

Marking Scheme – Advanced Higher Prelim – Maths 3

	Give one mark for each •	Illustrations for awarding each mark
1(a)	ans: $x^2 + 3xy + 2y^2$ 2 marks <ul style="list-style-type: none"> • multiplies first 2 matrices together correctly • multiplies final 2 correctly 	<ul style="list-style-type: none"> • $(x \ y) \begin{pmatrix} 1 & 6 \\ -3 & 2 \end{pmatrix} = (x - 3y \quad 6x + 2y)$ • $(x - 3y \quad 6x + 2y) \begin{pmatrix} x \\ y \end{pmatrix} = x^2 + 3xy + 2y^2$
1(b)	ans: $ax^2 + (b + c)xy + dy^2$ 2 marks <ul style="list-style-type: none"> • multiplies first 2 matrices together correctly • multiplies final 2 correctly 	<ul style="list-style-type: none"> • $(x \ y) \begin{pmatrix} a & b \\ c & d \end{pmatrix} = (ax + cy \quad bx + dy)$ • $(ax + cy \quad bx + dy) \begin{pmatrix} x \\ y \end{pmatrix} = ax^2 + bxy + cxy + dy^2$
1(c)	ans: $\begin{pmatrix} 3 & 2 \\ -5 & 4 \end{pmatrix}$ 4 marks <ul style="list-style-type: none"> • states p and s • squares matrix • equates coefficients • finds q and r 	<ul style="list-style-type: none"> • $p = 3, s = 4$ • $\begin{pmatrix} 3 & q \\ r & 4 \end{pmatrix} \begin{pmatrix} 3 & q \\ r & 4 \end{pmatrix} = \begin{pmatrix} 9 + qr & 7q \\ 7r & rq + 16 \end{pmatrix}$ • $7q = 14, 7r = -35$ • $q = 2, r = -5$
2.	ans: Proof 5 marks <ul style="list-style-type: none"> • show true for $n = 1$ • state inductive hypothesis • consider the case for $n = k + 1$ • carry out manipulation • state conclusion 	<ul style="list-style-type: none"> • $2^{3(1)-1} + 3 = 2^2 + 3 = 7$ which is divisible by 7 so true for $n = 1$ • Assume $2^{3k-1} + 3 = 7m$ for some $m \in \mathbf{N}$ • Consider $2^{3(k+1)-1} + 3 = 2^{3k+2} + 3$ • $2^{3k+2} + 3 = \dots = 8(2^{3k-1} + 3) - 21$ • $= 8(7m) - 21 = 7(8m - 3)$ which is divisible by 7 • So, if the formula is valid for n, it is valid for $n + 1$. Since it is valid for $n = 1$, it is therefore true for all $n \geq 1$.

Marking Scheme – Advanced Higher Prelim – Maths 3 cont.

	Give one mark for each •	Illustrations for awarding each mark
3(a)	<p>ans: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$ 5 marks</p> <ul style="list-style-type: none"> • finds values for $f(0), f'(0)$ • finds value for $f''(0)$ • finds values for higher order derivatives • method • final statement of terms 	<ul style="list-style-type: none"> • $f(0) = 0, f'(0) = 1$ • $f''(0) = -1$ • 2, -6, -24 • $f(x) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \dots$ • $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$
3(b)	<p>ans: $-2x - 2x^2 - \frac{8}{3}x^3 - 4x^4 - \frac{32}{5}x^5$ 2 marks</p> <ul style="list-style-type: none"> • knows to replace x in (a) with $(-2x)$ • all calculations correct 	<ul style="list-style-type: none"> • $-2x - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4} + \frac{(-2x)^5}{5}$ • $-2x - 2x^2 - \frac{8}{3}x^3 - 4x^4 - \frac{32}{5}x^5$
3(c)	<p>ans: $-x - \frac{5}{2}x^2 - \frac{7}{3}x^3 - \frac{17}{4}x^4 - \frac{31}{5}x^5$ 3 marks</p> <ul style="list-style-type: none"> • recognises connection between series • knows how to combine series • combines series correctly 	<ul style="list-style-type: none"> • $1 - x - 2x^2 = (1+x)(1-2x)$ • $\ln(1-x-2x^2) = \ln(1+x) + \ln(1-2x)$ • $-x - \frac{5}{2}x^2 - \frac{7}{3}x^3 - \frac{17}{4}x^4 - \frac{31}{5}x^5$
4(a)	<p>ans: Proof 4 marks</p> <ul style="list-style-type: none"> • knows how to calculate volume • calculates area of base correctly • calculates height correctly • proves formula 	<ul style="list-style-type: none"> • Volume = $\frac{1}{3}$ area of base \times height • $\frac{1}{2} \underline{a} \times \underline{b}$ • $\underline{c} \cos \theta$ • $\frac{1}{6} \underline{a} \times \underline{b} \cdot \underline{c}$
4(b)	<p>ans: $1\frac{1}{3}$ units³ 4 marks</p> <ul style="list-style-type: none"> • knows how to calculate vector product • calculates vector product correctly • calculates scalar product correctly • answer 	<ul style="list-style-type: none"> • $\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 4 \\ 4 & 3 & 5 \end{vmatrix}$ • $-2\underline{i} + \underline{j} + \underline{k}$ • $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} = 8$ • Volume = $\frac{1}{6} \times 8 = 1\frac{1}{3}$ units³

Marking Scheme – Advanced Higher Prelim – Maths 3 cont.

	Give one mark for each •	Illustrations for awarding each mark
5(a)	<p>ans: $y = Ae^{-2x} + Be^{-x} - 2xe^{-2x}$ 6 marks</p> <ul style="list-style-type: none"> • solves auxiliary equation • finds complimentary function • states correct form of Particular Integral • calculates 1st and 2nd derivatives of PI • calculates value of C • states general solution 	<ul style="list-style-type: none"> • $m^2 + 3m + 2 = 0 \Rightarrow m = -2, m = -1$ • $y = Ae^{-2x} + Be^{-x}$ • $y = Cxe^{-2x}$ • • $\frac{dy}{dx} = Ce^{-2x} - 2Cxe^{-2x}, \frac{d^2y}{dx^2} = -4Ce^{-2x} + 4Cxe^{-2x}$ • $C = -2$ • $y = Ae^{-2x} + Be^{-x} - 2xe^{-2x}$
5(b)	<p>ans: $y = 7e^{-x} - 2e^{-2x}(3+x)$ 4 marks</p> <ul style="list-style-type: none"> • calculates derivative of general solution • substitutes initial conditions into equations • solves equations simultaneously • states solution 	<ul style="list-style-type: none"> • $y' = -2Ae^{-2x} - Be^{-x} - 2e^{-2x} + 4xe^{-2x}$ • $1 = A + B, 5 = -2A - B$ • $A = -6, B = 7$ • $y = 7e^{-x} - 6e^{-2x} - 2xe^{-2x}$

Total 41 marks