Prelim Examination 2001 / 2002
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$
$$\sin 2A = 2 \sin A \cos A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

1.

2. A sequence is defined by the recurrence relation $U_{n+1} = 0 \cdot 6U_n + 8$.

(a) Explain why this sequence has a limit as $n \to \infty$. **(1)**

(b) Find the limit of this sequence. **(2)**

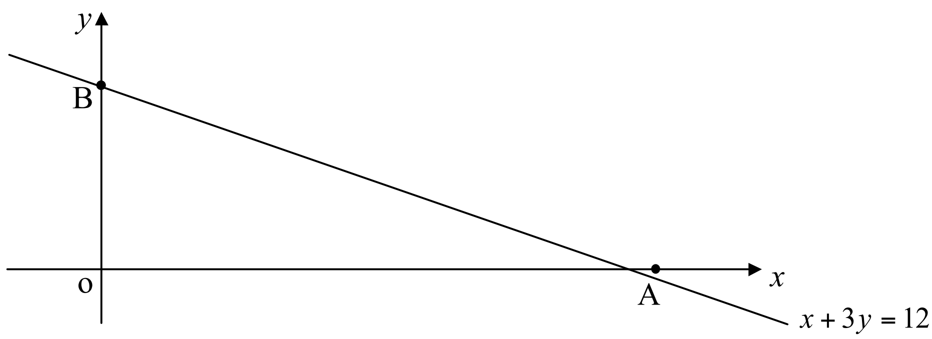
(c) Given that $L - U_1 = 3$, where L is the limit of this sequence, establish the value of U_0 , the intitial value. **(3)**

3. A function is defined on a suitable domain as $f(x) = \frac{1}{\sqrt{x}}\left(x^2 - x\right)$.

(a) Differentiate f with respect to x, expressing your answer with positive indices. **(4)**

(b) Hence find the rate of change of f when $x = 4$. **(2)**

4. The line with equation $x + 3y = 12$ meets the x and the y axes at the points A and B respectively.



Find the equation of the perpendicular bisector of AB. **(5)**

5. Two functions f and g are defined on the set of real numbers as follows :

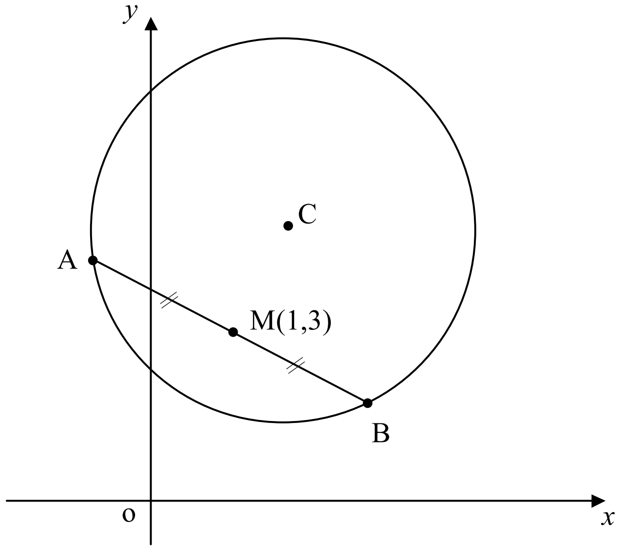
$$f(x) = 8 - 2x \quad , \quad g(x) = \tfrac{1}{2}(x + 8).$$

(a) Evaluate $f\big(g(2)\big)$. **(1)**

(b) Find an expression , in its simplest form, for $g\left(f(x)\right)$. **(2)**

(c) Hence prove that $f^{-1}(x) = \frac{1}{2}\left[g\big(f(x)\big)\right]$. **(3)**

6. The circle below, centre C, has as its equation $x^2 + y^2 - 4x - 10y + 19 = 0$.
M(1,3) is the mid-point of the chord AB.



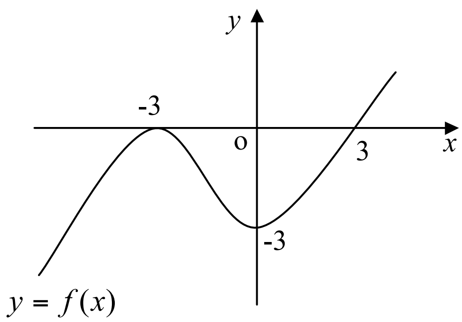
(a) Write down the coordinates of C, the centre of the circle. **(1)**

(b) Show that the equation of the chord AB can be written as $x = 7 - 2y$. **(3)**

(c) Hence find algebraically the coordinates of A and B. **(4)**

7. Part of the graph of $y = f(x)$ is shown in the diagram.

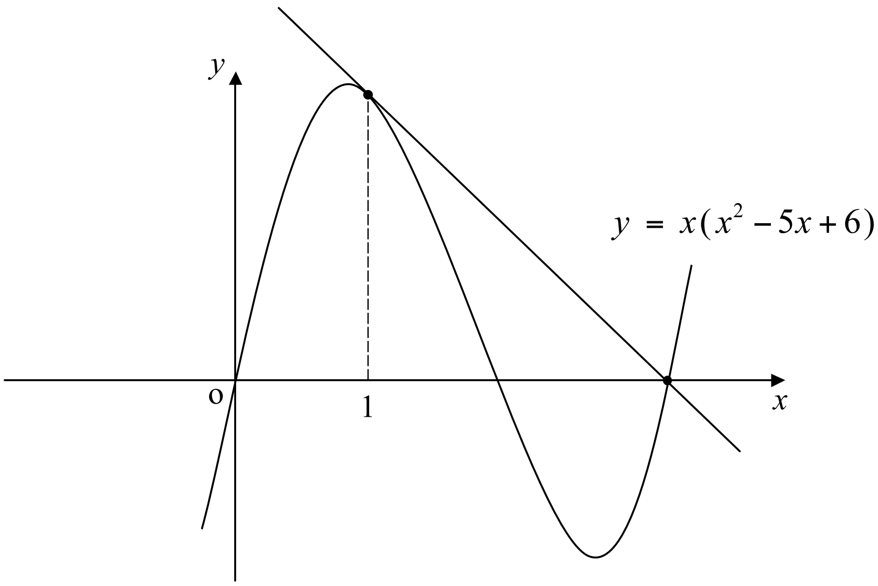
Sketch the graph of the related function $y = -f(x) - 3$, showing all the relevant points. **(3)**



8. Find a given that $\int_a^{2a} (10 - 2x)\ dx = 8$, where a is a positive whole number. **(5)**

9. Part of the graph of the curve $y = x(x^2 - 5x + 6)$ is shown in the diagram.

The tangent to the curve at the point where $x = 1$ is also shown.



(a) Find the equation of the tangent to the curve at the point where $x = 1$. **(4)**

(b) Show that this tangent also passes through one of the points where the curve crosses the x-axis. **(2)**

[END OF QUESTION PAPER]

Prelim Examination 2001 / 2002
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

1. **Calculators may be used in this paper.**
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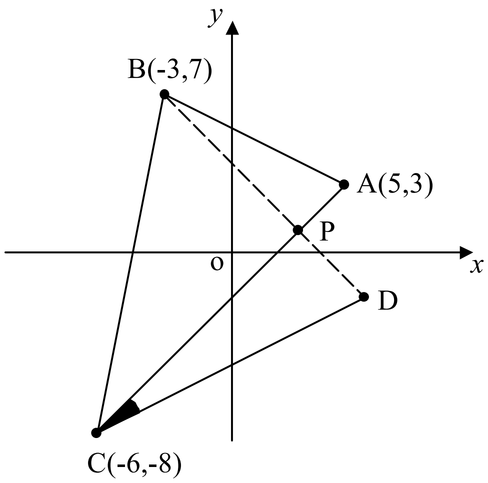
4. **This examination paper contains questions graded at all levels.**

1. Triangle ABC has vertices A(5,3), B(-3,7) and C(-6,-8) as shown.
The altitude through B meets AC at P.



(a) Find the equation of side AC and the equation of the altitude BP. **(4)**

(b) Hence find the coordinates of P. **(3)**

(c) BP is produced in such a way that PD = $\frac{1}{2}$ BP. Establish the coordinates of D. **(1)**

(d) By considering gradients, calculate the size of angle DCP to the nearest degree. **(3)**

2. Solve algebraically the equation

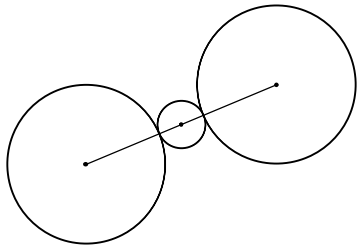
$$\sin x^\circ - 3\cos 2x^\circ + 2 = 0, \qquad 0 \le x < 360. \qquad \textbf{(5)}$$

3. Two functions are defined as $f(x) = (x+2)(x+1)$ and $g(x) = x(x-2)$.

(a) Given that $h(x) = f\big(g(x)\big)$, show clearly that $h(x) = x^4 - 4x^3 + 7x^2 - 6x + 2$. **(3)**

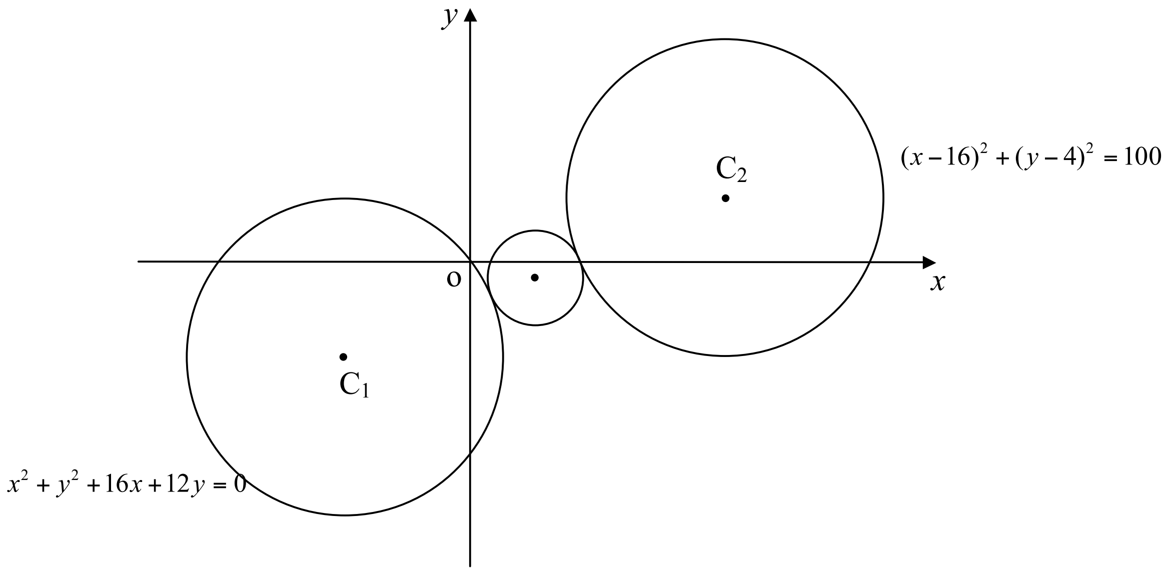
(b)

4. Three wheels are positioned in such a way that their centres are collinear.



When placed on a set of rectangular axes the equations of the two larger circles are

$x^2 + y^2 + 16x + 12y = 0$ and $(x-16)^2 + (y-4)^2 = 100$, as shown.

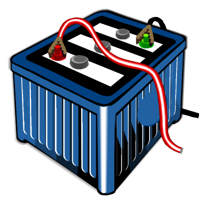


(a) Write down the coordinates of the two centres C_1 and C_2. **(2)**

(b) Calculate the radii of the two larger circles and the distance between the two centres C_1 and C_2.
(4)

(c) Hence establish the centre and radius of the small circle and write down its equation. **(3)**

5. A new 24 volt lead acid battery is being tested as a possible power source for a battery-powered wheelchair.



The battery, which has an initial capacity of 20 Ah (ampere hours), is being **artificially** drained over a 12 hour period to represent 1 month of use and an operating distance of 300 miles.
It has been found that by the end of each draining period the battery has lost 24% of its initial capacity at the start of that session.

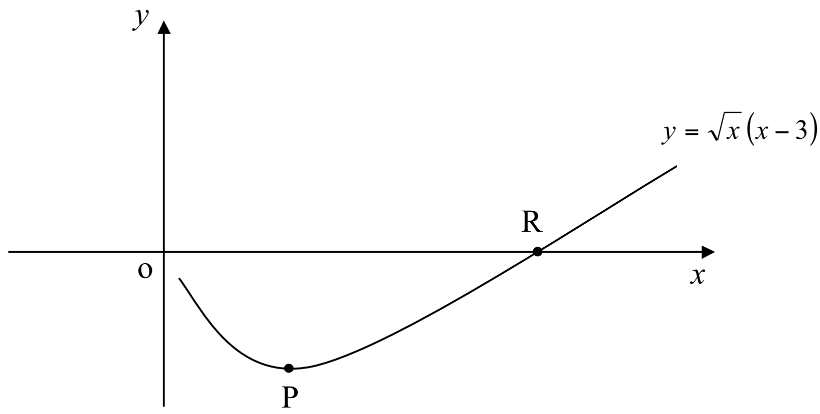
After each 12 hour draining period the battery is hooked up to a super-charger for 12 hours which allows it to regain 3 Ah of capacity.

(a) What is the capacity of the battery immediately after its fifth re-charging period? **(3)**

(b) The battery is unusable if its capacity falls below $12 \cdot 51$ Ah.
By considering the limit of a suitable sequence, make a comment on the durability and lifespan of the battery. **(3)**

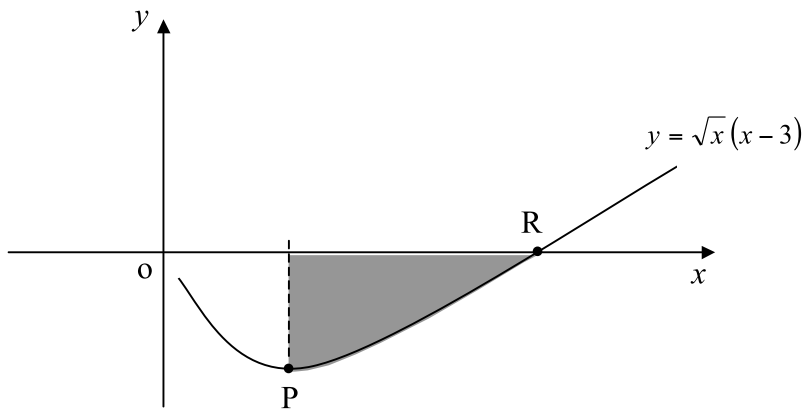
6. A function is defined as $f(x) = \sqrt{x}\left(x-3\right)$, where only the positive value of $\sqrt{x}$ is taken for each value of $x > 0$.

Part of the graph of $y = f(x)$ is shown below.



(a) Find the coordinates of the turning point at P and the root at R. **(6)**

(b) Hence calculate the shaded area below giving your answer **correct to 2 decimal places**.



(5)

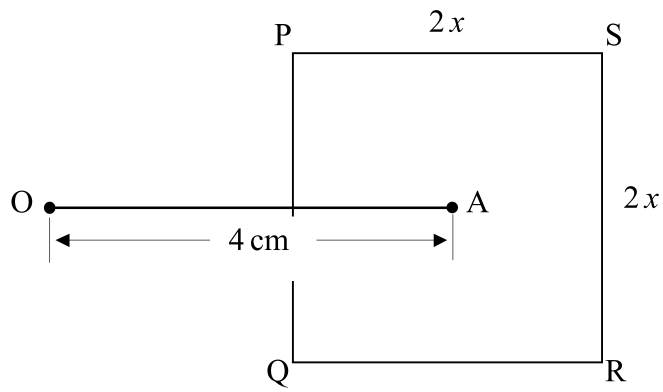
7. An equation is given as $\frac{3}{4}x^2 - (k+3)x - (k^2+2k) = 0$.

(a) Prove that this equation has **real roots** for <u>all</u> real values of k. **(4)**

(b) Hence write down the value of k which allows this equation to have equal roots and solve the equation for x when k takes this value. **(4)**

8. In the diagram below PQRS is a square of side $2x$ cm.

A straight line OA, measuring 4 cm, has been drawn in such a way that A lies at the centre of the square and OA is parallel to PS.



(a) Show that $OP^2 = 2x^2 - 8x + 16$. **(4)**

(b) Hence, by completing the square, or otherwise, find x for which the length of OP is at a minimum and state the minimum length of OP. **(4)**

[END OF QUESTION PAPER]