac{2}{3} \\ 1 &amp; -2 &amp; 0 \\ 1 &amp; \frac{-4}{3} &amp; \frac{-1}{3} \end{pmatrix}</math></li> </ul>

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6	<b>ans:</b> $\frac{\pi}{2} - \ln 2$ <b>4 marks</b> <ul style="list-style-type: none"> <li>• applies integration by parts formula correctly</li> <li>• evaluates correctly</li> <li>• integrates correctly</li> <li>• evaluates correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\left[2x \tan^{-1} x\right]_0^1 - \int_0^1 \frac{2x}{1+x^2} dx</math></li> <li>• <math>\frac{\pi}{2}</math></li> <li>• <math>\left[\ln 1+x^2 \right]_0^1</math></li> <li>• <math>\ln 2</math></li> </ul>
7	<b>ans:</b> $y = -x + 3$ <b>4 marks</b> <ul style="list-style-type: none"> <li>• applies correct formula</li> <li>• finds correct gradient</li> <li>• finds correct point</li> <li>• finds correct equation</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{dy}{dx} = \frac{y'}{x'} = \frac{-4t^3}{2t-2}</math></li> <li>• <math>m = -1</math></li> <li>• <math>(3, 0)</math></li> <li>• <math>y - 0 = -1(x - 3)</math></li> </ul>
8	<b>ans:</b> $f(x) = x + 5 + \frac{1}{x-1} - \frac{2}{(x-1)^2}$ <b>6 marks</b> <ul style="list-style-type: none"> <li>• applies algebraic long division</li> <li>• applies correctly</li> <li>• restates function in correct form</li> <li>• correct form for partial fractions</li> <li>• correct value for <math>A</math></li> <li>• correct value for <math>B</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>Q = x + 5, R = x - 3</math></li> <li>• <math>x + 5 + \frac{x-3}{x^2 - 2x + 1}</math></li> <li>• <math>\frac{A}{x-1} + \frac{B}{(x-1)^2}</math></li> <li>• 1</li> <li>• -2</li> </ul>

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9	<b>ans:</b> Proof <span style="float: right;"><b>5 marks</b></span> <ul style="list-style-type: none"> <li>• differentiate correctly</li> <li>• Change limits correctly</li> <li>• substitute correctly</li> <li>• Integrate correctly</li> <li>• Evaluate correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>dt = \sec^2 x dx</math></li> <li>• <math>0 \rightarrow 1, \frac{\pi}{4} \rightarrow 2</math></li> <li>• <math>\int_1^2 \frac{dt}{t}</math></li> <li>• <math>[\ln t ]_1^2</math></li> <li>• <math>\ln 2</math></li> </ul>
10(a)	<b>ans:</b> 1665 <span style="float: right;"><b>4 marks</b></span> <ul style="list-style-type: none"> <li>• identifies correct series</li> <li>• applies formula correctly</li> <li>• finds correct number of terms</li> <li>• evaluates sum correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>12 + 15 + 18 + \dots + 99</math></li> <li>• <math>S_n = \frac{n}{2}(2(12) + 3(n-1))</math></li> <li>• <math>n = 30</math></li> <li>• 1665</li> </ul>
10(b)	<b>ans:</b> $\theta = \frac{\pi}{4}$ <span style="float: right;"><b>5 marks</b></span> <ul style="list-style-type: none"> <li>• Identifies sum to infinity of geometric series</li> <li>• Applies formula correctly</li> <li>• Correct substitution (or equivalent)</li> <li>• Correct workings</li> <li>• Solves correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>r = \sin^2 \theta</math></li> <li>• <math>2 = \frac{1}{1 - \sin^2 \theta}</math></li> <li>• <math>2 = \frac{1}{\cos^2 \theta}</math></li> <li>• <math>\cos \theta = \frac{1}{\sqrt{2}}</math> or <math>\frac{-1}{\sqrt{2}}</math></li> <li>• <math>\theta = \frac{\pi}{4}</math></li> </ul>

	Give one mark for each •	Illustrations for awarding each mark
11 (a)	<b>ans:</b> $y = \ln \left  \frac{-1}{e^x + C} \right $ <b>4 marks</b> <ul style="list-style-type: none"> <li>• Rewrites RHS</li> <li>• Separates variables &amp; knows to integrate</li> <li>• Integrates correctly</li> <li>• Solves for y [<b>Note : this mark may be awarded within a candidates response to part (b)</b>]</li> </ul>	<ul style="list-style-type: none"> <li>• <math>e^x e^y</math></li> <li>• <math>\int e^{-y} dy = \int e^x dx</math></li> <li>• <math>-e^{-y} = e^x + C</math></li> <li>• <math>y = \ln \left( \frac{-1}{e^x + C} \right)</math></li> </ul>
11 (b)	<b>ans:</b> $y = \ln \left  \frac{-1}{e^x - 1 - e} \right $ <b>2 marks</b> <ul style="list-style-type: none"> <li>• Substitutes correctly</li> <li>• Correct solution</li> </ul>	<ul style="list-style-type: none"> <li>• <math>0 = \ln \left( \frac{-1}{e + C} \right)</math></li> <li>• <math>C = -1 - e</math></li> </ul>
12 (a) (i)	<b>Ans:</b> $x = -1$ <b>1 mark</b> <ul style="list-style-type: none"> <li>• correct equation</li> </ul>	<ul style="list-style-type: none"> <li>• <math>x = -1</math></li> </ul>
12 (a) (ii)	<b>Ans:</b> $y = x - 1$ <b>2 marks</b> <ul style="list-style-type: none"> <li>• correct division</li> <li>• correct equation</li> </ul>	<ul style="list-style-type: none"> <li>• <math>f(x) = x - 1 + \frac{4}{x + 1}</math></li> <li>• <math>y = x - 1</math></li> </ul>
12 (a) (iii)	<b>Ans:</b> (0, 3) <b>2 marks</b> <ul style="list-style-type: none"> <li>• Doesn't cross x-axis</li> <li>• Correct point</li> </ul>	<ul style="list-style-type: none"> <li>• <math>x^2 \neq -3</math></li> <li>• (0, 3)</li> </ul>
12 (b)	<b>Ans:</b> (1, 2) is a Min. T.P. & (-3, -6) is a Max. T.P. <b>5 marks</b> <ul style="list-style-type: none"> <li>• Differentiates correctly</li> <li>• Solves <math>f'(x) = 0</math> correctly</li> <li>• Correct y – coordinates</li> <li>• Finds second derivative correctly (or correct nature table)</li> <li>• Correct nature of both points</li> </ul>	<ul style="list-style-type: none"> <li>• <math>1 - 4(x + 1)^{-2}</math></li> <li>• 1 &amp; -3</li> <li>• 2 &amp; -6</li> <li>• <math>\frac{8}{(x + 1)^3}</math></li> <li>• (1,2) <math>\rightarrow</math> Min.T.P. &amp; (-3,-6) <math>\rightarrow</math> Max.T.P.</li> </ul>
12 (c)	<b>Ans:</b> Correct Sketch (see diagram below) <b>3 marks</b> <ul style="list-style-type: none"> <li>• both T.P.s indicated</li> <li>• both asymptotes shown</li> <li>• y-intercept shown &amp; correct shape</li> </ul>	<ul style="list-style-type: none"> <li>•</li> <li>•</li> <li>•</li> </ul>

**TOTAL MARKS = 73**

Q12 (c)

