Holiday Revision

1. The vertices of a triangle are P(-1,-1), Q(2,1) and R(-6,2).
   Find the equation of the altitude drawn from Q.

2. Simplify a) $2 \log_9 2 + 3 \log_9 3 - \log_9 36$

   b) $\log_2 3 + \log_2 4 + \log_2 5 - \log_2 30$

3. Find y' in each example

   a) $y = (2x+1)(x^2-2)$ b) $y = \dfrac{2x+1}{\sqrt{x}}$

4. Evaluate $\int_0^1 \left(2x^2 + 3\right)dx$.

5. Express $x^2 + 6x + 11$ in completed square form and state the minimum.
   Hence state the maximum value of $g(x) = \dfrac{1}{x^2 + 6x + 11}$.

6. Find the equation of the tangent to the circle $x^2 + y^2 = 29$ at the point (-5,2) on the circle.

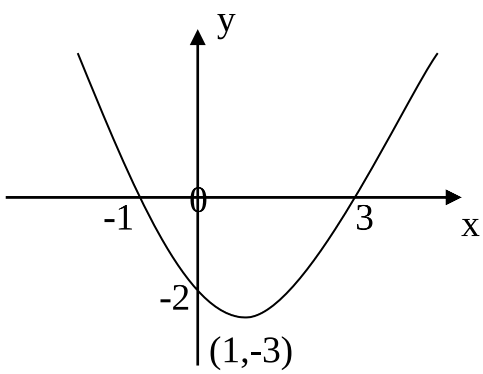
7. Find the equation of the tangent to the curve $y = x^3 - 4x - 5$ at $x = 1$.
   Find the angle which this tangent makes with the positive direction of the x axis.

8. Sketch the graph of $y = 8\cos(\theta - \frac{\pi}{4})$, $0 \le \theta \le 2\pi$

9. Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$.

10. A function is defined by the formula $f(x) = 4x^2(x-3)$.
    a) Write down the coordinates of the points where the curve cuts the coordinate axis.
    b) Find the stationary values and determine their nature.
    c) Sketch the curve $y = f(x)$.
    d) Find the area enclosed by the curve and the x axis.

11.

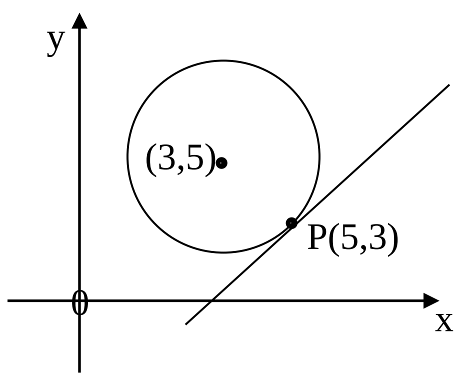


The diagram shows a sketch of the function y = f(x).
On separate diagrams draw the graphs of
a) -f(x) b) f(x+2) c) 3 + f(x) d) 2 – f(x)

12 Express f(x) = $5\cos x + 4\sin x$ in the form $k\cos(x-\alpha)$.
(i) State the max/min values of f and the values of x at which the max/min occur.

13. Find the equation of the tangent to the curve y = $4x^3 - 2$ at the point where x = -1.

14.



Find the equation of the tangent to the circle centre (3,5) at the point (5,3) on the circle.

15. The initial quantity of pollution in the loch is 25 tons, the Council remove 35% during the week and a factory discharges 8 tons into the loch each Sunday.

i) Find the amount of pollution after 1, 2, 3 and 4 weeks

ii) Establish a recurrence relation and hence find the long term state of the loch.

16. If f(x) = 2x + 1 and g(x) = 1 – 5x find

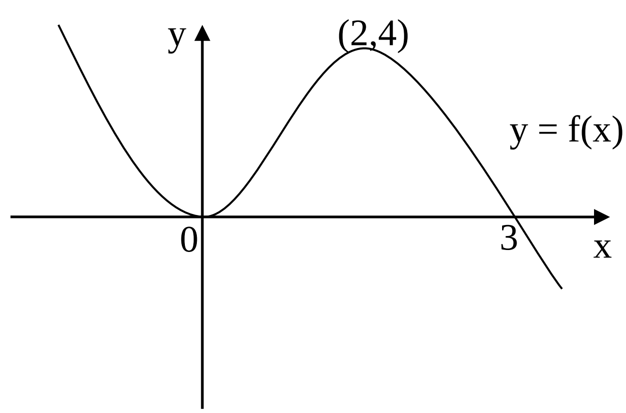
a) f(g(x)) b) g(f(x))

Hence solve the equation f(g(x)) - g(f(x)) = 8x + 7

17. Evaluate $\int_{-1}^{3}\left(x^2+2\right)dx$ and draw a sketch to illustrate the area represented by this integral.

18. For all points on the curve y = f(x) , f ′(x) = 1 – 4x.
If the curve passes through the point (1,-1), find the equation of the curve.

18.



The diagram shows a sketch of a cubic function f with stationary values at the origin and (2,4). Sketch the graph of the derived function.

20. Given that $\frac{x^2+4x+10}{2x+5}=n$, form a quadratic equation in x and hence show that if $n \le -3$ or $n \ge 2$ then the roots will be real.

21. If $y=\frac{(x+2)(x+1)}{\sqrt{x}}$, find y' when x = 4.

22. For what values of x is the function y = $\frac{1}{3}x^3-2x^2-5x-4$ increasing.

23. If sinA = $\frac{8}{17}$ and A is acute, find the exact values of

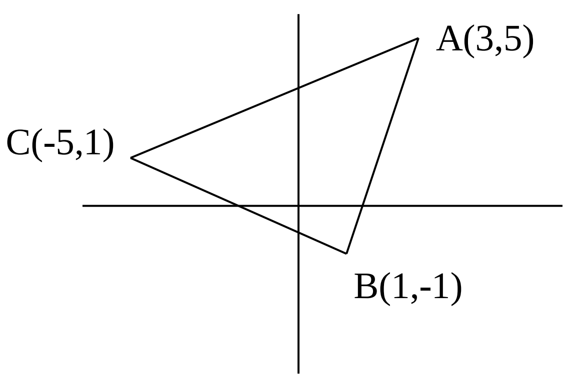
a) sin2A b) cos2A

24. Show that the point (3,-1) lies on the circle with equation

$$x^2 + y^2 - 4x + 6y + 8 = 0$$

and find the equation of the tangent to the circle at this point.

25. In the diagram shown, find the equation of the altitude from A and the median from B.



26. The number of bacteria present in a beaker, during an experiment can be measured using the formula $N(t) = 30e^{1.25t}$ where t is the number of hours passed.

(a) How many bacteria are in the beaker at the start of the experiment?

(b) Calculate the number of bacteria present after 5 hours.

(c) How long will it take for the number of bacteria present to treble?

27. Solve for $x > 0$

(a) $\log_a 5 + \log_a 2x = \log_a 60$ (b) $2\log_a 3 + \log_a x = \log_a 36$

(c) $\frac{1}{2}\log_x 64 + 2\log_x 2 = 5$ (d) $2\log_x 6 - \frac{2}{3}\log_x 8 = 2$

28. Find where the following curves cut the x-axis.

(a) $y = \log_4 x - 2$ (b) $y = \log_2 (x - 4) - 1$

29. Find where the following curves cut the y-axis.

(a) $y = \log_2 (x + 4) + 1$ (b) $y = \log_3 (x + 27) + 5$

30. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula $M = M_o e^{-kt}$ where Mo is the initial mass of the isotope.

In 5 years a mass of 10 grams of the isotope is reduced to 8 grams.

(a) Calculate k.

(b) Calculate the half-life of the substance i.e. the time taken for half the substance to decay.

31. If $f(x) = \frac{1}{2}x + 8$ find $f^{-1}(x)$

32. If $f(x) = x^3 - 6$ find $f^{-1}(x)$ and state the domain and range of f.

33. If $f(x) = 2\sqrt{x} + 5$, state a suitable domain for f. Find the inverse function.

34. Using $R\sin(x - \alpha)$ find the maximum values of f and g, and the corresponding values of x for $0 \leq x \leq 2\pi$.

(a) $f(x) = 1 + \sqrt{2}\cos x - \sqrt{2}\sin x$ (b) $g(x) = 2 + \sqrt{3}\sin x - \cos x$.