	Give 1 mark for each •	**Illustration(s) for awarding each mark**

1.	**(a) ans:** $y = 2x$ **4 marks**
	
	 •1 For mid-point of AB
	 •2 For gradient of AB
	 •3 For gradient of perpendicular
	 •4 For equation of bisector
	
	 (b) ans: C(4,8) **2 marks**
	
	 •1 For knowing to use x_A
	 •2 For sub. in equation to answer
	
	 (c) ans: $(x-4)^2 + (y-8)^2 = 225$ **2 marks**
	
	 •1 For finding radius
	 •2 For sub. in standard equ. to answer
		(a) •1 M(-2,-4)
		        •2 $m_{AB} = -\frac{1}{2}$
		        •3 $m_1 \times m_2 = -1$, $m_{bis.} = 2$
		        •4 $y + 4 = 2(x + 2) \Rightarrow y = 2x$
		
		 (b) •1 $x_C = x_A = 4$
		        •2 $y = 2(4) = 8 \therefore C(4,8)$
		
		 (c) •1 CA is vertical = radius = 15 units
		        •2 C(4,8) , $r = 15$ in equ…….
		              $(x-a)^2 + (y-b)^2 = r^2$

2.	**ans:** $p = -\frac{2}{9}$, $p = 2$ **4 marks** •1 For discr. = 0 (stated or implied) •2 For selecting a, b and c •3 Substituting and simplifying •4 Factorising to answers	•1 For equal roots $b^2 - 4ac = 0$ •2 $a = 4p + 1$, $b = -3p$, $c = 1$ •3 $(-3p)^2 - (4(4p+1).1) = 0$ $\Rightarrow 9p^2 - 16p - 4 = 0$ •4 $(9p + 2)(p - 2) = 0 \Rightarrow p = -\frac{2}{9}$ or 2
3.	**ans:** diagram **3 marks** •1 For reflection in y-axis •2 For translating …. 3 up •3 For annotating final sketch	
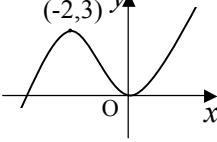
4.	**(a) ans:** $k = -5$ **3 marks**
	
	 •1 For setting up synthetic division
	 •2 For performing synth. div. correctly
	 •3 For solving to zero and finding k
	
	 (b) ans: $(x + 2)(x - 4)(x + 1)$ **2 marks**
	
	 •1 For sub. k into the quotient
	 •2 For factorising quad. to answer
		(a) •1 $\begin{array}{r|rrrr} -2 & 1 & -1 & 2k & -8 \\ & & & & \\ \hline & & & & \end{array}$
		
		        •2 $\begin{array}{r|rrrr} -2 & 1 & -1 & 2k & -8 \\ & & -2 & 6 & -4k-12 \\ \hline & 1 & -3 & 2k+6 & -4k-20 \end{array}$
		
		        •3 $-4k - 20 = 0 \therefore k = -5$
		
		 (b) •1 If k = -5 then quo. $\Rightarrow x^2 - 3x - 4$
		        •2 $x^2 - 3x - 4 \Rightarrow (x - 4)(x + 1)$

5.	**ans:** $P(\frac{5\pi}{18}, \frac{1}{2})$ **3 marks** •1 For equating •2 Angles for sin3x (choosing quad.) •3 Answer	•1 $\sin 3x = \frac{1}{2}$ •2 quad ½ $\therefore 3x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ •3 $x = \frac{5\pi}{18}$ ……. $P(\frac{5\pi}{18}, \frac{1}{2})$
6.	**ans:** $y = 4x^2 - 3x - 4$ **4 marks** •1 For setting up integral •2 For integrating •3 For substituting •4 Correct answer	•1 $y = \int 8x - 3 \ dx$ •2 $y = \frac{8x^2}{2} - 3x + c \Rightarrow 4x^2 - 3x + c$ •3 $-3 = 4(1^2) - 3(1) + c$ •4 $c = -4$ to answer

	Give 1 mark for each •	**Illustration(s) for awarding each mark**

7.	**(a) ans:** $k = \frac{1}{2}$, $c = 30$ **3 marks**
	
	•1 Setting up a system of equ.
	•2 Finding k
	•3 Finding c
	
	(b) ans: 60 **2 marks**
	
	•1 Knowing how to find limit
	•2 Calculating limit
	
	(c) ans: 2% **2 marks**
	
	•1 For calculating U_5
	•2 For percentage calculation
		(a) •1 $U_3 = kU_2 + c \implies 65 = 70k + c$
		           $U_4 = kU_3 + c \implies 62 \cdot 5 = 65k + c$
		    •2 $5k = 2 \cdot 5 \implies k = \frac{1}{2}$
		    •3 $c = 30$
		
		(b) •1 $L = \dfrac{b}{1-a}$, or equivalent
		
		    •2 $L = \dfrac{30}{1 - \frac{1}{2}} = 60$
		
		(c) •1 $U_5 = \frac{1}{2}(62 \cdot 5) + 30 = 61 \cdot 25$
		
		    •2 $\dfrac{1 \cdot 25}{60} \times 100 \approx 2\%$

8.	**(a) ans:** $a = 9$, $b = 1$, $c = 2$ **4 marks**
	
	•1 For differentiating
	•2 For common factor
	•3 For the square $(x-1)^2$
	•4 For $c = 2$ (no need to list a,b and c)
	
	(b) ans: explanation **2 marks**
	
	•1 For statement on solving to zero
	•2 Derivative always +ve, always increasing
		(a) •1 $f'(x) = 9x^2 - 18x + 27$
		    •2 $9\lfloor x^2 - 2x + 3 \rfloor$
		    •3 $9\lfloor (x-1)^2 \dots\dots \rfloor$
		    •4 $9[\dots\dots -1 + 3] = 9\lfloor (x-1)^2 + 2 \rfloor$
		     (no marks off if $b = -1$)
		
		(b) •1 $(x-1)^2 + 2 = 0$ has no solution
		    •2 $(x-1)^2 + 2$ always +ve , always incr.

9.	**(a) ans:** 3 **1 mark**
	
	•1 answer
	
	(b) ans: $f(h(x)) = 12x + 4x^2$ **2 marks**
	
	•1 For substitition
	•2 Simplifying to answer
	
	(c) ans: $x = -3$, $x = -1$ **2 marks**
	
	•1 For equating
	•2 For solving to answers
		(a) •1 $f(3) = 0$, $h(0) = 3$ (or equiv.)
		
		(b) •1 $f(h(x)) = (3 + 2x)^2 - 9$
		    •2 $f(h(x)) = 12x + 4x^2$
		
		(c) •1 $12x + 4x^2 = x^2 - 9$
		    •2 $3x^2 + 12x + 9 = 0$ … x = -3 or -1
		        $3(x + 3)(x + 1) = 0$

10.	**(a) ans:** proof **2 marks**
	
	•1 Using Pyth. to find BD
	•2 Lifting $\tan x^o$ from triangle
	
	(b) ans: proof **3 marks**
	
	•1 For using double angle expansion
	•2 For hypotenuse = $\sqrt{7}$
	•3 For sub. & manipulation to answer
		(a) •1 $BD^2 = (\sqrt{6})^2 - (\sqrt{2})^2$
		        $BD^2 = 4 \therefore BD = 2$
		
		    •2 $\tan x^o = \dfrac{BD}{DC} = \dfrac{2}{\sqrt{3}}$
		
		(b) •1 $\sin 2x = 2\sin x \cos x$
		    •2 $BC^2 = 2^2 + (\sqrt{3})^2 = 4 + 3 = 7$, $BC = \sqrt{7}$
		    •3 $\sin 2x = 2 \times \frac{2}{\sqrt{7}} \times \frac{\sqrt{3}}{\sqrt{7}} = \frac{4}{7}\sqrt{3}$
		
		          Total 50 marks

	Give 1 mark for each •		**Illustration(s) for awarding each mark**
1.	**(a) ans:** $y = 3x + 1$ **2 marks** •1 For using Q •2 For answer	(a)	•1 Q(3,10) , $m = 3$ •2 $y - 10 = 3(x - 3)$
	(b) ans: $k = -2$ **1 mark** •1 For subst. to answer	(b)	•1 $y = 3(-1) + 1 = -2 = k$
	(c) ans: $y = x - 1$ **3 marks** •1 For mid-point of QR •2 For calculating gradient •3 answer	(c)	•1 $M_{QR}(7,6)$ •2 $m_{med} = \dfrac{6 - (-2)}{7 - (-1)} = 1$ •3 $y - 6 = 1(x - 7)$
	(d) ans: proof , isosceles **3 marks** •1 For knowing $m_1.m_2 = -1$ •2 For calculation to prove •3 For isosceles (no explanation required)	(d)	•1 If perp… $m_{QR} \times m_{Pm} = -1$ (*stated or implied*) •2 $-1 \times 1 = -1$ •3 isosceles
2.	**ans:** $f'(4) = \dfrac{1}{32}$ **5 marks** •1 For preparing to differentiate •2 Differentiating first term •3 Differentiating second term •4 Subst. x = 4 in derivative •5 Calculating answer		•1 $f(x) = x^{-2}(x - 2x^{\frac{1}{2}})$ $= x^{-1} - 2x^{-\frac{3}{2}}$ •2 $f'(x) = -x^{-2}$ .... •3 $f'(x) = ... 3x^{-\frac{5}{2}}$ •4 $f'(x) = -\dfrac{1}{4^2} + \dfrac{3}{4^{\frac{5}{2}}}$ •5 $f'(4) = -\dfrac{2}{32} + \dfrac{3}{32} = \dfrac{1}{32}$
3.	**(a) ans:** proof **3 marks** •1 For sub 2 in f and h •2 For equating •3 For solving to required answer	(a)	•1 $f(2) = 4a - 2b$, $h(2) = \dfrac{4 - 6b}{3}$ •2 $4a - 2b = \dfrac{4 - 6b}{3}$ •3 $3(4a - 2b) = 4 - 6b$ $12a - 6b = 4 - 6b$ ....$a = \frac{1}{3}$
	(b) ans: proof **1 mark** •1 For sub. for a and b and adjusting to required answer	(b)	•1 $f(x) = \frac{1}{3}x^2 - 2(px - 6)$ $f(x) = \frac{1}{3}x^2 - 2px + 12$
	(c) ans: $-2 < p < 2$ **4 marks** •1 For discr. statement (or implied) •2 For values of a, b and c •3 For subst. and factorising •4 For final statement (worded ans. o.k.)	(c)	•1 for no real roots $b^2 - 4ac < 0$ •2 $a = \frac{1}{3}$, $b = -2p$, $c = 12$ •3 $(-2p)^2 - (4.\frac{1}{3}.12) < 0$ $4(p - 2)(p + 2) < 0$ •4 p has to lie between -2 and 2

	Give 1 mark for each ●	**Illustration(s) for awarding each mark**

4.	**(a) ans:** $a = 2$, $b = 14$ **3 marks**
	
	 ●1 For sub. 4 for y in order to solve
	 ●2 manipulating equation to zero
	 ●3 factorising and answers
	
	 (b) ans: proof **2 marks**
	
	 ●1 Strategy of line minus curve
	 ●2 Constant out + tidy to required answer
	
	 (c) ans: 16 m^2 **4 marks**
	
	 ●1 For integrating (all 3 terms)
	 ●2 For substituting
	 ●3 For simplifying each part
	 ●4 For calculating answer
	
	 (d) ans: 768 m^3 **2 marks**
	 ●1 For total surface area
	 ●2 For volume
		(a) ●1 $4 = \frac{1}{18}(x^2 - 16x + 100)$
		      ●2 $x^2 - 16x + 28 = 0$
		      ●3 $(x-2)(x-14) = 0 \Rightarrow x = 2$, $x = 14$
		
		 (b) ●1 $A = \int 4 - \frac{1}{18}(x^2 - 16x - 100)\,dx$
		      ●2 $A = \frac{1}{18}\int 72 - (x^2 - 16x + 100)\,dx$
		           $A = \frac{1}{18}\int_{2}^{14}(16x - x^2 - 28)\,dx$
		
		 (c) ●1 $A = \frac{1}{18}\left[8x^2 - \frac{x^3}{3} - 28x\right]_{2}^{4}$
		      ●2 ....
		 $A = \frac{1}{18}\left[(8(14^2) - \frac{14^3}{3} - 28(14)) - (8(2^2) - \frac{2^3}{3} - 28(2))\right]$
		      ●3 $A = \frac{1}{18}[1200 - 912]$
		      ●4 $A = \frac{1}{18}[288] = 16$
		
		 (d) ●1 $A = 16 + (4 \times 12) = 64\ m^2$
		      ●2 $V = 64 \times 12 = 768\ m^3$

5.	**(a) ans:** $(2 , 2)$ **4 marks**
	
	 ●1 For knowing to differentiate
	 ●2 For differentiating correctly
	 ●3 For solving deriv. to 1 to get $x = 2$
	 ●4 For y-coordinate
	
	 (b) ans: $y = x$ **1 mark**
	
	 ●1 For equation of tangent
		(a) ●1 $\frac{dy}{dx} = m = 1$, or equiv. (stated or imp.)
		
		       ●2 $\frac{dy}{dx} = \frac{1}{8}x^3$
		       ●3 $\frac{1}{8}x^3 = 1 \Rightarrow x^3 = 8 \therefore x = 2$
		       ●4 $y = \frac{1}{32}(2^4) + \frac{3}{2} = 2$ (or equivalent)
		 (b) ●1 $y - 2 = 1(x - 2) \Rightarrow y = x$

6.	**(a) ans:** Yes , $104 \cdot 08 > 100$ **3 marks**
	
	 ●1 For taking $a = 0 \cdot 97$
	 ●2 For calculation
	 ●3 For consistent answer
	
	 (b) ans: Plan o.k., over the long-term
	    between $113 \cdot 3$ *and* $163 \cdot 3$ mgs **4 marks**
	 ●1 For attempting lines of working
	 ●2 For finding the limit
	 ●3 For being aware of the lower limit as
	      well as the upper limit
	 ●4 Consistent comment on findings
		(a) ●1 $a = 0 \cdot 97$
		      ●2 $U_{12} = (0 \cdot 97)^{12} \times 150 = 104 \cdot 08$
		      ●3 Yes , since $U_{12} > 100$
		 (b) ●1 $U_1 \rightarrow U_5$ below ....
		        154·08, 156·91, 158·87, 160·23, 161·17
		      ●2 $L = \dfrac{50}{1 - (0 \cdot 97)^{12}} = 163 \cdot 31$
		      ●3 lower limit $= 163 \cdot 31 - 50 = 113 \cdot 31$
		      ●4 Over the long-term the amount present
		        would always be between 113·31 and
		        163·31 which is ideal.
		 ** *This is a guide - this questions marks allocation*
		    may be altered.

	Give 1 mark for each •	**Illustration(s) for awarding each mark**

7.	**(a) ans:** C(4,2) , $r = 5$ **2 marks**
	
	•1 For centre
	•2 For radius
	
	(b) ans: $4y + 3x = 45$ (or equiv.) **3 marks**
	
	•1 For gradient of radius
	•2 For gradient of tangent
	•3 For sub. using m_{tan} & T(7,6)
	
	(c) ans: S(-1,2) , P(-1,12) **3 marks**
	
	•1 For $x_c - 5 = x_s = -1$, then coord. of S
	•2 For knowing to sub. $x = -1$ into equat.
	•3 For coordinates of P
		(a) •1 C(4,2)
		     •2 $r = \sqrt{4^2 + 2^2 - (-5)} = \sqrt{25} = 5$
		
		(b) •1 $m_{CT} = \dfrac{6-2}{7-4} = \dfrac{4}{3}$
		     •2 $m_{tan} = -\frac{3}{4}$
		     •3 $y - 6 = -\frac{3}{4}(x - 7)$
		
		(c) •1 $4 - 5 = -1 \therefore S(-1,2)\ldots$ same y as C
		     •2 x-coordinate of P is same as S. Also on line $4y + 3x = 45$.
		         $4y + 3(-1) = 45$
		     •3 $y = 12 \ \therefore \ P(-1,12)$

8.	**ans:** $109 \cdot 5^\circ$, $250 \cdot 5^\circ$ **5 marks** •1 For substituting double angle formula •2 For re-arranging to zero and factorising •3 For roots & discarding one root •4 First angle (<u>or</u> give this mark for $70 \cdot 5^\circ$) •5 For related angles	•1 $3(2\cos^2 x - 1) - 7\cos x = 0$ •2 $6\cos^2 x - 7\cos x - 3 = 0$ $(3\cos x + 1)(2\cos x - 3) = 0$ •3 $\cos x = -\frac{1}{3}$ or $\cos x = \frac{3}{2}$ (discard) •4 $109 \cdot 5^\circ$ (70.5°) •5 $109 \cdot 5^\circ$ or $250 \cdot 5^\circ$

9.	**(a) ans:** $k = 9$ **4 marks**
	
	•1 For knowing that at A the deriv. = 0
	•2 For differentiating
	•3 For solving deriv = 0 and making x = 1
	•4 Calculating k
	
	(b) ans: B(3,4) **3 marks**
	
	•1 Solving deriv. to zero when $k = 9$
	•2 Finding x-coordinate of B
	•3 Sub. to find y-coordinate of B
		(a) •1 A is a stat. point $\therefore \dfrac{dy}{dx} = 0 \ at \ A$
		     •2 $\dfrac{dy}{dx} = 3x^2 - 12x + k$
		     •3 $3(1^2) - 12(1) + k = 0$
		     •4 $k = 9$
		
		(b) •1 solve $3x^2 - 12x + 9 = 0$
		     •2 $3(x - 1)(x - 3) = 0 \Rightarrow x = 3$ for B
		     •3 $y = 3^3 - 6(3^2) + 9(3) + 4 = 4$

		Total 65 marks