4

y

Higher 2000 - Paper 1 Solutions

1. On the coordinate diagram shown, A is the [SQA] y, A(6,8) point (6,8) and B is the point (12, -P) fragree placements AOC = p and angle COB = q. Find the exact value of sin(p+q). р C **⋆** 9] 0 B(12, -5)

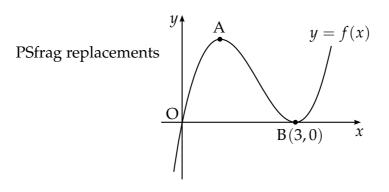
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	С	NC	T9	$\frac{63}{65}$	2000 P1 Q1
• ² • ³	ss: knc pd: pro ic: inte pd: pro	cess mis erpret da	sing sic		• $\sin p \cos q + \cos p$ • $10 \text{ and } 13$ • $\frac{8}{10} \cdot \frac{12}{13} + \frac{6}{10} \cdot \frac{5}{13}$ • $\frac{126}{130}$	sin q

replacements

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- x y

2. A sketch of the graph of y = f(x) where $f(x) = x^3 - 6x^2 + 9x$ is shown below. [SQA] The graph has a maximum at A and a minimum at B(3,0).



- (*a*) Find the coordinates of the turning point at A.
- (*b*) Hence sketch the graph of y = g(x) where g(x) = f(x+2) + 4. Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes.

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(c) V	Vrite dov	vn the 1	ange o	f values of k	for which $g(x) = k$ has 3 real roots.
Part	Marks	Level	Calc.	Content	Answer U1 OC3
<i>(a)</i>	4	С	NC	C8	A(1,4) 2000 P1 Q2
(b)	2	С	NC	A3	sketch (translate 4 up, 2 left)
(C)	1	A/B	NC	A2	4 < k < 8
•2 •3 •4 •5 •6	ss: kno pd: diff ss: kno pd: pro ic: inte ic: inte ic: inte	erentiate ow gradi cess erpret tra erpret tra	e correc ent $= 0$ ansform	tly nation	•1 $\frac{dy}{dx} =$ •2 $\frac{dy}{dx} = 3x^2 - 12x + 9$ •3 $3x^2 - 12x + 9 = 0$ •4 A = (1,4) translate $f(x)$ 4 units up, 2 units left •5 sketch with coord. of A'(-1,8) •6 sketch with coord. of B'(1,4) •7 4 < k < 8 (accept 4 ≤ k ≤ 8)

replacements

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х y

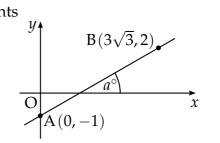
Ox

y

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Higher Mathematics

[SQA] 3. Find the size of the angle a° that the line joining the points A(0, -1) and B($3\sqrt{3}$, 2) makes with the positive direction of the *x*-axis.



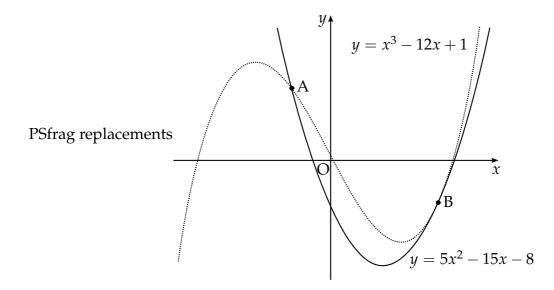
Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	С	NC	G2	30	2000 P1 Q3
	ss: kno equ. pd: pro ic: inte			d gradient or 1e	•1 $\frac{2-(-1)}{3\sqrt{3-0}}$ •2 $\tan a = \text{gradient}$ stated o •3 $a = 30$	r implied by

replacements

O x

[SQA] 4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



- (*a*) (i) Find the *x*-coordinates of the point of the curves where the gradients are equal.
 - (ii) By considering the corresponding *y*-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i).
- (b) The point A is (-1, 12) and B is (3, -8).

Find the area enclosed between the two curves.

		т 1	<u>C</u> 1		
Part	Marks	Level	Calc.	Content	Answer U2 OC2
(ai)	4	С	NC	C4	$x = \frac{1}{3}$ and $x = 3$ 2000 P1 Q4
(aii)	1	С	NC	CGD	parallel and coincident
<i>(b)</i>	5	С	NC	C17	$21\frac{1}{3}$
•2 •3 •4 •5 •6 •7 •8 •9	ss: kno pd: diff pd: forr ic: inte ic: inte ss: kno curves ic: inte pd: forr pd: pro pd: pro	erentiate n equati erpret so erpret di ow how erpret lir n integr cess inte	e ion Iution agram to find nits al egration	area between	• ¹ find derivatives and equate • ² $3x^2 - 12$ and $10x - 15$ • ³ $3x^2 - 10x + 3 = 0$ • ⁴ $x = 3, x = \frac{1}{3}$ • ⁵ tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident • ⁶ $\int (\text{cubic} - \text{parabola})$ or $\int (\text{cubic}) - \int (\text{parabola})$ • ⁷ $\int_{-1}^{3} \cdots dx$ • ⁸ $\int (x^3 - 5x^2 + 3x + 9) dx$ or equiv. • ⁹ $[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x]_{-1}^{3}$ or equiv. • ¹⁰ $21\frac{1}{3}$

replacements

x y

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[SQA] 5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of *a* and evaluate the limit.

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
	4	С	NC	A13	$a = \frac{3}{5}, L = 25$	2000 P1 Q5
	1	A/B	NC	A12		
\bullet^2 \bullet^3 \bullet^4	ss: knc pd: pro pd: pro ic: inte pd: pro	cess cess erpret co			• ¹ $L = aL + 10$ or $L = L = \frac{b}{1-a}$ • ² $L = \frac{10}{1-a}$ or $L = \frac{16}{1-a^2}$ • ³ $\frac{10}{1-a}$ or $\frac{16}{1-a^2}$ • ⁴ $10a^2 - 16a + 6 = 0$ • ⁵ $a = \frac{3}{5}$ and $L = 25$	<i>a</i> ² <i>L</i> + 16 or

[SQA] 6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle?

U2 OC4 Part Marks Level Calc. Content Answer 5 NC G9, A17 for all *k* 2000 P1 Q6 А \bullet^1 ss: know to examine radius •¹ g = 2k, f = -k, c = -k - 2stated or implied by \bullet^2 •² pd: process pd: process • $r^2 = 5k^2 + k + 2$ •3 •⁴ ic: interpret quadratic inequation •³ (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (accept \geq) •⁴ use discr. **or** complete sq. **or** diff. interpret quadratic inequation \bullet^5 ic: •⁵ true for all k

replacements

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x

y

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Higher Mathematics

PSfrag replacements

^[SQA] 7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

- $\overrightarrow{\text{VA}}$ represents -7i 13j 11k
- \overrightarrow{AB} represents 6i + 6j 6k
- $\overrightarrow{\text{AD}}$ represents 8i 4j + 4k.

K divides BC in the ratio 1:3.

Find \overrightarrow{VK} in component form.

Part	Marks	Level	Calc.	Content	Answer U3 OC1
	3	C	CN	G25, G21, G20	$\begin{pmatrix} 1 \end{pmatrix}$
• ²	ss: reco ic: inte pd: pro	erpret ra	tio	1	• ¹ $\overrightarrow{VK} = \overrightarrow{VB} + \overrightarrow{BK}$ $\overrightarrow{AB} + \overrightarrow{AB} + \overrightarrow{BK}$ or • ² $\overrightarrow{BK} = \frac{1}{4}\overrightarrow{BC}$ or $\frac{1}{4}\overrightarrow{AD}$ or $\begin{pmatrix} 2\\-1\\1 \end{pmatrix}$ or $\begin{pmatrix} -1\\-7\\-17 \end{pmatrix}$ • ³ $\overrightarrow{VK} = \begin{pmatrix} 1\\-8\\-16 \end{pmatrix}$

[SQA] 8. The graph of y = f(x) passes through the point $(\frac{\pi}{9}, 1)$.

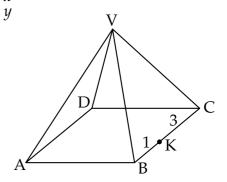
If $f'(x) = \sin(3x)$ express *y* in terms of *x*.

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C18, C23	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	2000 P1 Q8
\bullet^2 \bullet^3	ss: kno pd: inte ic: inte pd: pro	egrate erpret (7	U		• ¹ $y = \int \sin(3x) dx$ stated • ² • ² $-\frac{1}{3}\cos(3x)$ • ³ $1 = -\frac{1}{3}\cos(\frac{3\pi}{9}) + c$ or e • ⁴ $c = \frac{7}{6}$	

replacements

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y y



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Higher Mathematics

 $[\text{SQA}] \qquad 9. \ Evaluate \ \log_5 2 + \log_5 50 - \log_5 4.$

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	С	NC	A28	2	2000 P1 Q9
	1	A/B	NC	A28		
• ²	pd: use pd: use pd: use	$\log_a x -$	$\log_a y$	$= \log_a xy$ $= \log_a \frac{x}{y}$	• ¹ $\log_5 100 - \log_5 4$ • ² $\log_5 25$ • ³ 2	

[SQA] 10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$.

Part	Marks	Level	Calc.	Content	Answer U3 OC4
	6	A/B	CN	T14	max_value $\sqrt{2}$ when 2000 P1 Q10
					$x = \frac{7\pi}{4}$
• ² • ³ • ⁴ • ⁵	ss: use ic: exp pd: con pd: pro pd: pro ic: inte	and cho npare co cess cess	sen rule efficien	e ts	• ¹ e.g. use $k \cos(x + a)$ • ² $k \cos x \cos a - k \sin x \sin a$ • ³ $k \cos a = 1$ and $k \sin a = 1$ • ⁴ $k = \sqrt{2}$ • ⁵ $\tan a = 1, a = \frac{\pi}{4}$ (45° <i>is bad form</i>) • ⁶ max. value = $\sqrt{2}$ when $x = \frac{7\pi}{4}$ (do not accept 45°)

[END OF QUESTIONS]

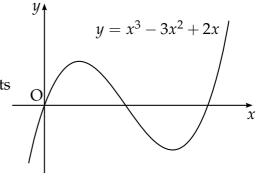
replacements

O x

x y 3

Higher 2000 - Paper 2 Solutions

- [SQA] 1. The diagram shows a sketch of the graph of $y = x^3 3x^2 + 2x$.
 - (*a*) Find the equation of the tangent to this curve at the point where x = 1.PSfrag replacements
 - (*b*) The tangent at the point (2,0) has equation y = 2x 4. Find the coordinates of the point where this tangent meets the curve again.



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Part	Marks	Level	Calc.	Content	Answer	U2 OC1
<i>(a)</i>	5	С	CN	C5	x + y = 1	2000 P2 Q1
<i>(b)</i>	5	С	CN	A23, A22, A21	(-1,-6)	
•2 •3 •4 •5 •6 •7 •8 •9	ss: knc pd: diff ss: knc ss: knc ic: stat ss: equ pd: arra ss: knc pd: pro ic: inte	erentiate ow that g ow that g e equ. o ate equa inge in s ow how cess	e correc gradient /-coord f line ations standard	tly f = f'(1) f = f(1) d form	• ¹ $y' =$ • ² $3x^2 - 6x + 2$ • ³ $y'(1) = -1$ • ⁴ $y(1) = 0$ • ⁵ $y - 0 = -1(x - 1)$ • ⁶ $2x - 4 = x^3 - 3x^2 + 2x$ • ⁷ $x^3 - 3x^2 + 4 = 0$ • ⁸ • ⁹ identify $x = -1$ from work • ¹⁰ $(-1, -6)$	

replacements

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x

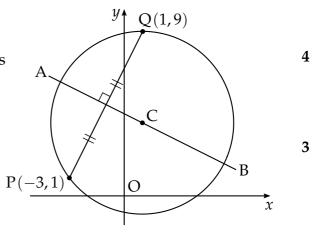
Ο x

y

- 2. (a) Find the equation of AB, the [SQA] perpendicular bisector of the line joing the points P(-3,1)and Q(1,9). PSfrag replacements
 - (b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the *y*-axis, determine the equation of the circle.
 - (c) The tangents at P and Q intersect at T.

Write down

- (i) the equation of the tangent at Q
- (ii) the coordinates of T.



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Part	Marks	Level	Calc.	Content	Answer	U2 OC4
<i>(a)</i>	4	С	CN	G7	x + 2y = 9	2000 P2 Q2
<i>(b)</i>	3	С	CN	G10	$(x-1)^2 + (y-4)^2 = 25$	
(C)	2	С	CN	G11, G8	(i) $y = 9$, (ii) T(-9,9)	
•2 •3 •4 •5 •6 •7 •8	ic: stat	cess gra ow how t e equ. o erpret "p cess rad e equ. o erpret di	dient of to find p f line parallel f ius f circle agram	PQ perp. gradient to y-axis"	• ¹ midpoint = $(-1,5)$ • ² $m_{PQ} = \frac{9-1}{1-(-1)}$ • ³ $m_{\perp} = -\frac{1}{2}$ • ⁴ $y - 5 = -\frac{1}{2}(x - (-1))$ • ⁵ $y_{C} = 4$ stated or implied by • • ⁶ radius = 5 or equiv. stated or implied by • ⁷ • ⁷ $(x - 1)^{2} + (y - 4)^{2} = 25$ • ⁸ $y = 9$ • ⁹ T= $(-9,9)$	•7

replacements

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[SQA]

3. f(x) = 3 - x and $g(x) = \frac{3}{x}, x \neq 0$.

(a) Find p(x) where p(x) = f(g(x)).

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(b) If
$$q(x) = \frac{3}{3-x}$$
, $x \neq 3$, find $p(q(x))$ in its simplest form

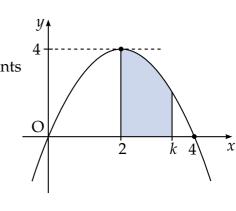
Part	Marks	Level	Calc.	Content	Answer	U1 OC2
<i>(a)</i>	2	С	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
<i>(b)</i>	2	С	CN	A4	x	
<i>(b)</i>	1	A/B	CN	A4		
 •¹ ic: interpret composite func. •² pd: process •³ ic: interpret composite func. •⁴ pd: process •⁵ pd: process 					•1 $f\left(\frac{3}{x}\right)$ stated or implied by •2 $3 - \frac{3}{x}$ •3 $p\left(\frac{3}{3-x}\right)$ stated or implied b •4 $3 - \frac{3}{\frac{3}{3-x}}$ •5 x	

[SQA] 4. The parabola shown crosses the *x*-axis at (0,0) and (4,0), and has a maximum at (2,4). PSfrag replacements

The shaded area is bounded by the parabola, the *x*-axis and the lines x = 2 and x = k.

- (*a*) Find the equation of the parabola.
- (*b*) Hence show that the shaded area, *A*, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}k^3 + \frac{16}{3}k^$$



2

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Part	Marks	Level	Calc.	Content	Answer	U2 OC2
<i>(a)</i>	2	С	CN	A19	$y = 4x - x^2$	2000 P2 Q4
(<i>b</i>)	3	С	CN	C16	proof	
• ² • ³ • ⁴	ic: stat pd: pro ss: knc pd: inte pd: proof	cess for ow to int egrate co	x ² coeff regrate prrectly		• ¹ $ax(x-4)$ • ² $a = -1$ • ³ $\int_{2}^{k} (function from (a))$ • ⁴ $-\frac{1}{3}x^{3} + 2x^{2}$ • ⁵ $-\frac{1}{3}k^{3} + 2k^{2} - (-\frac{8}{3} + 8)$	

replacements

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[SQA]	5. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \le x \le 360$.
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Part	Marks	Level	Calc.	Content	Answer U2 OC3
	5	A/B	CR	T10	60, 131.8, 228.2, 300 2000 P2 Q5
• ² • ³ • ⁴	pd: pro	cess w to/ar cess		$= 2\cos^2 x - 1$ rise quadratic	• ¹ $3(2\cos^2 x^\circ - 1)$ • ² $6\cos^2 x^\circ + \cos x^\circ - 2 = 0$ • ³ $(2\cos x^\circ - 1)(3\cos x^\circ + 2)$ • ⁴ $\cos x^\circ = \frac{1}{2}, x = 60, 30$ • ⁵ $\cos x^\circ = -\frac{2}{3}, x = 132, 228$

[SQA] 6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2} \left(x^2 + \frac{16}{x} \right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	A/B	CN	C11	<i>x</i> = 2	2000 P2 Q6
•2 •3 •4 • ⁵	ss: knc pd: pro ss: knc pd: dea pd: pro ic: che	cess ow to set l with <i>x</i> cess	f'(x) = -2	= 0	• ¹ $A'(x) =$ • ² $\frac{3\sqrt{3}}{2}(2x - 16x^{-2})$ or $3\sqrt{3}x$ • ³ $A'(x) = 0$ • ⁴ $-\frac{16}{x^2}$ or $-\frac{24\sqrt{3}}{x^2}$ • ⁵ $x = 2$ • ⁶ $\frac{x 2^- 2 2^+}{A'(x) -ve 0 +ve}$ so $x = 2$ is min.	$-24\sqrt{3}x^{-2}$

replacements

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PSfrag replacements

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y

Higher Mathematics

[SQA] 7. For what value of *t* are the vectors $\boldsymbol{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	2	С	CN	G27	t = 4	2000 P2 Q7
	ss: knc ic: inte			1	• ¹ $u.v = 2t - 20 + 3t$ • ² $u.v = 0 \Rightarrow t = 4$	

[SQA] 8. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate f'(4).

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	С	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		
• ¹ pd: differentiate power • ² pd: differentiate 2nd function • ³ pd: evaluate $f'(x)$					• ¹ $\frac{1}{2}(5x-4)^{-\frac{1}{2}}$ • ² ×5 • ³ $f'(4) = \frac{5}{8}$	

replacements

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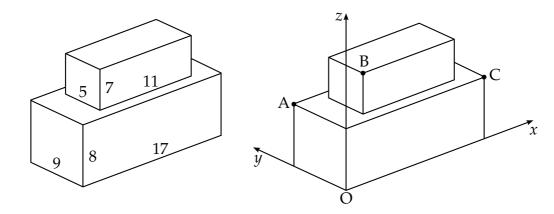
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PSfrag replacements of measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.



- (*a*) The point A has coordinates (0,9,8) and C has coordinates (17,0,8). Write down the coordinates of B.
- (*b*) Calculate the size of angle ABC.

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
<i>(a)</i>	1	С	CN	G22	B(3,2,15)	2000 P2 Q9
<i>(b)</i>	6	С	CR	G28	92·5°	
• ² • ³ • ⁴ • ⁵ • ⁶	ic: inte ss: knc pd: pro pd: pro pd: pro pd: pro pd: eva	ow to use cess vec cess vec cess leng cess scal	e scalar tors tors gths lar proc	luct	• ¹ B= (3, 2, 15) treat $\begin{pmatrix} 3\\2\\15 \end{pmatrix}$ • ² cos A $\widehat{B}C = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{ \overrightarrow{BA} \overrightarrow{BC} }$ • ³ $\overrightarrow{BA} = \begin{pmatrix} -3\\7\\-7 \end{pmatrix}$ • ⁴ $\overrightarrow{BC} = \begin{pmatrix} 14\\-2\\-7 \end{pmatrix}$ • ⁵ $ \overrightarrow{BA} = \sqrt{107}, \overrightarrow{BC} = \sqrt{24}$ • ⁶ $\overrightarrow{BA}.\overrightarrow{BC} = -7$ • ⁷ A $\widehat{B}C = 92.5^{\circ}$	

replacements

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- x
- y

PStrag replacements

Higher Mathematics

2

[SQA] 10. Find
$$\int \frac{1}{(7-3x)^2} dx$$
.

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\frac{1}{3(7-3x)}+c$	2000 P2 Q10
• ¹ • ²	pd: inte pd: dea	egrate fu 1 with fu	nction inction	of function	• ¹ $\frac{1}{-1}(7-3x)^{-1}$ • ² $\times \frac{1}{-3}$	

replacements

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Higher Mathematics

PSfrag replacements

x y

- [SQA] 11. The results of an experiment give rise to the graph shown.
 - (*a*) Write down the equation of the line in terms of *P* and *Q*.

It is given that $P = \log_e p$ and $Q = \log_e q$.

(b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b.

		T 1	<u> </u>			•	
Part	Marks	Level	Calc.	Content		Answer	U3 OC3
<i>(a)</i>	2	A/B	CR	G3		P = 0.6Q + 1.8	2000 P2 Q11
<i>(b)</i>	4	A/B	CR	A33		a = 6.05, b = 0.6	
• ² • ³ • ⁴ • ⁵	$x \log y$	e equ. o erpret str ow how	f line raight li 7 to de to expre	al with x of ess number as	M	• ¹ $m = \frac{1 \cdot 8}{3} = 0.6$ • ² $P = 0.6Q + 1.8$ ethod 1 • ³ $\log_e p = 0.6 \log_e q + 1.8$ • ⁴ $\log_e q^{0.6}$ • ⁵ $\log_e 6.05$ • ⁶ $p = 6.05q^{0.6}$ ethod 2 $p = \ln aq^b$ • ³ $\ln p = \ln a + b \ln q$ • ⁴ $\ln p = 0.6 \ln q + 1.8$ state $by \bullet^5 \text{ or } \bullet^6$ • ⁵ $\ln a = 1.8$ • ⁶ $a = 6.05, b = 0.6$	ed or implied

[END OF QUESTIONS]

replacements

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- x
- y

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