

All questions should be attempted

1. Differentiate with respect to x , simplifying your answer as far as possible:

(a) $y = \tan^{-1}\left(\frac{x+1}{x-1}\right)$ (4)

(b) $y = \ln(\sec x)$ (3)

2. Use Gaussian Elimination to solve the system

$$\begin{aligned} 2x + 3y - 4z &= -3 \\ x + 2y + 3z &= 3 \\ 3x - y - z &= 6 \end{aligned} \quad (5)$$

3. Prove by induction $\frac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers, n . (5)

4. Using the substitution $x = \sqrt{t}$, evaluate the integral

$$\int_{\frac{1}{3}}^3 \frac{1}{t + \sqrt{t}} dt \quad (6)$$

5. Find the coefficient of x^5 in the expansion of $\left(x^3 + \frac{2}{x}\right)^7$. (3)

6. (a) Find partial fractions for $\frac{2x^2 + 6x + 36}{(x^2 + 9)(x + 3)}$. (4)

(b) Hence evaluate the integral $\int_{-2}^0 \frac{2x^2 + 6x + 36}{(x^2 + 9)(x + 3)} dx$. (3)

7. Suppose that x and y are differentiable functions of t and that

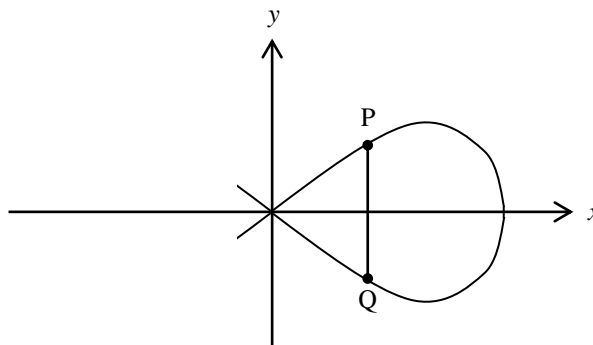
$$\frac{d^2y}{dx^2} = t^2 + 1, \quad \frac{dy}{dx} = t^3 + 3t.$$

Find $x(t)$ given that $x(1) = 4$. (6)

8. PQ is a chord of the loop of the curve $y^2 = x^2(8 - x^2)$, $x > 0$.

PQ is parallel to the y -axis.

Calculate the maximum possible length of PQ.



(6)

9. (a) Find two numbers x and y whose sum is 4 and whose product is 8. (4)
 (b) Plot the solutions on an Argand diagram. (2)

10. Use integration by parts to show that

$$\int x^3 \cos x \, dx = 3(x^2 - 2)\cos x + (x^3 - 6x)\sin x + C. \quad (5)$$

11. (a) Find an expression for the sum of n terms of the series

$$2 + \frac{2}{3} + \frac{2}{9} + \dots$$

in its simplest form. (4)

- (b) If $S_n = \frac{242}{81}$, find the value of n . (2)

12. An investor has £2000 with which to open an account and plans to add a further £1000 each year.

All funds in the account will earn compound interest at a rate of 10% p.a. .

Let $x(t)$ be the amount of money in the account at time t years.

- (a) Write down a first order differential equation representing the rate of change of money in the account each year. (2)

- (b) Hence show that $t = 10 \ln \frac{(1000 + 0.1x)}{1200}$. (7)

- (c) How many years would it take to save £100 000 ? (2)

13. A function $f(x)$ is defined by

$$f(x) = \left| \frac{x^2 - 2x + 2}{x - 1} \right| .$$

- (a) Write down the equation of the vertical asymptote of $f(x)$. (1)

- (b) For the function $g(x) = \frac{x^2 - 2x + 2}{x - 1}$, show that there is a non-vertical asymptote and find its equation. (3)

- (c) Find the coordinates of the stationary points of $g(x)$ and determine their nature. (5)

- (d) By first considering the graph of $g(x)$, sketch the graph of $f(x)$ showing all its main features. (4)

14. The semi-circle $y = \sqrt{a^2 - x^2}$ is rotated about the x-axis to generate a sphere.

- (a) Find an expression for the volume of the sphere. (8)

- (b) Find the volume of the sphere with equation $y = \sqrt{25 - x^2}$. (2)

END OF QUESTION PAPER