Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2

	Give one mark for each •	**Illustrations for awarding each mark**
1.	**ans:** $\pi/6$ **5 marks**	
	• knowing to complete the square	• $\int_{-1}^{0} \frac{dx}{\sqrt{4-(x+1)^2}}$
	• knowing to use a suitable substitution	• $u = x+1,\ du = dx,$ $x=-1: u=0$
	• finds new limits	• $x=0: u=1$
	• integrates correctly	• $\sin^{-1}\frac{u}{2}\Big]_0^1$
	• answer	• $\pi/6$
2.	**ans:** $2i, -2i, 2+3i, 2-3i$ **5 marks**	
	• substitutes $2i$ correctly into equation and proves result	• $(2i)^4 - 4(2i)^3 + 17(2i)^2 - 16(2i) + 52$ $= 16 + 32i - 68 - 32i + 52 = 0$
	• states complex conjugate is a solution	• $-2i$ is a solution
	• finds first quadratic factor	• $(z-2i)(z+2i) = z^2+4$
	• finds second quadratic factor by dividing original equation by first quadratic factor	• $(z^4 - 4z^3 + 17z^2 - 16z + 52) \div (z^2+4)$ $= z^2 - 4z + 13$
	• uses quadratic formula to find other 2 solutions	• $2+3i, 2-3i$
3.	**ans:** $x = -3, y = -2, z = 4$ **5 marks**	
	• method	• $\begin{bmatrix} 1 & -1 & 2 & 7 \\ 3 & 2 & 1 & -9 \\ 2 & 1 & -3 & -20 \end{bmatrix}$
	• first modified system	• $\begin{bmatrix} 1 & -1 & 2 & 7 \\ 0 & 5 & -5 & -30 \\ 0 & 3 & -7 & -34 \end{bmatrix}$
	• second modified system	• $\begin{bmatrix} 1 & -1 & 2 & 7 \\ 0 & 5 & -5 & -30 \\ 0 & 0 & -4 & -16 \end{bmatrix}$
	• method of back-substitution	• $-4z = -16 \Rightarrow z = 4$
	• values of x, y and z	• $y = -2,\ x = -3$

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4(a)	**ans:** $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ **2 marks** • all powers in expansion • all coefficients in expansion	• and • $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
4(b)	**ans:** $16x^2 + \frac{64}{x^2}$ **5 marks** • correctly substituting x and $\frac{2}{x}$ in place of a & b • correct expression • substitutes x and $-\frac{2}{x}$ in place of a and b • correct expression • answer	• $x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$ • $x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ • $x^4 + 4x^3\left(-\frac{2}{x}\right) + 6x^2\left(-\frac{2}{x}\right)^2 + 4x\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$ • $x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$ • $16x^2 + \frac{64}{x^2}$
5(a)	**ans:** $-\frac{2}{(x-1)^2}\, e^{\frac{x+1}{x-1}}$ **4 marks** • knowing how to differentiate e • chain rule factor • using quotient rule correctly • answer	• $e^{\frac{x+1}{x-1}}$ • $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$ • $\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = -\frac{2}{(x-1)^2}$ • $-\frac{2}{(x-1)^2}\, e^{\frac{x+1}{x-1}}$
5(b)	**ans:** $y = x$ **4 marks** • differentiating correctly • finding expression for $\frac{dy}{dx}$ • finding gradient • finding equation of tangent	• $4x - 3y - 3x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = \frac{3y - 4x}{2y - 3x}$ • $m = 1$ • $y - 1 = 1(x - 1)$

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6.	**ans:** $a = 20$, $b = 2$ **4 marks** • finding complex conjugate • finding $\frac{z}{\bar{z}}$ • equating real and imaginary parts • Solving simultaneous equations	• $\bar{z} = 3 + i$ • $\frac{z}{\bar{z}} = \frac{4}{5} - \frac{3}{5}i$ • $\frac{4}{5}a + 3b = 22, -\frac{3}{5}a - b = -14$ • $a = 20, b = 2$
7.	**ans:** $k = 3$ **4 marks** • Finding general term • knowing to put $r = 2$ • substituting $r = 2$ correctly • finding value of k	• $\binom{6}{r}k^{6-r}(-4x)^r$ • $r = 2$ • $\binom{6}{2}k^4(-4)^2 = 19440 \Rightarrow 240k^4 = 19440$ • $k = 3$
8.	**ans:** $2x - 14y + 3 = 0$ **5 marks** • changing the subject of x to t • changing the subject of y to t • knowing to equate expressions • finding equation of line • proving point lies on line	• $t = \frac{3 + 2x}{1 - 4x}$ • $t = \frac{2y}{1 - 4y}$ • $\frac{3 + 2x}{1 - 4x} = \frac{2y}{1 - 4y}$ • $2x - 14y + 3 = 0$ • $2\left(\frac{11}{2}\right) - 14(1) + 3 = 11 - 14 + 3 = 0$ as required
9(a)	**ans:** $8n$, $4n(n + 1)$ **4 marks** • knowing how to find general term • finding general term • knowing how to find sum to n terms • finding sum to n terms	• $u_n = a + (n-1)d$ • $u_n = 8 + (n-1)8 = 8n$ • $S_n = \frac{n}{2}[2a + (n-1)d]$ • $S_n = \frac{n}{2}[16 + (n-1)8] = 4n(n+1)$
9(b)	**ans:** 62000 **3 marks** • knows how to find n • finds n correctly • finds sum of terms	• $8n < 1000 \Rightarrow n < 125$ • $n = 124$ • $S_{124} = 4 \times 124\ (124 + 1) = 62000$

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10.	**ans:** $-8i$, $8i$ **6 marks** • finds modulus • finds argument • writes complex number in polar form • deals with power correctly in polar form • finds first solution • finds second solution	• $\sqrt{2^2+\left(-2\sqrt{3}\right)^2}=4$ • $\tan\theta=\frac{-2\sqrt{3}}{2}=-\sqrt{3}\Rightarrow\theta=-\frac{\pi}{3}$ • $4\left(\cos\left(-\frac{\pi}{3}+2k\pi\right)+i\sin\left(-\frac{\pi}{3}+2k\pi\right)\right)$ • $4^{3/2},\ \frac{3}{2}\left(-\frac{\pi}{3}+2k\pi\right)$ • $k=0\Rightarrow -8i$ • $k=0\Rightarrow 8i$
11.	**ans:** $\frac{7}{30}$ **3 marks** • writes 0·2333333…. As a geometric series • finds sum to infinity • answer	• $\frac{2}{10}+\left(\frac{3}{100}+\frac{3}{1000}+\frac{3}{10000}+...\right)$ • $S_\infty=\frac{a}{1-r}=\frac{\frac{3}{100}}{1-\frac{1}{10}}=\frac{1}{30}$ • $\frac{2}{10}+\frac{1}{30}=\frac{7}{30}$
12.	**ans:** 1, i, -1, $-i$ **5 marks** • writes 1 in polar form • takes 4th root correctly • knows values of k to use • finds roots • proves roots sum to zero	• $1=\cos 2k\pi+i\sin 2k\pi$ • $z=\cos\frac{2k\pi}{4}+i\sin\frac{2k\pi}{4}$, • $k=0,\ 1,\ 2,\ 3$ • $z=1,\ i,\ -1,\ -i$ • $1+i+(-1)+(-i)=0$ as required
13(a)	**ans:** Proof **5 marks** • knowing to use integration by parts • correct application of integration by parts • knows to use trigonometric identity • combining terms • answer	• $\int\sin x\sin^{n-1}x\,dx$ • $-\cos x\sin^{n-1}x-\int(n-1)\cos x\sin^{n-2}x(-\cos x)dx$ $=-\cos x\sin^{n-1}x+(n-1)\int\cos^2 x\sin^{n-2}x\,dx$ • $\cos^2 x=1-\sin^2 x$ • $[1+(n-1)]\int\sin^n x\,dx=-\cos x\sin^{n-1}x+(n-1)\int\sin^{n-2}x\,dx$ • $\int\sin^n x\,dx=-\frac{1}{n}\cos x\sin^{n-1}x+\frac{n-1}{n}I_{n-2}$

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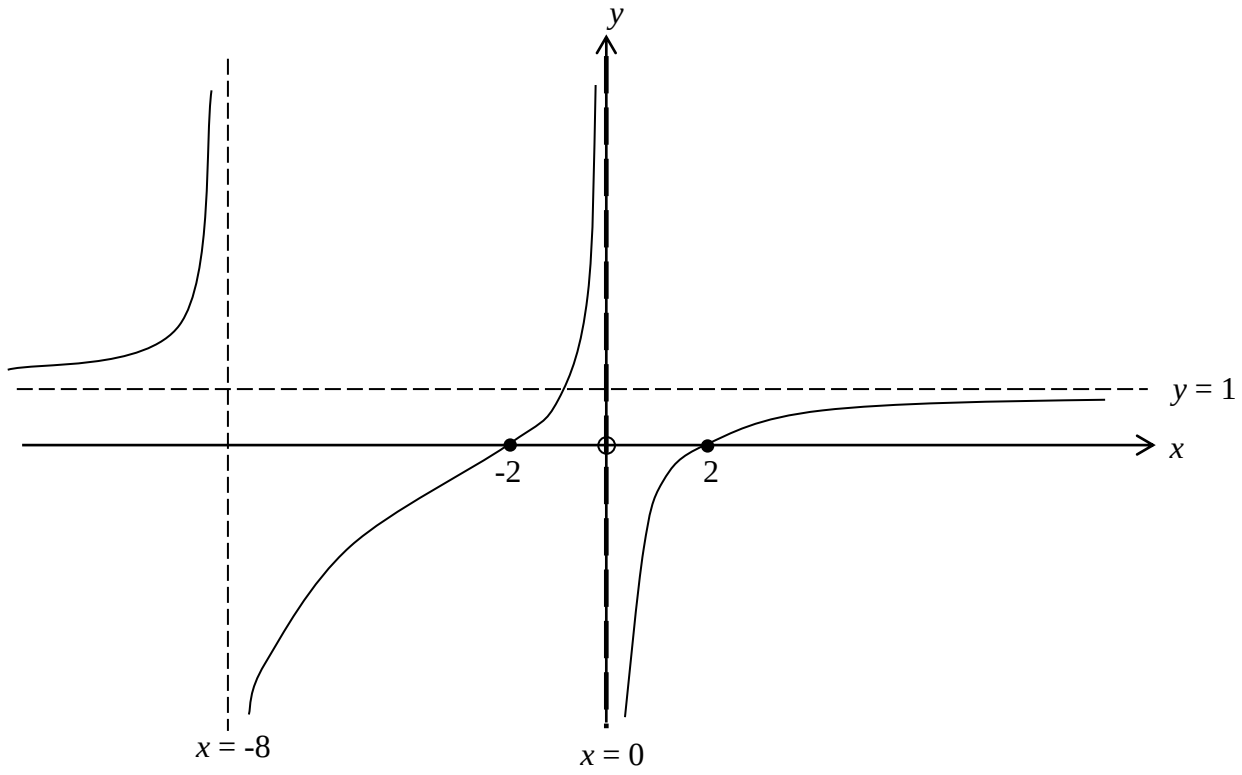
13(b)	**ans:** $\frac{n-1}{n}I_{n-2}$ **2 marks** • applying limits to reduction formula • answer	• $\left(-\frac{1}{n}\cos\frac{\pi}{2}\sin^{n-1}\frac{\pi}{2}\right)-\left(-\frac{1}{n}\cos 0\sin^{n-1}0\right)+\frac{n-1}{n}\int_0^{\pi/2}\sin^{n-2}x\,dx$ • $\frac{n-1}{n}I_{n-2}$
13(c)	**ans:** $\frac{35\pi}{256}$ **3 marks** • knowing to use reduction formula repeatedly • correctly integrating $\sin^2 x$ • answer	• $\int_0^{\pi/2}\sin^8 x\,dx=\frac{8-1}{8}\int_0^{\pi/2}\sin^6 x\,dx=\frac{7}{8}I_6$ etc. • $\sin^2 x=\frac{1}{2}(1-\cos 2x)$ • $\frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$
14(a)	**ans:** $h=\frac{40}{r}-\frac{3r}{2}$ **2 marks** • finds surface area correctly • finds h in terms of r	• cylinder = $2\pi rh+\pi r^2$, hemisphere = $2\pi r^2$ $\Rightarrow\ 80\pi=2\pi rh+3\pi r^2$ • $h=\frac{40}{r}-\frac{3r}{2}$
14(b)	**ans:** 4 cm, 4 cm, 80π cm^3 **8 marks** • finds correct expression for volume • substitutes expression for h • differentiates volume • puts derivative = 0 • finds value for r • checks nature is maximum • finds value for h • finds value for V	• $V=\pi r^2h+\frac{2}{3}\pi r^3$ • $V=40\pi r-\frac{5}{6}\pi r^3$ • $40\pi-\frac{5}{2}\pi r^2$ • $40\pi-\frac{5}{2}\pi r^2=0$ • r = 4 cm • nature table • h = 4 cm • $V=\frac{320\pi}{3}$ cm^3
15(a)	**ans:** x = 0, x = -8, y = 1 **2 marks** • finds equations of vertical asymptotes • finds equation of horizontal asymptote	• $x^2+8x=0\Rightarrow x=0,\ x=-8$ • $f(x)=1-\frac{8x+4}{x^2+8x}\Rightarrow y=1$

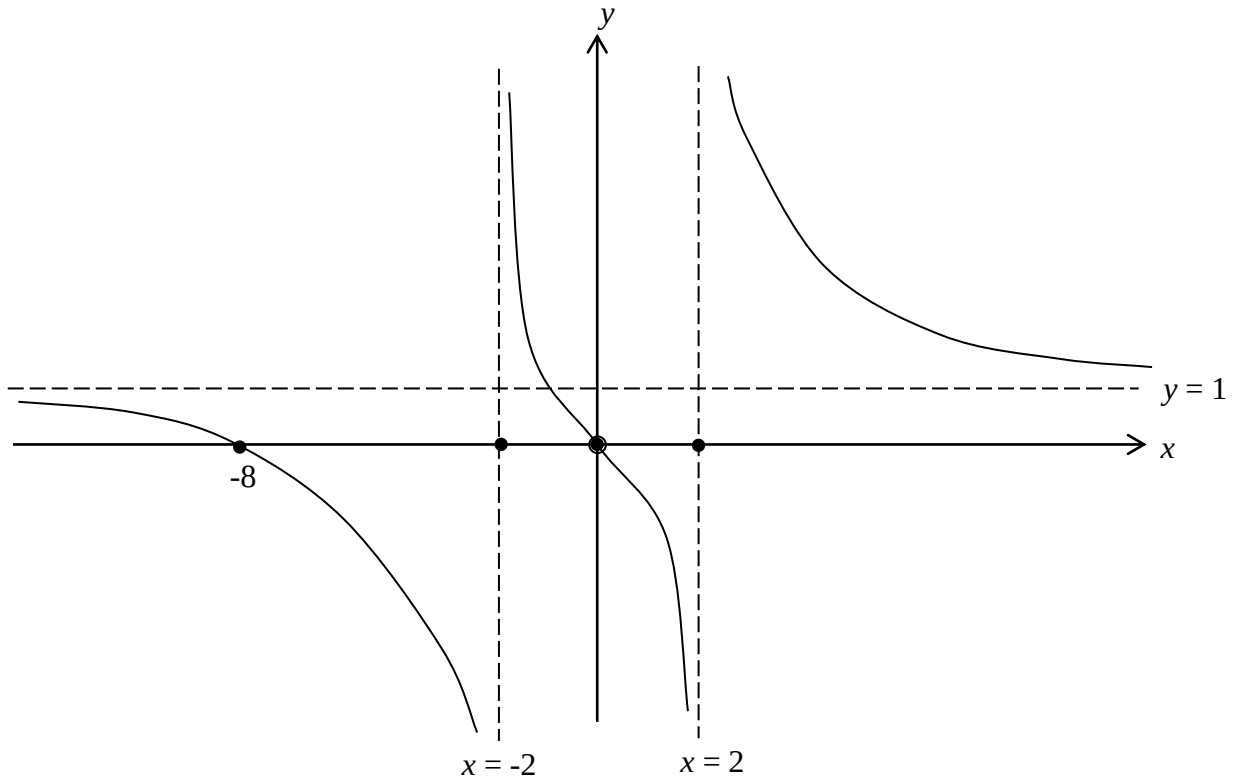
	Give one mark for each •	**Illustrations for awarding each mark**
15(b)	**ans:** Proof **3 marks** • calculates $f'(x)$ • knows to put $f'(x) = 0$ • uses discriminant to prove there is no solution	• and • $f'(x) = \dfrac{8x^2 + 8x + 32}{(x^2 + 8x)^2} = 0$ • $b^2 - 4ac = 64 - 4 \times 8 \times 32 < 0$ $\therefore$ no solution
15(c)	**ans:** see graph on next page **3 marks** • x intercepts clearly shown • approaches asymptotes correctly • completing graph	See next page
15(d)	**ans:** see graph on next page **4 marks** • asymptotes in (c) become roots • roots in (c) become asymptotes • approaches to asymptotes correct • completing graph	See next page

Total 96 marks

15(c)



15(d)



Total Marks: 96