

X100/701

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. There are five Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2

Section B assesses the optional unit Mathematics 3

Section C assesses the optional unit Statistics 1

Section D assesses the optional unit Numerical Analysis 1

Section E assesses the optional unit Mechanics 1.

Candidates must attempt Section A (Mathematics 1 and 2) **and one** of the following Sections:

Section B (Mathematics 3)

Section C (Statistics 1)

Section D (Numerical Analysis 1)

Section E (Mechanics 1).

3. **Candidates must use a separate answer book for each Section.** Take care to show clearly the optional section chosen. On the front of the answer book, in the top right hand corner, write B, C, D or E.
4. A booklet of Mathematical Formulae and Statistical Tables is supplied for all candidates. It contains Numerical Analysis formulae and Statistical formulae and tables.
5. **Full credit will be given only where the solution contains appropriate working.**



Section A (Mathematics 1 and 2)

All candidates should attempt this Section.

Marks

Answer all the questions.

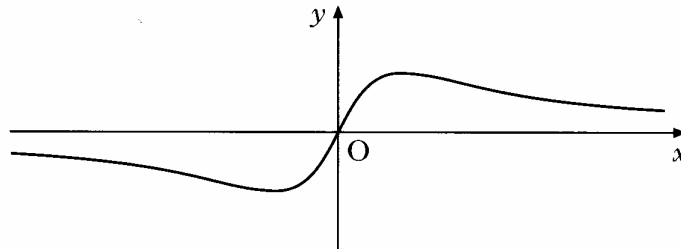
- A1.** (a) Given $f(x) = x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer. 3
 (b) Given $y = 3^x$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x . 3
- A2.** Given that $u_k = 11 - 2k$, ($k \geq 1$), obtain a formula for $S_n = \sum_{k=1}^n u_k$. 3
 Find the values of n for which $S_n = 21$. 2
- A3.** The equation $y^3 + 3xy = 3x^2 - 5$ defines a curve passing through the point $A(2, 1)$. Obtain an equation for the tangent to the curve at A . 4
- A4.** Identify the locus in the complex plane given by $|z + i| = 2$. 3
- A5.** Use the substitution $x = 1 + \sin \theta$ to evaluate $\int_0^{\pi/2} \frac{\cos \theta}{(1 + \sin \theta)^3} d\theta$. 5
- A6.** Use elementary row operations to reduce the following system of equations to upper triangular form

$$\begin{aligned} x + y + 3z &= 1 \\ 3x + ay + z &= 1 \\ x + y + z &= -1. \end{aligned} \quad \text{2}$$

Hence express x , y and z in terms of the parameter a . 2

Explain what happens when $a = 3$. 2

A7.



The diagram shows the shape of the graph of $y = \frac{x}{1+x^2}$. Obtain the stationary points of the graph. 4

Sketch the graph of $y = \left| \frac{x}{1+x^2} \right|$ and identify its three critical points. 3

A8. Given that $p(n) = n^2 + n$, where n is a positive integer, consider the statements:

- A $p(n)$ is always even
 B $p(n)$ is always a multiple of 3.

For each statement, prove it if it is true or, otherwise, disprove it.

4

A9. Given that $w = \cos \theta + i \sin \theta$, show that $\frac{1}{w} = \cos \theta - i \sin \theta$.

1

Use de Moivre's theorem to prove $w^k + w^{-k} = 2\cos k\theta$, where k is a natural number.

3

Expand $(w + w^{-1})^4$ by the binomial theorem and hence show that

$$\cos^4 \theta = \frac{1}{8} \cos 4\theta + \frac{1}{2} \cos 2\theta + \frac{3}{8}.$$

5

A10. Define $I_n = \int_0^1 x^n e^{-x} dx$ for $n \geq 1$.

(a) Use integration by parts to obtain the value of $I_1 = \int_0^1 x e^{-x} dx$.

3

(b) Similarly, show that $I_n = nI_{n-1} - e^{-1}$ for $n \geq 2$.

4

(c) Evaluate I_3 .

3

A11. The volume $V(t)$ of a cell at time t changes according to the law

$$\frac{dV}{dt} = V(10 - V) \quad \text{for } 0 < V < 10.$$

Show that

$$\frac{1}{10} \ln V - \frac{1}{10} \ln (10 - V) = t + C$$

for some constant C .

4

Given that $V(0) = 5$, show that

$$V(t) = \frac{10e^{10t}}{1 + e^{10t}}.$$

3

Obtain the limiting value of $V(t)$ as $t \rightarrow \infty$.

2

[END OF SECTION A]

Candidates should now attempt ONE of the following

Section B (Mathematics 3) on Page four

Section C (Statistics 1) on Pages five and six

Section D (Numerical Analysis 1) on Pages seven and eight

Section E (Mechanics 1) on Pages nine, ten and eleven.

Section B (Mathematics 3)

Marks

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

- B1.** Find the point of intersection of the line

$$\frac{x-3}{4} = \frac{y-2}{-1} = \frac{z+1}{2}$$

and the plane with equation $2x + y - z = 4$.

4

- B2.** The matrix A is such that $A^2 = 4A - 3I$ where I is the corresponding identity matrix. Find integers p and q such that

$$A^4 = pA + qI.$$

4

- B3.** A recurrence relation is defined by the formula

$$x_{n+1} = \frac{1}{2} \left\{ x_n + \frac{7}{x_n} \right\}.$$

Find the fixed points of this recurrence relation.

3

- B4.** Obtain the Maclaurin series for $f(x) = \sin^2 x$ up to the term in x^4 .
Hence write down a series for $\cos^2 x$ up to the term in x^4 .

4

1

- B5.** (a) Prove by induction that for all natural numbers $n \geq 1$

$$\sum_{r=1}^n 3(r^2 - r) = (n-1)n(n+1).$$

4

- (b) Hence evaluate $\sum_{r=11}^{40} 3(r^2 - r)$.

2

- B6.** Solve the differential equation

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^x,$$

given that $y = 2$ and $\frac{dy}{dx} = 1$, when $x = 0$.

10

[END OF SECTION B]