Prelim Examination 2002 / 2003
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

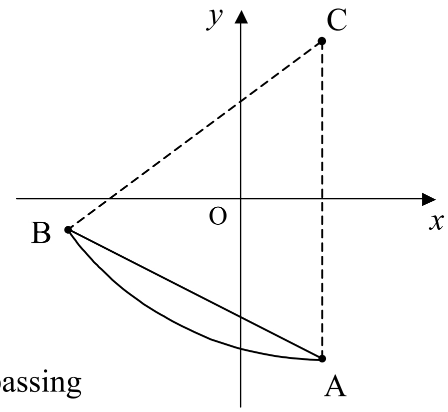
The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

All questions should be attempted

1. The diagram shows an arc AB of the circle with its centre at C. The coordinates of A and B are $(4, -7)$ and $(-8, -1)$ respectively.



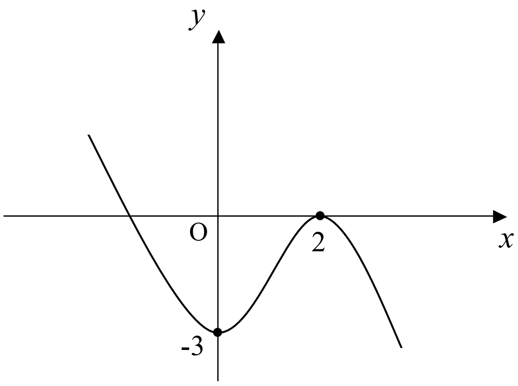
(a) Find the equation of the perpendicular bisector of the chord AB. **4**

(b) Hence establish the coordinates of C, given that C is vertically above A. **2**

(c) Write down the equation of the circle, centre C, passing through the points A and B. **2**

2. For what value(s) of p does the equation $(4p+1)x^2 - 3px + 1 = 0$ have equal roots? **4**

3. The diagram below shows part of the the graph of $y = g(x)$.
The function has stationary points at $(0, -3)$ and $(2, 0)$ as shown.

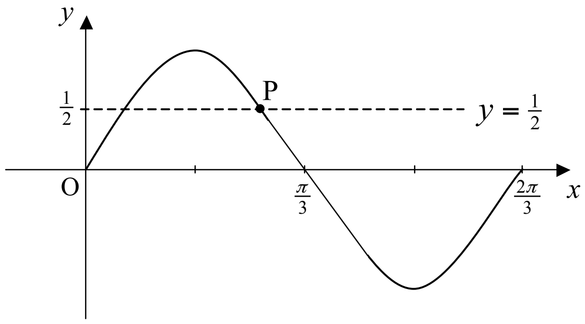


Sketch the graph of the related function $y = g(-x) + 3$. **3**

4. *(a)* For what value of k is $x + 2$ a factor of $x^3 - x^2 + 2kx - 8$? **3**

(b) Hence fully factorise the expression $x^3 - x^2 + 2kx - 8$ when k takes this value. **2**

5. The daigram shows the graph of $y = \sin 3x$, for $0 \leq x \leq \frac{2\pi}{3}$, and the line with equation $y = \frac{1}{2}$.



Establish the coordinates of the point P. **3**

6. A curve has as its derivative $\dfrac{dy}{dx} = 8x - 3$.

Given that the point $(1, -3)$ lies on this curve, express y in terms of x. **4**

7. A sequence of numbers is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.

(a) Given that $U_2 = 70$, $U_3 = 65$ and $U_4 = 62 \cdot 5$, find **algebraically**, the values of k and c. **3**

(b) Hence find the limit of this sequence. **2**

(c) Express the difference between the fifth term and the limit of this sequence as a percentage of the limit, correct to the nearest percent. **2**

8. A function is given as $f(x) = 3x^3 - 9x^2 + 27x$ and is defined on the set of real numbers.

(a) Show that the derivative of this function can be expressed in the form $f'(x) = a\left[(x-b)^2 + c\right]$ and write down the values of *a*, *b* and *c*. **4**

(b) Explain why this function has no stationary points and is in fact increasing for **all** values of x. **2**

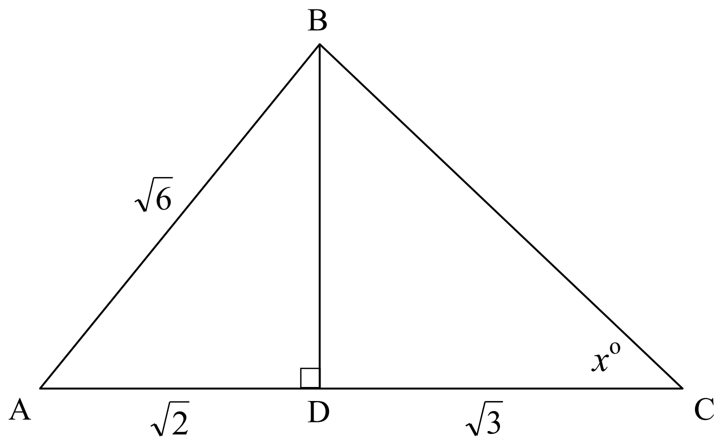
9. The functions $f(x) = x^2 - 9$ and $h(x) = 3 + 2x$ are defined on the set of real numbers.

(a) Evaluate $h\big(f(3)\big)$. **1**

(b) Find an expression , in its simplest form, for $f\big(h(x)\big)$. **2**

(c) For what value(s) of x does $f\big(h(x)\big) = f(x)$? **2**

10. In the diagram below $\text{AB} = \sqrt{6}$, $\text{AD} = \sqrt{2}$ and $\text{DC} = \sqrt{3}$. Angle $\text{BCD} = x^\circ$.



(a) Show clearly that $\tan x^\circ = \dfrac{2}{\sqrt{3}}$. **2**

(b) Hence show that $\sin 2x^\circ = \frac{4}{7}\sqrt{3}$. **3**

[END OF QUESTION PAPER]

Prelim Examination 2002 / 2003
(Assessing Units 1 & 2)

MATHEMATICS
Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

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FORMULAE LIST

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Trigonometric formulae:

$$\begin{aligned}
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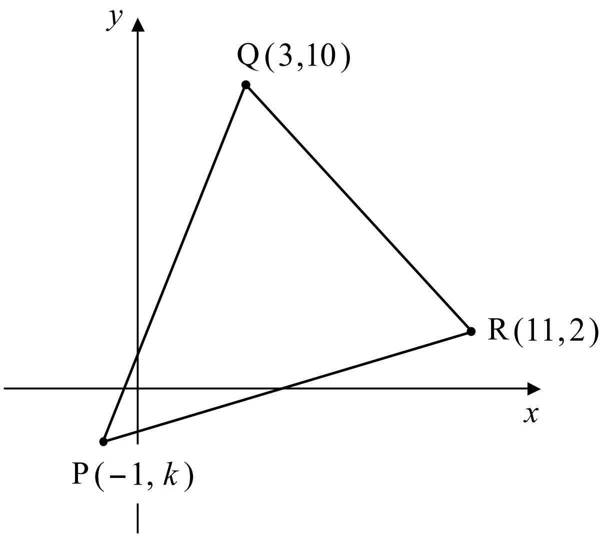
\end{aligned}$$

All questions should be attempted

1. Triangle PQR has vertices $P(-1, k)$, $Q(3,10)$ and $R(11,2)$ as shown.



(a) Given that the gradient of side PQ is 3, find the equation of PQ. **2**

(b) Hence find k, the y-coordinate of vertex P. **1**

(c) Find the equation of the median from P to QR. **3**

(d) Show that this median is at right-angles to side QR.
What type of triangle is PQR? **3**

2. Evaluate $f'(4)$ when $f(x) = \dfrac{x - 2\sqrt{x}}{x^2}$. **5**

3. Two functions are defined as $f(x) = ax^2 - 2b$ and $h(x) = \dfrac{2x - 6b}{3}$, where a is a constant .

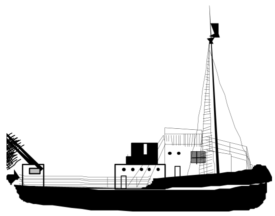
(a) Given that $f(2) = h(2)$, show clearly that $a = \frac{1}{3}$. **3**

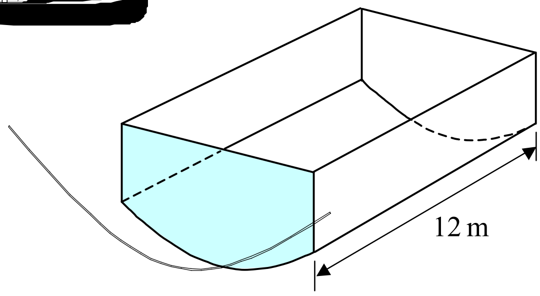
(b) If $b = px - 6$, show that $f(x) = \frac{1}{3}x^2 - 2px + 12$. **1**

(c) Hence state the values of p for which $f(x) = 0$ has no real roots. **4**

4. A fishing boat's fish hold is in the shape of the prism shown opposite.

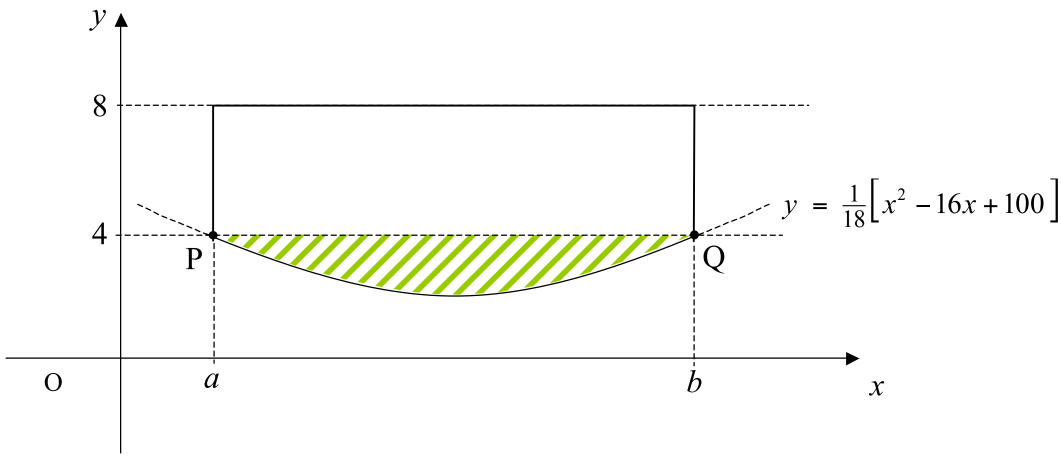
The length of the hold is 12 metres.





The cross-section of the hold is represented in the coordinate diagram below.

All the units are in metres, with the floor of the hold represented by the curve $y = \frac{1}{18}\left[x^2 - 16x + 100\right]$.



(a) Find the values of a and b, the x-coordinates of P and Q. **3**

(b) Show clearly that the area between the line PQ and the curve $y = \frac{1}{18}\left[x^2 - 16x + 100\right]$ can be calculated by evaluating the integral: $A = \frac{1}{18}\int_a^b (16x - x^2 - 28)\, dx$. **2**

(c) Calculate this area in square metres. **4**

(d) Hence calculate the **volume** of the hold, in cubic metres, by first establishing the **total** cross-sectional area of the hold. **2**

5. A curve has as its equation $y = \frac{1}{32}(x^4 + 48)$.

(a) At what point on the curve is the gradient of the tangent equal to 1? **4**

(b) Write down the equation of this tangent. **1**

6. Certain radioisotopes are used as *tracers*, to track down diseased tissue within the body, and then be absorbed, to act as a long-term radio-therapy treatment. Their passage through the body and mass is ascertained by means of a Geiger-Müller counter.

During trials of a particular radioisotope the following information was obtained.

- *the isotope loses* 3% *of its mass every hour*
- *the maximum recommended mass in the bloodstream is* 165mgs
- 100mgs *is the smallest mass detectable by the Geiger-Müller counter*

(a) An intial dose of 150mgs of the isotope is injected into a patient.

Would the mass remaining after 12 hours still be detectable by the Geiger-Müller counter? **3**

Your answer must be accompanied by appropriate working.

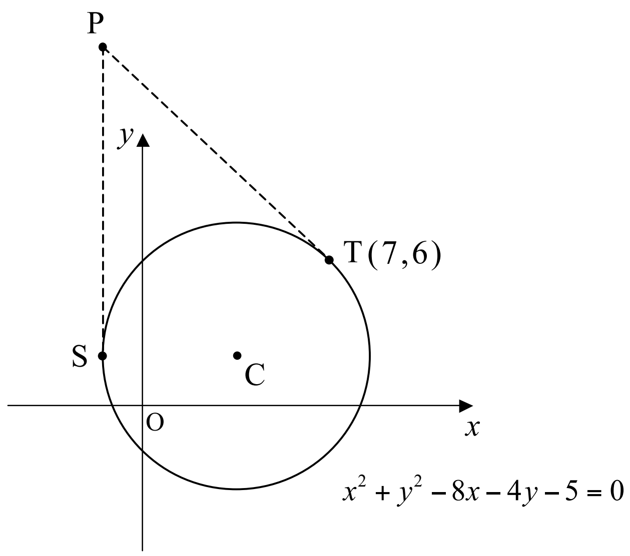
(b) **After the initial dose**, top-up injections of 50mgs are given every 12 hours.

Comment on the long-term suitabilty of this plan. **4**

Your answer must be accompanied by appropriate working.

7. The diagram shows a circle, centre C, with equation $x^2 + y^2 - 8x - 4y - 5 = 0$.

Two common tangents have been drawn from the point P to the points S and T$(7,6)$ on the circle.



(a) Find the centre and radius of the circle. **2**

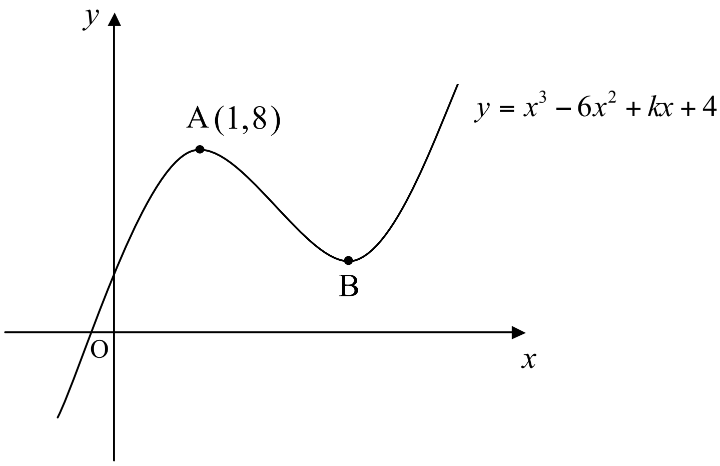
(b) Hence find the equation of the tangent PT. **3**

(c) Given now that the tangent PS is parallel to the y-axis, determine the coordinates of S and P. **3**

8. Solve algebraically the equation

$$3\cos 2x^o = 7\cos x^o, \text{ where } 0 \le x < 360.$$ **5**

9. The curve below has as its equation $y = x^3 - 6x^2 + kx + 4$, where k is a constant.
$A(1,8)$ is a stationary point.



(a) Using the x-coordinate of A, to help you, find the value of k. **4**

(b) Hence find the coordinates of the other stationary point at B. **3**

[END OF QUESTION PAPER]