

Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2

	Give one mark for each •	Illustrations for awarding each mark
1.	ans: $\pi/6$ 5 marks <ul style="list-style-type: none"> • knowing to complete the square • knowing to use a suitable substitution • finds new limits • integrates correctly • answer 	<ul style="list-style-type: none"> • $\int_{-1}^0 \frac{dx}{\sqrt{4-(x+1)^2}}$ • $u = x+1, du = dx,$ $x = -1: u = 0$ • $x = 0: u = 1$ • $\sin^{-1} \frac{u}{2} \Big _0^1$ • $\pi/6$
2.	ans: $2i, -2i, 2 + 3i, 2 - 3i$ 5 marks <ul style="list-style-type: none"> • substitutes $2i$ correctly into equation and proves result • states complex conjugate is a solution • finds first quadratic factor • finds second quadratic factor by dividing original equation by first quadratic factor • uses quadratic formula to find other 2 solutions 	<ul style="list-style-type: none"> • $(2i)^4 - 4(2i)^3 + 17(2i)^2 - 16(2i) + 52$ $= 16 + 32i - 68 - 32i + 52 = 0$ • $-2i$ is a solution • $(z - 2i)(z + 2i) = z^2 + 4$ $(z^4 - 4z^3 + 17z^2 - 16z + 52) \div (z^2 + 4)$ $= z^2 - 4z + 13$ • $2 + 3i, 2 - 3i$
3.	ans: $x = -3, y = -2, z = 4$ 5 marks <ul style="list-style-type: none"> • method • first modified system • second modified system • method of back-substitution • values of x, y and z 	<ul style="list-style-type: none"> • $\begin{bmatrix} 1 & -1 & 2 & 7 \\ 3 & 2 & 1 & -9 \\ 2 & 1 & -3 & -20 \end{bmatrix}$ • $\begin{bmatrix} 1 & -1 & 2 & 7 \\ 0 & 5 & -5 & -30 \\ 0 & 3 & -7 & -34 \end{bmatrix}$ • $\begin{bmatrix} 1 & -1 & 2 & 7 \\ 0 & 5 & -5 & -30 \\ 0 & 0 & -4 & -16 \end{bmatrix}$ • $-4z = -16 \Rightarrow z = 4$ • $y = -2, x = -3$

Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2 cont.

	Give one mark for each •	Illustrations for awarding each mark
4(a)	<p>ans: $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ 2 marks</p> <ul style="list-style-type: none"> • all powers in expansion • all coefficients in expansion 	<ul style="list-style-type: none"> • and • $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
4(b)	<p>ans: $16x^2 + \frac{64}{x^2}$ 5 marks</p> <ul style="list-style-type: none"> • correctly substituting x and $\frac{2}{x}$ in place of a & b • correct expression • substitutes x and $-\frac{2}{x}$ in place of a and b • correct expression • answer 	<ul style="list-style-type: none"> • $x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$ • $x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ • $x^4 + 4x^3\left(-\frac{2}{x}\right) + 6x^2\left(-\frac{2}{x}\right)^2 + 4x\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$ • $x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$ • $16x^2 + \frac{64}{x^2}$
5(a)	<p>ans: $-\frac{2}{(x-1)^2} e^{\frac{x+1}{x-1}}$ 4 marks</p> <ul style="list-style-type: none"> • knowing how to differentiate e • chain rule factor • using quotient rule correctly • answer 	<ul style="list-style-type: none"> • $e^{\frac{x+1}{x-1}}$ • $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$ • $\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = -\frac{2}{(x-1)^2}$ • $-\frac{2}{(x-1)^2} e^{\frac{x+1}{x-1}}$
5(b)	<p>ans: $y = x$ 4 marks</p> <ul style="list-style-type: none"> • differentiating correctly • finding expression for $\frac{dy}{dx}$ • finding gradient • finding equation of tangent 	<ul style="list-style-type: none"> • $4x - 3y - 3x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = \frac{3y - 4x}{2y - 3x}$ • $m = 1$ • $y - 1 = 1(x - 1)$

Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2 cont.

	Give one mark for each •	Illustrations for awarding each mark
6.	ans: $a = 20$, $b = 2$ 4 marks <ul style="list-style-type: none"> finding complex conjugate finding $\frac{z}{\bar{z}}$ equating real and imaginary parts Solving simultaneous equations 	<ul style="list-style-type: none"> $\bar{z} = 3 + i$ $\frac{z}{\bar{z}} = \frac{4}{5} - \frac{3}{5}i$ $\frac{4}{5}a + 3b = 22$, $-\frac{3}{5}a - b = -14$ $a = 20$, $b = 2$
7.	ans: $k = 3$ 4 marks <ul style="list-style-type: none"> Finding general term knowing to put $r = 2$ substituting $r = 2$ correctly finding value of k 	<ul style="list-style-type: none"> $\binom{6}{r} k^{6-r} (-4x)^r$ $r = 2$ $\binom{6}{2} k^4 (-4)^2 = 19440 \Rightarrow 240k^4 = 19440$ $k = 3$
8.	ans: $2x - 14y + 3 = 0$ 5 marks <ul style="list-style-type: none"> changing the subject of x to t changing the subject of y to t knowing to equate expressions finding equation of line proving point lies on line 	<ul style="list-style-type: none"> $t = \frac{3 + 2x}{1 - 4x}$ $t = \frac{2y}{1 - 4y}$ $\frac{3 + 2x}{1 - 4x} = \frac{2y}{1 - 4y}$ $2x - 14y + 3 = 0$ $2\left(\frac{11}{2}\right) - 14(1) + 3 = 11 - 14 + 3 = 0$ as required
9(a)	ans: $8n$, $4n(n + 1)$ 4 marks <ul style="list-style-type: none"> knowing how to find general term finding general term knowing how to find sum to n terms finding sum to n terms 	<ul style="list-style-type: none"> $u_n = a + (n - 1)d$ $u_n = 8 + (n - 1)8 = 8n$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_n = \frac{n}{2}[16 + (n - 1)8] = 4n(n + 1)$
9(b)	ans: 62000 3 marks <ul style="list-style-type: none"> knows how to find n finds n correctly finds sum of terms 	<ul style="list-style-type: none"> $8n < 1000 \Rightarrow n < 125$ $n = 124$ $S_{124} = 4 \times 124 (124 + 1) = 62000$

Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2 cont.

	Give one mark for each •	Illustrations for awarding each mark
10.	<p>ans: $-8i$, $8i$ 6 marks</p> <ul style="list-style-type: none"> finds modulus finds argument writes complex number in polar form deals with power correctly in polar form finds first solution finds second solution 	<ul style="list-style-type: none"> $\sqrt{2^2 + (-2\sqrt{3})^2} = 4$ $\tan \theta = \frac{-2\sqrt{3}}{2} = -\sqrt{3} \Rightarrow \theta = -\frac{\pi}{3}$ $4 \left(\cos \left(-\frac{\pi}{3} + 2k\pi \right) + i \sin \left(-\frac{\pi}{3} + 2k\pi \right) \right)$ $4^{3/2}, \frac{3}{2} \left(-\frac{\pi}{3} + 2k\pi \right)$ $k = 0 \Rightarrow -8i$ $k = 1 \Rightarrow 8i$
11.	<p>ans: $\frac{7}{30}$ 3 marks</p> <ul style="list-style-type: none"> writes $0.2333333\dots$ As a geometric series finds sum to infinity answer 	<ul style="list-style-type: none"> $\frac{2}{10} + \left(\frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \dots \right)$ $S_{\infty} = \frac{a}{1-r} = \frac{\frac{3}{100}}{1-\frac{1}{10}} = \frac{1}{30}$ $\frac{2}{10} + \frac{1}{30} = \frac{7}{30}$
12.	<p>ans: $1, i, -1, -i$ 5 marks</p> <ul style="list-style-type: none"> writes 1 in polar form takes 4th root correctly knows values of k to use finds roots proves roots sum to zero 	<ul style="list-style-type: none"> $1 = \cos 2k\pi + i \sin 2k\pi$ $z = \cos \frac{2k\pi}{4} + i \sin \frac{2k\pi}{4}$, $k = 0, 1, 2, 3$ $z = 1, i, -1, -i$ $1 + i + (-1) + (-i) = 0$ as required
13(a)	<p>ans: Proof 5 marks</p> <ul style="list-style-type: none"> knowing to use integration by parts correct application of integration by parts knows to use trigonometric identity combining terms answer 	<ul style="list-style-type: none"> $\int \sin x \sin^{n-1} x dx$ $-\cos x \sin^{n-1} x - \int (n-1) \cos x \sin^{n-2} x (-\cos x) dx$ $= -\cos x \sin^{n-1} x + (n-1) \int \cos^2 x \sin^{n-2} x dx$ $\cos^2 x = 1 - \sin^2 x$ $[1 + (n-1)] \int \sin^n x dx = -\cos x \sin^{n-1} x$ $+ (n-1) \int \sin^{n-2} x dx$ $\int \sin^n x dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} I_{n-2}$

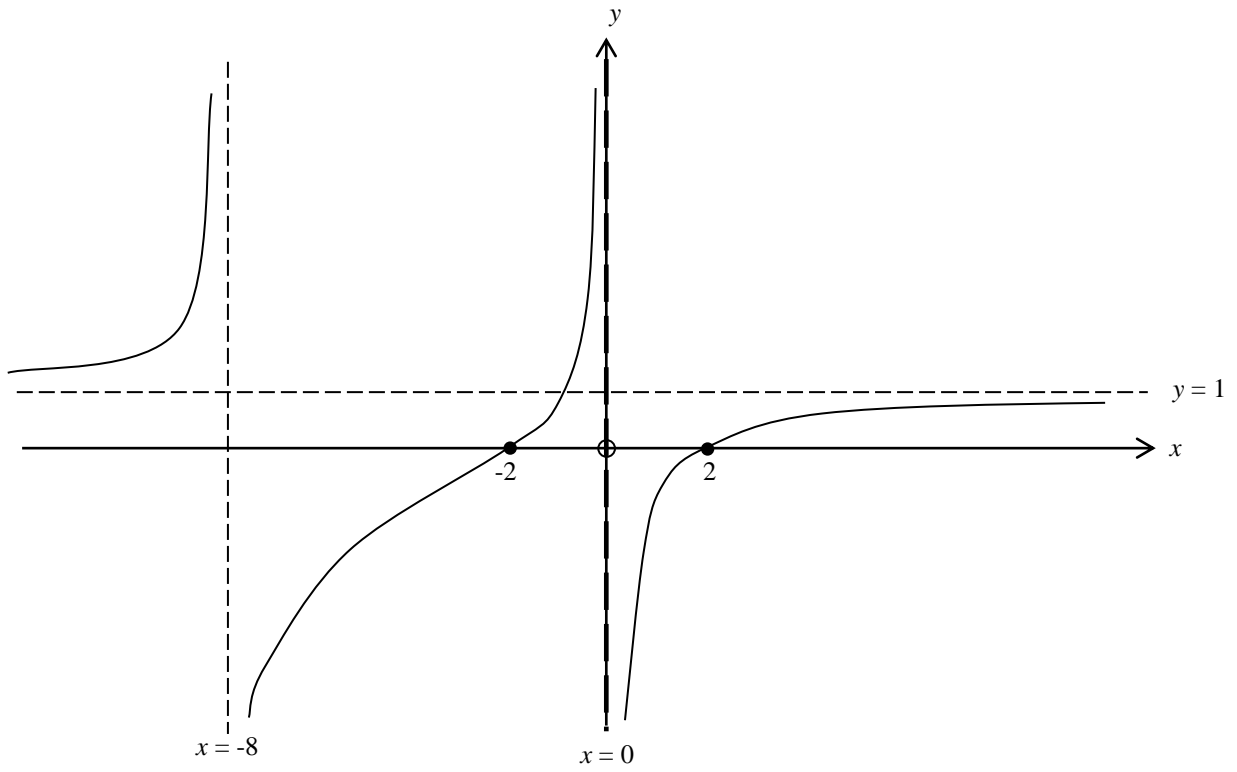
Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2 cont.

	Give one mark for each •	Illustrations for awarding each mark
13(b)	<p>ans: $\frac{n-1}{n} I_{n-2}$ 2 marks</p> <ul style="list-style-type: none"> • applying limits to reduction formula • answer 	<ul style="list-style-type: none"> • $\left(-\frac{1}{n} \cos \frac{\pi}{2} \sin^{n-1} \frac{\pi}{2}\right) - \left(-\frac{1}{n} \cos 0 \sin^{n-1} 0\right)$ $+ \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x dx$ • $\frac{n-1}{n} I_{n-2}$
13(c)	<p>ans: $\frac{35\pi}{256}$ 3 marks</p> <ul style="list-style-type: none"> • knowing to use reduction formula repeatedly • correctly integrating $\sin^2 x$ • answer 	<ul style="list-style-type: none"> • $\int_0^{\pi/2} \sin^8 x dx = \frac{8-1}{8} \int_0^{\pi/2} \sin^6 x dx = \frac{7}{8} I_6$ etc. • $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ • $\frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$
14(a)	<p>ans: $h = \frac{40}{r} - \frac{3r}{2}$ 2 marks</p> <ul style="list-style-type: none"> • finds surface area correctly • finds h in terms of r 	<ul style="list-style-type: none"> • cylinder = $2\pi rh + \pi r^2$, hemisphere = $2\pi r^2$ $\Rightarrow 80\pi = 2\pi rh + 3\pi r^2$ • $h = \frac{40}{r} - \frac{3r}{2}$
14(b)	<p>ans: 4 cm, 4 cm, $80\pi \text{ cm}^3$ 8 marks</p> <ul style="list-style-type: none"> • finds correct expression for volume • substitutes expression for h • differentiates volume • puts derivative = 0 • finds value for r • checks nature is maximum • finds value for h • finds value for V 	<ul style="list-style-type: none"> • $V = \pi r^2 h + \frac{2}{3} \pi r^3$ • $V = 40\pi r - \frac{5}{6} \pi r^3$ • $40\pi - \frac{5}{2} \pi r^2$ • $40\pi - \frac{5}{2} \pi r^2 = 0$ • $r = 4 \text{ cm}$ • nature table • $h = 4 \text{ cm}$ • $V = \frac{320\pi}{3} \text{ cm}^3$
15(a)	<p>ans: $x = 0, x = -8, y = 1$ 2 marks</p> <ul style="list-style-type: none"> • finds equations of vertical asymptotes • finds equation of horizontal asymptote 	<ul style="list-style-type: none"> • $x^2 + 8x = 0 \Rightarrow x = 0, x = -8$ • $f(x) = 1 - \frac{8x+4}{x^2+8x} \Rightarrow y = 1$

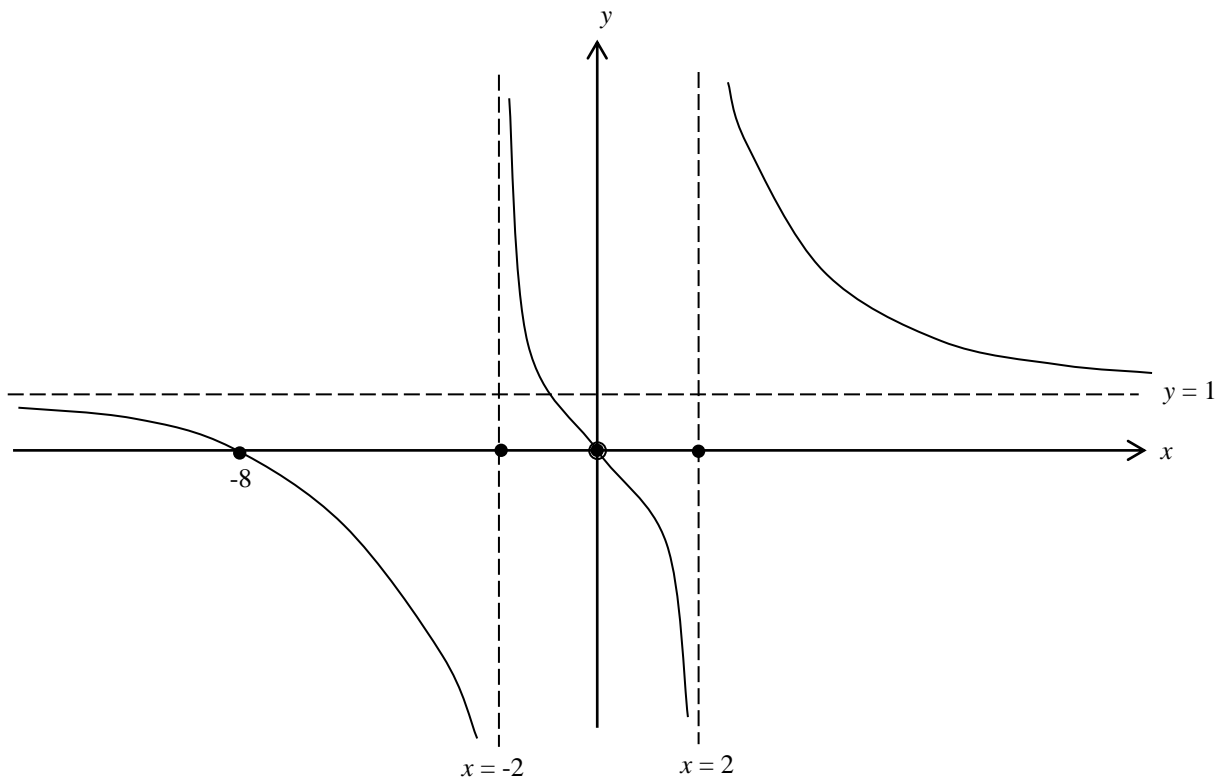
Marking Scheme – Advanced Higher Prelim – Maths 1 & Maths 2 cont.

	Give one mark for each •	Illustrations for awarding each mark
15(b)	ans: Proof 3 marks <ul style="list-style-type: none"> • calculates $f'(x)$ • knows to put $f'(x) = 0$ • uses discriminant to prove there is no solution 	<ul style="list-style-type: none"> • and • $f'(x) = \frac{8x^2 + 8x + 32}{(x^2 + 8x)^2} = 0$ • $b^2 - 4ac = 64 - 4 \times 8 \times 32 < 0 \therefore$ no solution
15(c)	ans: see graph on next page 3 marks <ul style="list-style-type: none"> • x intercepts clearly shown • approaches asymptotes correctly • completing graph 	
15(d)	ans: see graph on next page 4 marks <ul style="list-style-type: none"> • asymptotes in (c) become roots • roots in (c) become asymptotes • approaches to asymptotes correct • completing graph 	

15(c)



15(d)



Total Marks: 96