Cumbernauld Academy

Mathematics Department

4th Level (Upper)

Block 4 - homework booklet

Factorisation

1. Factorise each of the following by first removing a common factor.

(a)	$3x-15$	(b)	$5y+35$	(c)	$8a+48$	(d)	$18+6t$
(e)	x^2+9x	(f)	$3y-y^2$	(g)	b^2-4b	(h)	$5p+p^2$
(i)	$ab+ac$	(j)	x^2-xy	(k)	$pq-pr$	(l)	a^2+ax
(m)	$8a+20$	(n)	$21y-28$	(o)	$24x+22$	(p)	$36c-63$

2. Factorise each of the following by first removing the highest common factor.

(a)	$2a^2+6a$	(b)	$5x^2-40x$	(c)	$30y-y^2$	(d)	$3t^2+18t$
(e)	$6x^2-27x$	(f)	$14y-10y^2$	(g)	$12b^2-32b$	(h)	$25x^2+20x$

3. Factorise (each expression contains a difference of squares):

(a)	a^2-3^2	(b)	x^2-2^2	(c)	p^2-9^2	(d)	c^2-5^2
(e)	b^2-1	(f)	y^2-16	(g)	m^2-25	(h)	a^2-9
(i)	$36-d^2$	(j)	$4-q^2$	(k)	$49-w^2$	(l)	x^2-64
(m)	$81-4g^2$	(n)	$36w^2-y^2$	(o)	$4a^2-1$	(p)	g^2-81h^2
(q)	$49x^2-y^2$	(r)	$9c^2-16d^2$	(s)	$4p^2-9q^2$	(t)	b^2-100c^2

4. Factorise each of the following quadratic expressions:

(a)	$b^2+7b+12$	(b)	$x^2+14x+45$	(c)	$s^2+11s+24$	(d)	$y^2+11y+28$
(e)	b^2+6b+9	(f)	$c^2+13c+42$	(g)	$a^2+12a+32$	(h)	y^2+6y+8
(i)	x^2-6x+5	(j)	c^2-6c+8	(k)	$y^2-10y+21$	(l)	$b^2-14b+48$
(m)	$x^2-7x+10$	(n)	$s^2-13s+40$	(o)	$y^2-11y+18$	(p)	$a^2-8a+16$

5. Factorise:

(a)	x^2+4x-5	(b)	$a^2-4a-21$	(c)	t^2-t-20	(d)	$y^2+4y-32$
(e)	$c^2-5c-14$	(f)	x^2-5x-6	(g)	$b^2+7b-18$	(h)	$p^2-8p-20$
(i)	y^2-y-56	(j)	$z^2-2z-24$	(k)	$x^2-3x-28$	(l)	$a^2-13a-30$
(m)	c^2+c-20	(n)	p^2-6p-7	(o)	$b^2+5b-50$	(p)	$x^2+3x-18$

Functions & Graphs (2)

1. (a) **Copy** and complete the table below for the function with formula $f(x) = x^2 - 2x - 3$.

x	-2	-1	0	1	2	3	4
$f(x)$	5				-3		

(b) Draw the graph the function for $-2 \le x \le 4$ where x is a real number.

(c) Write down the nature and the coordinates of the turning point.

(d) For what value(s) of x is $f(x) = 0$?

2. (a) **Copy** and complete the table below for the function with formula $g(x) = x^2 - 8x + 12$.

x	0	1	2	3	4	5	6	7	8
$g(x)$		5				-3			

(b) Draw the graph the function for $0 \le x \le 8$ where x is a real number.

(c) Write down the nature and the coordinates of the turning point.

(d) For what value(s) of x is $g(x) = 0$?

3. (a) **Copy** and complete the table below for the function with formula $h(x) = 8x - x^2$.

x	-1	0	1	2	3	4	5	6	7	8	9
$h(x)$			7				15				

(b) Draw the graph the function for $-1 \le x \le 9$ where x is a real number.

(c) Write down the nature and the coordinates of the turning point.

(d) For what value(s) of x is $h(x) = 0$?

4. (a) **Copy** and complete the table below for the function with formula $f(x) = 3x^2 - x^3$.

x	-1	0	1	2	3	4
$f(x)$	4					-16

(b) Draw the graph the function for $-1 \le x \le 4$ where x is a real number.

(c) Write down the natures and the coordinates of any turning points.

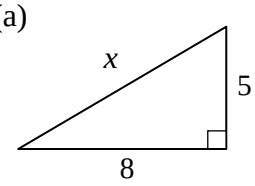
(d) For what value(s) of x is $f(x) = 0$?

Pythagoras (Revision)

You need a calculator for this worksheet.
Round your answers to one decimal place where necessary.

1. Calculate the length of the side marked x in each triangle below

(a)



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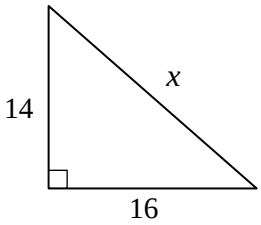

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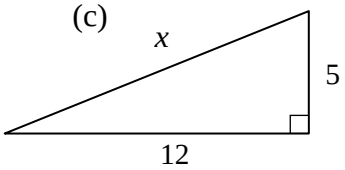

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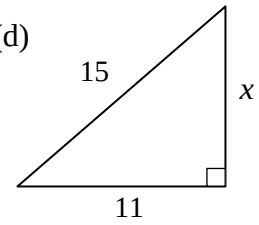

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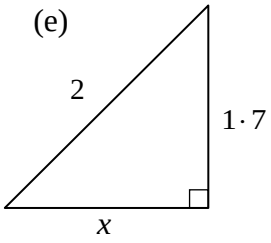

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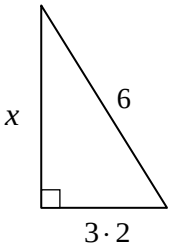

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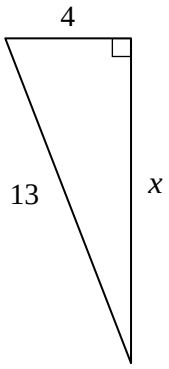

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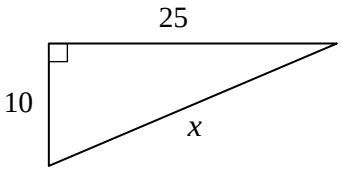
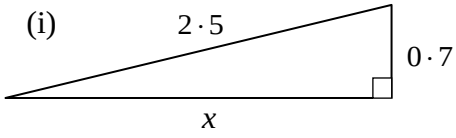
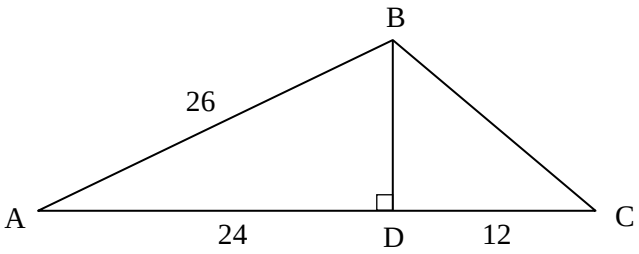


(i)



2. Consider the framework opposite.

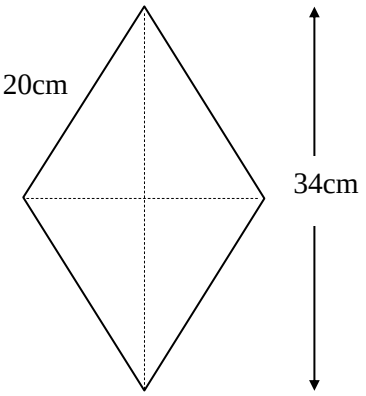


   (a) Calculate the length of BD.

   (b) Hence calculate the length of BC.

   (c) Calculate the area of triangle ABC.

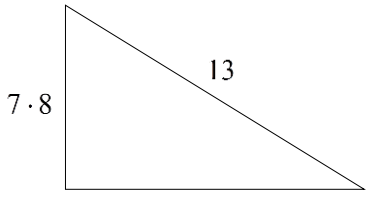
3. A rhombus has sides of 20cm and its longest diagonal measuring 34cm.



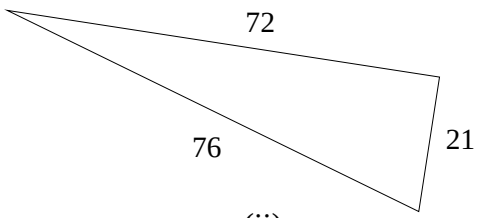
   (a) Calculate the length of the shorter diagonal.

   (b) Calculate the area of the rhombus

4. Which of the following triangles are right-angled?



(i)



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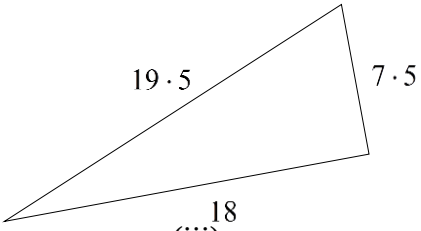


(iii)

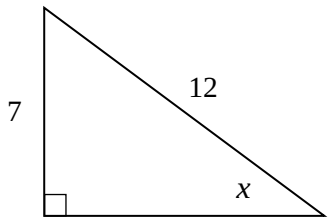
Trigonometry (1)

You need a scientific calculator for this worksheet.
Round all answers to 1 d.p. where necessary.

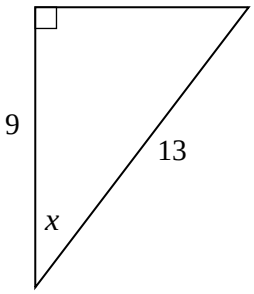
Remember
SOH CAH TOA

1. Find the size of angle x in each diagram.

(a)



(b)



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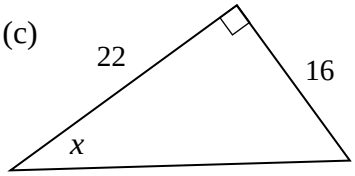

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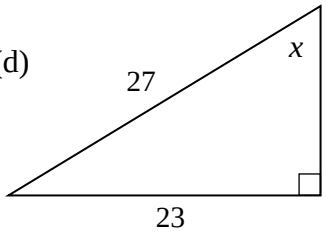

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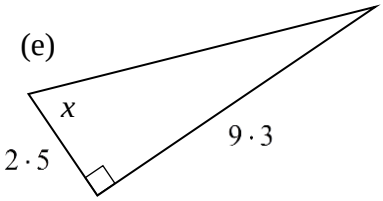

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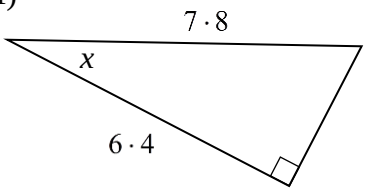
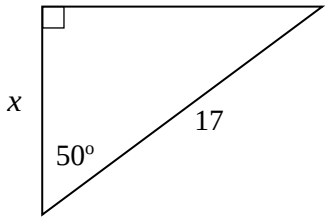


2. Calculate the length of the side marked x in each triangle below.

(a)



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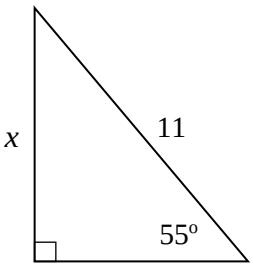
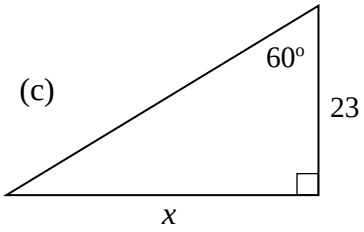
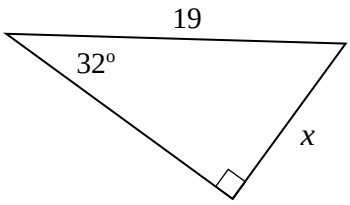


(c)



(d)



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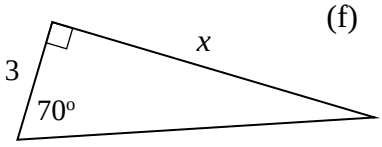

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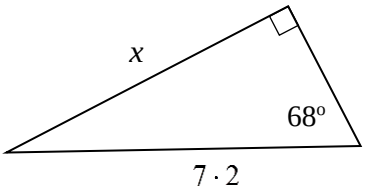
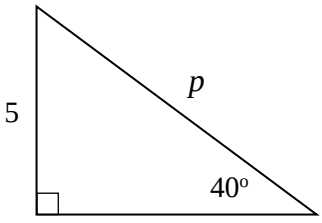


3. Calculate the length of the side marked p in each triangle below.

(a)



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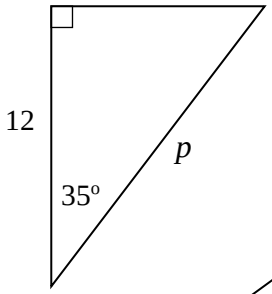

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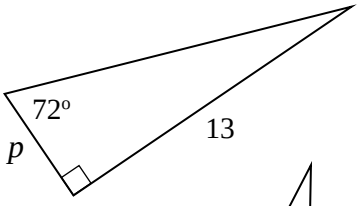

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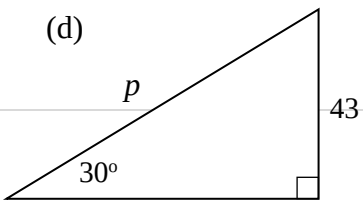

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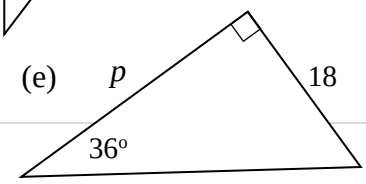

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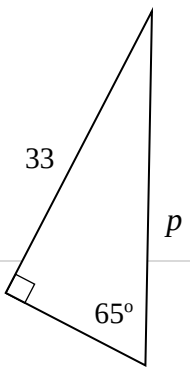


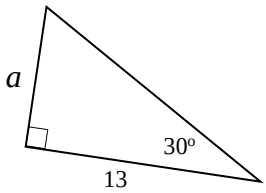
Trigonometry (2) – More Practice

You need a scientific calculator for this worksheet.

Remember
SOH CAH TOA

Find the angles and sides marked with letters.

1.



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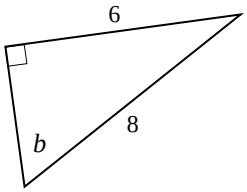

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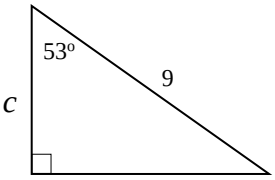

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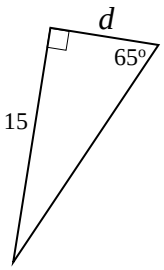

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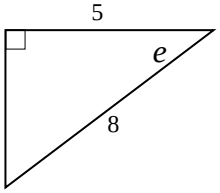

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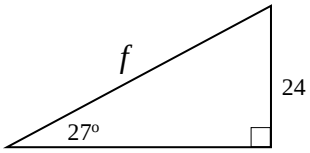

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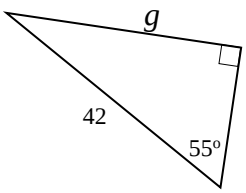

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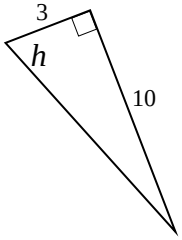

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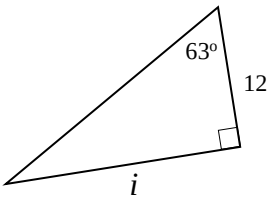

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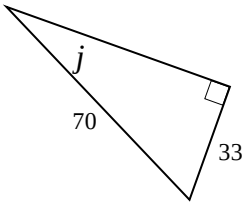

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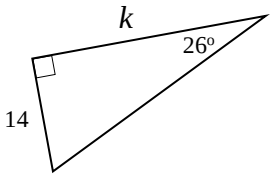

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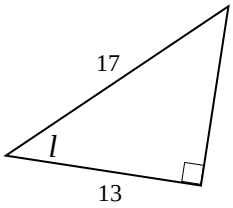

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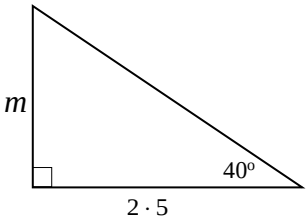
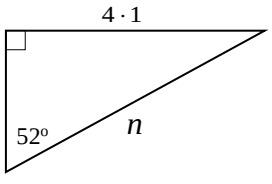


14.



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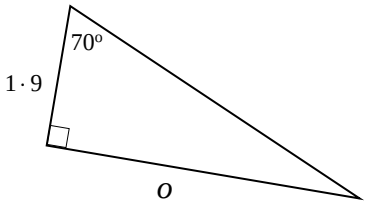

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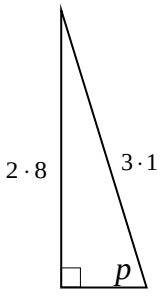

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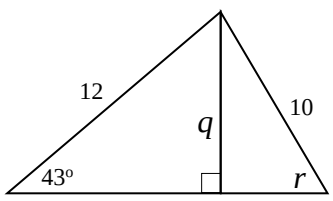

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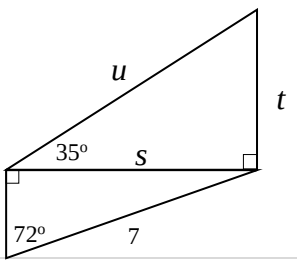

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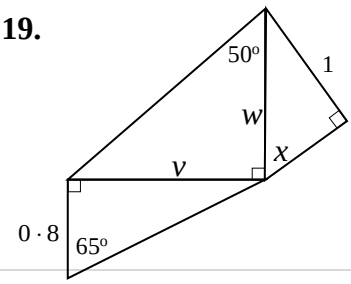
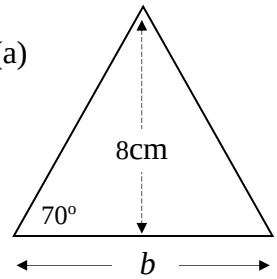


Trigonometry (3) - Isosceles Triangles & Frameworks

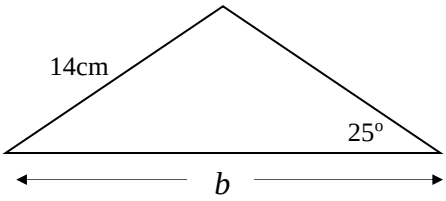
You need a scientific calculator for this worksheet.

1. Calculate the length of the base, b , of each isosceles triangle below.

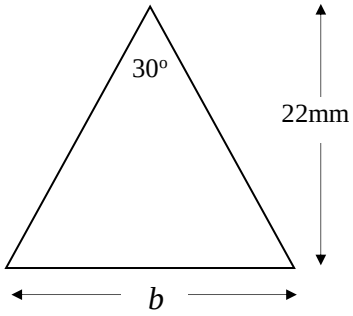
(a)



(b)

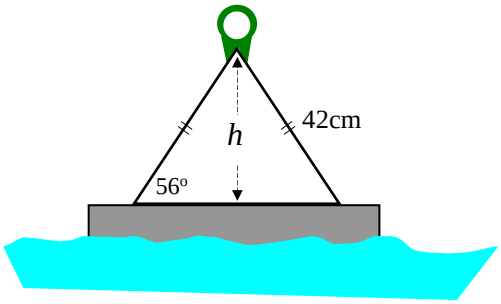


(c)



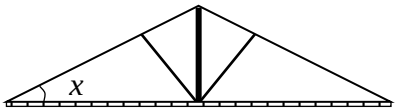
2. The diagram opposite represents a buoy used for tying-up small boats.

Calculate the vertical height (h) of its triangular frame.



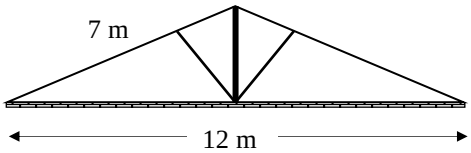
3. To comply with building regulations a roof must have an angle of between 21° and 28° to the horizontal (see diagram).

Which, if any, of the two roofs below comply with the building regulations?

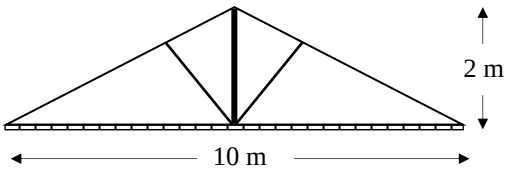


x must lie between 21° and 28°

(a)



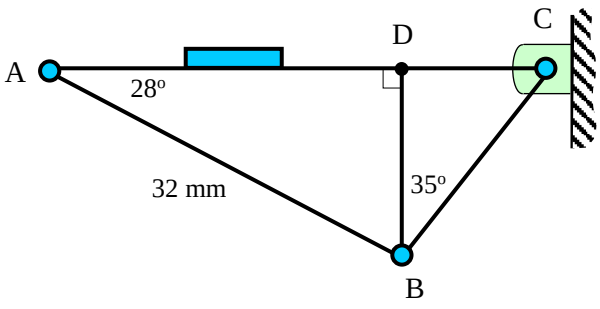
(b)



4. The diagram opposite shows part of the framework for a small hinged bracket.

(a) Calculate the length of DB.

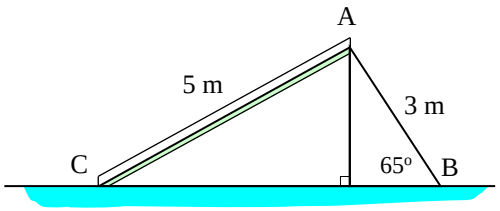
(b) Hence calculate the length of BC.



5. The diagram opposite represents a playground chute. AC represents the slide and AB the stairs.

A local council ruling states " *for a slide to be safe the* ***maximum*** *permissible angle between the slide and the ground is 35°* ".

Does this diagram represent a "safe" slide?



Trigonometry (4) - Problems

You need a scientific calculator for this worksheet.

Most of the problems below will require you to draw a neat sketch before attempting to answer the question.

1. A ladder of length 4m rests against a vertical wall so that the base of the ladder is $1 \cdot 5$ m from the wall.

   Calculate the angle between the ladder and the ground.

2. A ladder of length 5m leans against a vertical wall so that the base of the ladder is 2m from the wall.

   Calculate the angle between the ladder and the wall.

3. A vertical telegraph pole has a wire support of length 9m stretching from the top of the pole to the ground so that the angle between the wire support and the ground is 65^o.

   How far is the end of the wire support from the base of the pole?

4. A vertical aerial mast has a wire support of length 12m stretching from the top of the mast to the ground so that the angle between the wire support and the ground is 78^o.

   Calculate the height of the mast.

5. A ladder of length $4 \cdot 8$ m rests against a vertical wall so that it reaches up the wall to a height of $4 \cdot 3$ m.

   Calculate the angle between the ladder and the ground.

6. A tall thin tree has a height of 15m. A rope support stretches from the top of the tree to a point on the ground out from the base of the tree.

   If the angle between the rope and the ground is 62^o, calculate the length of the rope support.

7. From a distance of 20m from the base of a tower the angle of elevation to the top of the tower is 38^o. How high is the tower?

8. P is a point 30m from the base of a building. The building has a height of 18m.

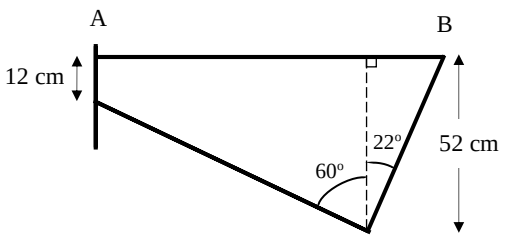
   Calculate the angle of elevation to the top of the building from P.

9. A girl is flying a kite from a string of length 46m.
   The string is taut, and is being held 1m above the ground.

   Calculate the height of the kite above the ground if the angle of elevation is 36^o between the string and the horizontal.

10. The frame of a bicycle is shown in the diagram opposite.

    Find the length of the cross bar, AB.



Changing the Subject of a Formula

1. Change the subject of each formula to x.

a.	$y = x + 3$	**b**.	$y = x - 5$	**c**.	$y = x + a$
d.	$y = x - b$	**e**.	$y = 3x$	**f**.	$y = 10x$
g.	$y = kx$	**h**.	$y = ax$	**i**.	$y = 3p + x$
j.	$y = x - 5t$	**k**.	$y = 2x + 1$	**l**.	$y = 3x - 7$
m.	$y = 7x + 4a$	**n**.	$y = 3b + 4x$	**o**.	$y = 8 + 10x$

2. Make x the subject of each formula below.

a.	$y = ax + b$	**b**.	$y = mx + c$	**c**.	$t = sx - r$
d.	$p = qx + 2r$	**e**.	$m = fx - 3n$	**f**.	$a = b + cx$
g.	$k = h - mx$	**h**.	$d = 3b + cx$	**i**.	$g = kc - hx$

3. Change the subject of each formula to the letter shown in brackets.

a.	$P = 4l$	(l)	**b**.	$V = IR$	(I)	**c**.	$S = DT$	(T)
d.	$A = lb$	(b)	**e**.	$C = \pi d$	(d)	**f**.	$G = UT$	(U)
g.	$v = u + at$	(t)	**h**.	$P = 2l + 2b$	(l)	**i**.	$H = xy + 5m$	(y)

4. Make x the subject of each formula. **(EXTENSION)**

a.	$y = \frac{3}{x}$	**b**.	$d = \frac{c}{x}$	**c**.	$m = \frac{y}{x}$
d.	$s = \frac{a+2}{x}$	**e**.	$w = \frac{z-1}{x}$	**f**.	$a = \frac{b+c}{x}$
g.	$a = \frac{x+8}{9}$	**h**.	$k = \frac{x-5}{2}$	**i**.	$p = \frac{3-x}{4}$
j.	$y = \frac{2}{x} + 1$	**k**.	$z = \frac{6}{x} - 7$	**l**.	$h = \frac{m}{x} + k$

5. Change the subject of each formula to the letter shown in brackets. **(EXTENSION)**

a.	$v^2 = u^2 + 2as$	(s)	**b**.	$v^2 = u^2 + 2as$	(u)
c.	$V = \pi r^2 h$	(h)	**d**.	$V = \pi r^2 h$	(r)
e.	$r = \sqrt{\frac{A}{\pi}}$	(A)	**f**.	$L = 3 + \sqrt{6a}$	(a)
g.	$2k = \sqrt{(p+4)}$	(p)	**h**.	$x^2 = \frac{4yz}{t}$	(y)
i.	$ar = \frac{1}{2}\sqrt{\frac{x}{b}}$	(b)	**j**.	$st = A^2(x - 3y)$	(A)
k.	$R = A^2(x - 3y)$	(x)	**l**.	$na = \sqrt{(1-n^2)}$	(n)
m.	$d = \frac{t(n-1)}{n}$	(n)	**n**.	$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$	(R)

Simultaneous Equations (1)

1. Solve the following systems of equations by "**elimination**".

(a) $2x + 4y = 24$, $7x - 2y = 4$ (b) $4a - 3b = 18$, $2a + 6b = -6$ (c) $2e + 7f = 26$, $8e - 5f = 38$

(d) $5x + y = -2$, $3x + 2y = 3$ (e) $2x - 3y = 10$, $3x - 6y = 18$ (f) $4p + 3q = 1$, $8p + 5q = -1$

(g) $2g + 3h = 1$, $5g - 2h = -26$ (h) $-2x + 3y = 6$, $9x - 7y = -1$ (i) $2u + 4v = -16$, $11u - 7v = -1$

(j) $2x - 8y = 0$, $5x - 5y = 15$ (k) $3p + 2q = -11$, $4p + 3q = -14$ (l) $10a - 3b = 46$, $6a - 8b = 40$

2. Solve the following systems of equations by "**substitution**".

(a) $x + 3y = 17$, $3x - 2y = -4$ (b) $a - 3b = 6$, $3a + b = 8$ (c) $2e + f = 1$, $5e - 2f = -20$

(d) $5x + 3y = 7$, $4x + y = 0$ (e) $2x - 5y = -14$, $x - 2y = -5$ (f) $2p + 3q = 6$, $4p + q = -8$

(g) $2g + h = 11$, $7g - 8h = 96$ (h) $3x - 2y = 25$, $x + 5y = -3$ (i) $u - 4v = 10$, $9u - 2v = 22$

(j) $2x = 3y + 5$, $x + 5y = 9$ (k) $3p - 2q + 7 = 0$, $4p + q = -2$ (l) $4a + b - 30 = 0$, $6a + 5b - 38 = 0$

3. Solve the following simultaneous equations "**graphically**".

(a) $x + y = 6$, $2x + y = 8$
Draw axes with x and y from 0 to 8

(b) $x + 2y = 8$, $3x + y = 9$
Draw axes with x and y from 0 to 9

(c) $x + 3y = 6$, $x - y = 2$
Draw axes with x from 0 to 8 and y from –2 to 4

(d) $2x + 3y = 12$, $x + y = 5$
Draw axes with x and y from 0 to 7

(e) $3x + 4y = 24$, $3x + 2y = 18$
Draw axes with x and y from 0 to 9

(f) $5x + y = 10$, $x - y = -4$
Draw axes with x from -4 to 4 and y from 0 to 10

Simultaneous Equations (2)

Problems Leading to Simultaneous Equations

Note: each question must begin with the construction of two separate equations each with two stated unknowns.

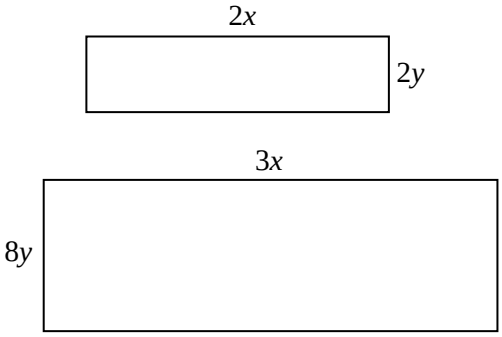
1. Find two numbers whose sum is 56 and whose difference is 16.

2. Find two numbers whose sum is 22 and where twice the big one minus three times the small one is 24.

3. Two numbers are such that twice the smaller plus the larger is equal to 18 and the difference between twice the larger and the smaller is 11.
Find the two numbers.

4. Two numbers are such that three times the larger plus twice the smaller is equal to 31 and the sum of twice the smaller plus the larger is 13.
Find the two numbers.

5. Consider the two rectangles opposite.

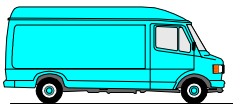


The smaller one has a perimeter of 60cm.
The larger one has a perimeter of twice the smaller.

(a) Form two equations and solve them simultaneously to find the values of x and y.

(b) Hence calculate the area of the smaller rectangle.

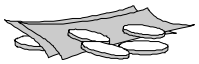
6. A van is carrying eight identical boxes and five identical parcels.



(a) If 3 boxes and 2 parcels weigh a total of 22kg and 4 boxes and 3 parcels weigh 30kg, find the weight of an individual box and a single parcel.

(b) What is the total weight carried by the van?

7. 3 pounds of butter and 4 pints of milk costs £3.84.
5 pounds of butter and 7 pints of milk costs £6.48.
Find the cost of a pound of butter and a single pint of milk.



8. In a certain factory, the basic rate of pay is £4.50 per hour, with overtime at £6.40.
His total wage for a certain week was £215.80.
If he worked a total of 45 hours in all, how many hours did he work at the basis rate?

9. At a concert 500 tickets were sold. Cheap tickets cost £5 whereas more expensive ones cost £9.
If the total receipts were £3 220, how many cheap tickets were sold?

10. John saves money by putting every 50p and every 20p coin he recieves in a box. After a while he discover's that he has 54 coins amounting to £17.10. How many 50p coins does he have?