

X100/701

NATIONAL
QUALIFICATIONS
2004

FRIDAY, 21 MAY
1.00 PM - 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

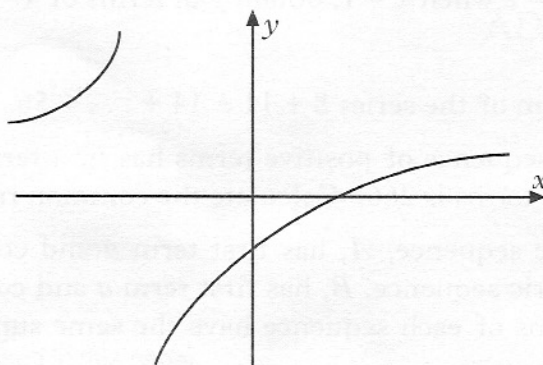
1. Calculators may be used in this paper.
2. Candidates should answer all questions.
3. **Full credit will be given only where the solution contains appropriate working.**



Answer all the questions.

1. (a) Given $f(x) = \cos^2 x e^{\tan x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, obtain $f'(x)$ and evaluate $f'(\frac{\pi}{4})$. 3,1
- (b) Differentiate $g(x) = \frac{\tan^{-1} 2x}{1+4x^2}$. 3
2. Obtain the binomial expansion of $(a^2 - 3)^4$. 3
3. A curve is defined by the equations
 $x = 5 \cos \theta$, $y = 5 \sin \theta$, $(0 \leq \theta < 2\pi)$.
 Use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ . 2
 Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{4}$. 3
4. Given $z = 1 + 2i$, express $z^2(z + 3)$ in the form $a + ib$. 2
 Hence, or otherwise, verify that $1 + 2i$ is a root of the equation
 $z^3 + 3z^2 - 5z + 25 = 0$. 2
 Obtain the other roots of this equation. 2
5. Express $\frac{1}{x^2 - x - 6}$ in partial fractions. 2
 Evaluate $\int_0^1 \frac{1}{x^2 - x - 6} dx$. 4
6. Write down the 2×2 matrix M_1 associated with an anti-clockwise rotation of $\frac{\pi}{2}$ radians about the origin. 2
 Write down the matrix M_2 associated with reflection in the x -axis. 1
 Evaluate $M_2 M_1$ and describe geometrically the effect of the transformation represented by $M_2 M_1$. 2
7. Obtain the first three non-zero terms in the Maclaurin expansion of $f(x) = e^x \sin x$. 5
8. Use the Euclidean algorithm to show that $(231, 17) = 1$ where (a, b) denotes the highest common factor of a and b .
 Hence find integers x and y such that $231x + 17y = 1$. 4
9. Use the substitution $x = (u - 1)^2$ to obtain $\int \frac{1}{(1 + \sqrt{x})^3} dx$. 5

10. Determine whether the function $f(x) = x^4 \sin 2x$ is odd, even or neither.
Justify your answer. 3
11. A solid is formed by rotating the curve $y = e^{-2x}$ between $x = 0$ and $x = 1$ through 360° about the x -axis. Calculate the volume of the solid that is formed. 5
12. Prove by induction that $\frac{d^n}{dx^n} (xe^x) = (x+n)e^x$ for all integers $n \geq 1$. 5
13. The function f is defined by $f(x) = \frac{x-3}{x+2}$, $x \neq -2$, and the diagram shows part of its graph.



- (a) Obtain algebraically the asymptotes of the graph of f . 3
- (b) Prove that f has no stationary values. 2
- (c) Does the graph of f have any points of inflexion? Justify your answer. 2
- (d) Sketch the graph of the inverse function, f^{-1} . State the asymptotes and domain of f^{-1} . 3
14. (a) Find an equation of the plane π_1 containing the points $A(1, 0, 3)$, $B(0, 2, -1)$ and $C(1, 1, 0)$. 4
- Calculate the size of the acute angle between π_1 and the plane π_2 with equation $x + y - z = 0$. 3
- (b) Find the point of intersection of plane π_2 and the line

$$\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2}. \quad 3$$

[Turn over for Questions 15 and 16 on Page four

15. (a) A mathematical biologist believes that the differential equation $x \frac{dy}{dx} - 3y = x^4$ models a process. Find the general solution of the differential equation. 5
- Given that $y = 2$ when $x = 1$, find the particular solution, expressing y in terms of x . 2
- (b) The biologist subsequently decides that a better model is given by the equation $y \frac{dy}{dx} - 3x = x^4$.
- Given that $y = 2$ when $x = 1$, obtain y in terms of x . 4
16. (a) Obtain the sum of the series $8 + 11 + 14 + \dots + 56$. 2
- (b) A geometric sequence of positive terms has first term 2, and the sum of the first three terms is 266. Calculate the common ratio. 3
- (c) An arithmetic sequence, A , has first term a and common difference 2, and a geometric sequence, B , has first term a and common ratio 2. The first four terms of each sequence have the same sum. Obtain the value of a . 3
- Obtain the smallest value of n such that the sum to n terms for sequence B is more than **twice** the sum to n terms for sequence A . 2

[END OF QUESTION PAPER]