ELGIN ACADEMY

Prelim Examination 2006 / 2007
(Assessing Units 1 & 2)

MATHEMATICS

Advanced Higher Grade

Time allowed - 2 hours

Read Carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (*a*) Given $f(x) = e^{-2x}\tan 4x$, $0 < x < \frac{\pi}{8}$, obtain $f'(x)$. **3**

(*b*) For $y = \frac{\ln 5x}{x-1}$, where $x > 1$, determine $\frac{dy}{dx}$ in its simplest form. **3**

2. For what value of t does the system of equations

$$\begin{aligned} x + 2y - 3z &= -7 \\ 4x - y + 2z &= 9 \\ 3x - 2y + tz &= 13 \end{aligned}$$

have no solution? **5**

3. Verify that $1 - 3i$ is a solution of $z^4 - 4z^3 + 11z^2 - 14z - 30 = 0$.

Hence express $z^4 - 4z^3 + 11z^2 - 14z - 30$ in the form $(z + a)(z + b)(z^2 + cz + d)$, where a, b, c and d are real numbers. **5**

4. Use the substitution $x = 3\cos\theta$ to show that

$$\int_{\frac{3}{2}}^{3} \frac{dx}{\sqrt{9 - x^2}} = \frac{\pi}{3}$$ **6**

5. Obtain the binomial expansion of $\left(3a^2 - \frac{4}{b}\right)^5$. **3**

6. Use integration by parts to evaluate $\int_0^1 x^2 e^{-x}\, dx$. **5**

7. Determine whether the function $f(x) = x^2\cos x + x^3$ is odd, even or neither.

Justify your answer. **3**

8. A spherical balloon is being inflated.

Its volume, V cm^3, is increasing at the rate of $\frac{30\pi}{7}$ cm^3 per second.

Find the rate at which the radius is increasing with respect to time when the volume is $\frac{36\pi}{5}$ cm^3.

[**Note**: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.] **5**

9. Prove that if n is odd then $n^4 - 1$ is divisible by 8. **3**

10. (*a*) Obtain partial fractions for

$$\frac{9}{x^2 - 9}$$ **2**

(*b*) Hence evaluate

$$\int_0^1 \frac{x^2}{x^2 - 9}\, dx.$$ **4**

11. The function f is defined by

$$f(x) = \frac{x^2}{x+3} \quad , x \neq -3.$$

(*a*) Obtain algebraically the asymptotes of the graph of f. **3**

(*b*) Find the stationary points of f and justify their nature. **5**

(*c*) Sketch the curve showing clearly the features found in (*a*) and (*b*). **2**

(*d*) Write down the coordinates of the stationary points of the graph of $g(x) = 10 + |f(x)|$. **2**

12. The first two terms of a series are $1 + \sqrt{2}$ and $1 + \frac{1}{\sqrt{2}}$.

(*a*) If the series is arithmetic, show that the common difference is $-\frac{1}{2}\sqrt{2}$.

Show also that the sum of the first ten terms is $\frac{5}{2}\left(4 - 5\sqrt{2}\right)$. **4**

(*b*) If the series is geometric, show that the sum to infinity exists.

Show also that $S_\infty = 4 + 3\sqrt{2}$. **5**

13. A solid is formed by rotating the curve $y = x^2 + 4$ between $x = 1$ and $x = t$, $t > 1$, through 360^0 about the y – axis.

Find the value of t given that the volume of the solid formed is 40π units^3. **6**

[*END OF QUESTION PAPER*]