Prelim Examination 2002 / 2003
(Assessing Unit 3)

MATHEMATICS

Advanced Higher Grade

Time allowed - 1 hour

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used in this paper.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (*a*) Evaluate the product of the quadratic form

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} 1 & 6 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

(2)

(*b*) Find the general result of the quadratic form

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

(2)

(*c*) Find the matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ given that

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 3x^2 - 3xy + 4y^2 \text{ and } \begin{pmatrix} p & q \\ r & s \end{pmatrix}^2 = \begin{pmatrix} -1 & 14 \\ -35 & 6 \end{pmatrix}.$$

(4)

2. Prove by induction that $2^{3n-1} + 3$ is divisible by 7 for all positive integers *n*. (5)

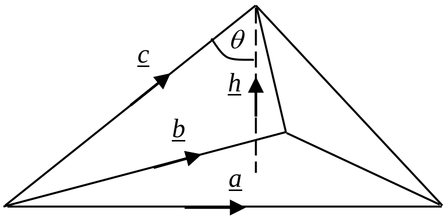
3. (*a*) Find the first five non-zero terms of the Maclaurin series for $\ln(1+x)$. (5)

(*b*) Deduce the Maclaurin series for $\ln(1-2x)$. (2)

(*c*) Hence find the first five terms of the Maclaurin series for $\ln(1-x-2x^2)$. (3)

4. (*a*) Prove that the volume of a tetrahedron is given by the formula

Volume = $\frac{1}{6}\left|\underline{a} \times \underline{b} \cdot \underline{c}\right|$ (4)



(*b*) Find the volume of tetrahedron OABC where O is the origin and A, B and C are the points (3, 2, 4), (4, 3, 5) and (0, 5, 3). (4)

5. (*a*) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^{-2x}.$$

(6)

(*b*) Hence determine the solution which satisfies the conditions $y(0) = 1,\ y'(0) = 3$. (4)

End of Question Paper